THE ROLE OF PHYSICS IN STUDENTS' CONCEPTUALIZATIONS OF CALCULUS CONCEPTS: IMPLICATIONS OF RESEARCH ON TEACHING PRACTICE

Karen MARRONGELLE
Portland State University
Department of Mathematical Sciences
P. O. Box 751
Portland, OR 97207-0751
e-mail: marrongelle@mth.pdx.edu

ABSTRACT

This paper discusses the implications of research on undergraduate calculus learning for calculus teaching practice. In particular, this paper addresses the challenges of using research results and adapting instructional materials in diverse classroom settings, and aligning research-based conceptions of teaching with practice. In a previous research study, the author investigated students' use of physics experiences and concepts as they construct calculus concepts in an interdisciplinary calculus and physics course. The results of this study suggest that students frequently draw upon physics experiences and concepts as they develop understandings of average rate of change, but that students less frequently make use of physics experiences as they develop understandings of derivative and antiderivative. In addition, other researchers have alluded to the importance of prior physics experiences on students' conceptualization of the average rate of change concept (e.g. Nemirovsky & Noble, 1997). However, implications of these research results for calculus teaching practice have received little attention. The combination of the author's research findings and those of other researchers suggest four major implications for calculus teaching practice. Research results: 1. Modify pre-existing instructional design theory. 2. Influence the design of classroom activities and development of learning sequences. In particular, research provides information about the potential mismatch between the experiences students bring with them to the classroom and teachers' assumptions about students' past experiences. 3. Influence how the teacher conceptualizes the role of the students in the classroom community. 4. Alter the role of technology in the classroom. The results of the author's previous research as well as other research results are discussed relative to these four outcomes for undergraduate calculus teaching practice.

1. Introduction

This paper discusses how the results of recent research on calculus learning inform new ways of teaching undergraduate calculus. The broad question that this paper discusses is: What do we know about the learning of calculus and how does that knowledge inform calculus teaching? Specifically, this paper presents a discussion of recent research on the role of physics understanding in the learning of calculus concepts and the implications of research for the calculus curriculum. This paper is divided into two sections and reflects the continuum of research informing practice and practice informing research. The first part of this paper describes research that grew out of the author's evaluation of an interdisciplinary calculus/physics program (practice informing research). The second part of this paper discusses the implications for practice of the results of the research described in the first part of the paper, along with other recent research in this area (research informing practice).

2. The role of Physics in Students' Conceptualizations of Calculus Concepts

Background and Purpose

Students' understanding of calculus concepts lays a foundation for their future study of advanced mathematics, science, and engineering courses. The idea of change – both how things change and the rate at which things change – plays a particularly important role in students' conceptualizations of calculus concepts. Students must understand the concept of rate of change in order to understand the derivative and differential equations. Furthermore, students must understand the idea of total change to understand the integral. Finally, students must understand the relationship between rate of change and total change in order to understand the relationship between derivatives and integrals outlined by the Fundamental Theorem of Calculus.

In order to grasp abstract ideas of rate of change, students might rely on physical interpretations of change (Nemirovsky, Tierney, & Ogonowski, 1992). Students may have encountered some of the underlying calculus concepts informally in everyday life; thus students often enter the calculus classroom with some intuition about concepts such as rate of change and derivative (Nemirovsky & Rubin, 1992; Nemirovsky & Noble, 1997). Furthermore, many students experience the mathematical concepts of average rate of change, derivative, and integral in physics classes as they study concepts such as motion, force, and electricity.

Physics, a typical introductory course for most engineering, science, and mathematics students, provides a context for which students can study change in a concrete setting. Research has shown that mathematics understanding enhances the learning of physics concepts (Hudson & McIntire, 1977; Champagne, Klopfer et al., 1980), and more recent research has begun to examine how physics understanding affects the learning of calculus concepts (Thompson, 1994; Marrongelle, 2001).

During the late 1990's the National Science Foundation of the United States of America funded several initiatives aimed at exploiting connections between mathematics and other disciplines at the undergraduate level. One such project, which took place at a large, public research university in the Northeast, integrated the curriculums of differential and integral calculus and introductory calculus-based physics (NSF-DUE 9752650). The integrated Calculus/Physics program was offered to first-year students as an alternative to enrolling in separate calculus and physics classes. The program was developed during the spring and summer of 1998 and first offered to students during the fall of 1998. The Calculus/Physics program development was informed by recent research in the areas of calculus and physics learning (c.f. McDermott, 1984; Ferrini-Mundy & Graham, 1994), ideas from the work in Cognitively Guided Instruction (Greeno, 1997), and research in the area of problem-solving (Larkin, 1980; Schoenfeld, 1985; Arcavi, Kessel, Meira, & Smith, 1998).

The ordering of the calculus and physics topics contributes greatly to the integrated nature of the curriculum. The curriculum is designed for the students to see the applicability of the calculus as they learn it and conversely that they have all the mathematics they need to solve physics problems. In order to coordinate the calculus and physics topics in the class, the presentation of calculus topics was reordered. The four basic threads of calculus (function, continuity, derivative, and integral) are discussed first for polynomial functions only and then again for other classes of functions (logarithmic, exponential, trigonometric) as they arise in the physics curriculum. This reordering of the calculus curriculum allows for the presentation of the physics and calculus content in a more unified way and gives the mathematics a rich context.

The author began examining students' learning in this context in her role as an evaluator of the calculus/physics program. As part of the evaluation of the Calculus/Physics program, the author conducted clinical interviews with students enrolled in the Calculus/Physics class and student enrolled in a traditional[1] calculus course. An analysis of the clinical interview data uncovered differences between the manner in which Calculus/Physics students and traditional calculus students approached average rate of change and derivative tasks. The Calculus/Physics students tended to use physics terminology and concepts as they solved average rate of change and derivative tasks. The traditional calculus students, who were either concurrently enrolled in a physics class or had completed a physics class, tended to rely on their memorization of mathematical formulas and processes as they solved average rate of change and derivative tasks. The Calculus/Physics students seemed to make more connections to their knowledge of physics as they solved the average rate of change and derivative tasks than the traditional students.

Research Question and Theoretical Perspective

As a result of her work evaluating the Calculus/Physics program, the author designed a research study to investigate the question: How do students draw upon physics concepts to inform their understanding of rate of change, derivative, and integral? The study was guided by the basic constructivist view that knowledge is constructed through a process of experience and reflective abstraction (Noddings, 1990). The consequence of holding a constructivist perspective is the assumption that mathematics is built from human activity; thus students informally experience mathematical ideas in their day-to-day, culturally situated experiences.

The theory of *transitional tools* was employed as a means to analyze and discuss role of physics in the students' constructive activity Nemirovsky and Noble (1997) put forth the notion of

[1] 'Traditional' is used here to describe those students who were not enrolled in the integrated Calculus/Physics program at the university.

transitional tools as part of their emerging psychological perspective that allows for the analysis of an individual's constructive activity by challenging the convention that any given object or picture must reside either inside or outside a person's mind. By rejecting the notion that a visualization must be either internal or external, Nemirovsky and Noble (1997) overcome the common difficulty that arises from the need to describe whether a visual image is internal or external to the student. Transitional tools are experiences or objects in the environment that both separate the learner from another physical object and strengthen his/her understanding of the object using mathematical contexts such as symbols and graphs. For example, a student who talks about the motion of a cart on a track to help him/her conceptualize properties of the derivative is using the cart and track as transitional tools. Note that the cart and track are tools that reside both internally (in the student's memory of the cart's motion on the track) and externally (the physical existence of the cart and track).

Recent Research in Calculus Learning

Much of the research on calculus learning has shown that students are able to successfully carry out methods of differentiation and integration but sometimes lack the conceptual underpinnings necessary to explain procedures, work though problems using multiple strategies, and make connections between concepts (Orton, 1983; Vinner, 1989; Ferrini-Mundy & Graham, 1994). Throughout the literature, researchers have alluded to the importance of prior experiences on students' conceptualizations of calculus concepts (Thompson, 1994; Nemirovsky & Noble, 1997; Noble, Nemirovsky, Tierney, & Wright, 1998). These experiences refer to both mathematical and non-mathematical episodes and situations encountered both in and out of the classroom. Additionally, some researchers have stressed the need for investigations into the effects of introducing substantial physical examples and applications in the calculus course (Ferrini-Mundy & Graham, 1991).

More recently, Ricardo Nemirovsky has undertaken a number of projects aimed at investigating the effects of physical graphing devices on students' calculus learning. Nemirovsky, Tierney, & Wright (1998) found that students will broaden their use of motion graphing devices as they become more familiar with the graphing devices. Steve Monk, Ricardo Nemirovsky, and Paul Wagoner have developed a set of computer controlled interactive physical devices. The devices enable students to explore calculus concepts in a physical context. Underscoring these projects is the assumption that students' physics experiences and knowledge will shape how and what they learn.

Methodology

Over a two-semester period, eight first-year students enrolled in the Calculus/Physics program participated in this study. Students were invited to participate in this study based on their reported backgrounds in mathematics and physics. The goal of the participant selection was to generate a sample of students whose range in abilities spanned the abilities represented in the Calculus/Physics class. Seven males and one female participated in the study. The gender balance in the study reflected the gender balance in the Calculus/Physics class.

This study utilized a multiple case study design with analysis by and across cases. Data sources included: four audio taped, task-based interviews with each student; classroom participant-observation; and photocopies of students' class notes, in-class activities, homework assignments, and examinations. The interview tasks were designed to help reveal students' ways of thinking about average rate of change, derivative, and integral. All of the interviews were transcribed and all data was coded into categories using micro-analytic and constant comparison methods (Strauss & Corbin, 1998).

Transcripts of the students' initial interviews were selected as the primary data source for the microanalysis since these were the earliest pieces of data collected. It was necessary to conduct a microanalysis on early pieces of data in order to generate a scheme by which to classify the ways in which the students used physics as they solved calculus problems. The classification scheme was further tested and refined with data collected later in the year. The classification scheme will be discussed in more detail in the following section.

A within-case analysis of each student was conducted in order to test the stability of the classification scheme that emerged during the microanalysis. The classification scheme (See Table 1) was used to analyze transcript episodes, students' homework assignments, classroom activities, and examinations. Selected pieces of the data were re-coded by three independent raters to check for inter-rater reliability.

Results

The results of this study indicate that when students used physics as transitional tools, they used physics in one of four (not necessarily disjoint) ways: Contextualizers, Example-Users, Mis-Users, and Language-Mixer (see Table 1).

Physics Use	**Description**
Contextualizer	Student works and talks through calculus problems as if it were a physics problem. Majority of technical vocabulary used to solve problem is physics terminology. There is evidence that student is thinking about the problem in terms of physics.
Example-User	Student uses physics examples to justify solutions to problems or to help make sense of part of the problem. Actual problem at hand is solved using mathematical concepts. Student does not submerge the problem in a physics context. Majority of technical vocabulary is mathematical terminology.
Mis-User	Student's use of physics misconceptions interferes with student's solution to the problem. Student uses physics misconception to incorrectly solve the problem at hand.
Language-Mixer	Student intersperses physics and calculus terminology as he/she solves problem. Student does not immerse problem in physics context or use physics examples to justify solutions or help make sense of problem. Rather, student intermingles physics and mathematical language as he/she solves the problem.
Non-User	Student does not use physics concepts to language to solve calculus problems.

Table 1: Physics Use Classification Scheme

Contextualizers show evidence of immersing calculus problems in a physical context in order to solve them. Example-Users use physics examples to justify solutions to calculus problems or help make sense of part of a problem, but do not show evidence of immersing the problem in a physical context. Mis-Users allow physics misconceptions to interfere with the solution to calculus problems. Language-Mixers intersperse mathematics and physics terminology as they solve calculus problems but show no evidence of using physics other than a communication tool. Another interesting finding is that students frequently used physics concepts as transitional tools to construct meaningful conceptualizations of average rate of change but less frequently drew upon physics concepts as transitional tools to aid in their understanding of derivative and integral.

A discussion of the implications of these research findings for teaching and curriculum development will be presented in the next section. The results of the research study described above, as well as the results of previously reported research have been synthesized in order to identify areas of possible modification in undergraduate teaching practice and curriculum development.

3. Implications for Practice

A number of observations arise here that suggest implications for calculus teaching practice and curriculum development.

1. Instructional design theory should reflect the notion that students build their mathematical understanding from human activity. Students often encounter calculus concepts through participation in physical situations in their environments. The 'human' aspect of calculus concepts is often ignored in the design of calculus course. Instructional design theory needs to incorporate students' prior experiences with calculus concepts.

For example, the instructional design theory of Realistic Mathematics Education is rooted in Freudenthal's interpretation of mathematics as a "human activity" (Freudenthal, 1991). From this perspective, students should learn mathematics by mathematizing subject matter from realistic situations (i.e. from context problems or from mathematically real objects for students) and by mathematizing their own mathematical activity. As a global, guiding theory, Realistic Mathematics Education provides a framework for considering the use of context problems and the role of mathematization in the learning process.

2. Curriculum changes should be made to address the close connection between calculus and physics as well as students' reliance on physics to help them make sense of calculus concepts. While integrating calculus and physics in a single course is not always possible or appropriate, the physical context of calculus cannot be ignored. The calculus curriculum should reflect that students' experiences with physics are valued and an important part of the learning process. Currently, many calculus textbooks continue to use physical examples only as 'applications' of calculus or as a follow-up to discussions about calculus concepts. Physics concepts and examples should be used to *initiate* discussions about calculus concepts, especially in calculus classes designed for science and engineering students. Drawing on students' previous experiences with physics will help students create more meaningful conceptualizations of calculus concepts. This suggestion should not lead to the exclusion of other calculus applications from the curriculum, such as biology, business, and economics. Because the results of the current research project were based on data collected from students with a predisposition to physics, further research is necessary to determine if the results of the present study are generalizable to a larger student population. Specifically, future research should investigate the effects of a physics-based approach to calculus on students majoring in such areas as business, economics, and biology.

3. Students' previous experiences and knowledge should actively shape the classroom learning environment. The research on calculus learning underscores the idea that "calculus students will actively formulate their own theories, build their own connections, and readily construct meaning for problem situations" (Ferrini-Mundy & Graham 1994, p. 43). Thus, the calculus curriculum should be informed by the experiences and knowledge that students bring with them to the classroom. In particular, students must be afforded opportunities to link their past experiences with physical phenomena to calculus concepts. Students should be given opportunities to share their experiences with and knowledge about calculus concepts, validating the student's role as a learner. Additionally, as students discuss their experiences and knowledge, they will begin to consider calculus concepts from multiple perspectives.

4. The role of technology in the classroom should be under continual modification. Students in the present study used motion detectors, graphical interfaces, and graphical software to create, analyze, and explore properties of derivative and anti-derivative functions. The use of such technological learning tools, while standard in many physics laboratories, has only recently been explored in mathematics classrooms (Nemirovsky, Tierney, & Wright, 1998; Huetinck, 1992). Motion detectors allow students to visually, audibly, and kinesthetically engage with calculus concepts. However, using motion detectors and related hardware and software requires additional funds and space for students to move about the classroom. Not every school or classroom can accommodate technology such as motion detectors. New technological advances as well as web-based programs are allowing for easier interactions with powerful teaching tools similar to the motion detector. As new technological tools are made available, educators need to consider their appropriate use in the classroom.

4. Summary and Conclusions

The combination of the author's research findings and those of other researchers suggest four major implications for calculus teaching practice. Research results suggest: 1. The modification of pre-existing instructional design theory. 2. Influence the design of classroom activities and development of learning sequences. In particular, research provides information about the potential mismatch between the experiences students bring with them to the classroom and teachers' assumptions about students' past experiences. 3. Influence how the teacher conceptualizes the role of the students in the classroom community. 4. Alter the role of technology in the classroom.

REFERENCES

Arcavi, A., Kessel, C., Meira, L., & Smith, J. (1998). Teaching mathematical problem solving: An analysis of an emergent classroom community. In Schoenfeld, A., Kaput, J., & Dubinsky, E. (Eds.), *CBMS Issues in Mathematics Education, Vol. 7: Research in Collegiate Mathematics Education III* (pp. 1-70). Providence, RI: American Mathematical Society.

Champagne, A. B., Klopfer, L. E., & Anderson, J. H. (1980). Factors influencing the learning of classical mechanics. *American Journal of Physics*, **48**, 1074-1079.

Ferrini-Mundy, J. & Graham, K. (1991). An overview of the calculus curriculum reform effort: Issues for learning, teaching, and curriculum development. *American Mathematical Monthly*, **98**, 627-635.

Ferrini-Mundy, J. & Graham, K. (1994). Research in calculus learning: Understanding of limits, derivatives, and integrals. In Kaput, J. & Dubinsky, E. (Eds.) *Research Issues in Undergraduate Mathematics Learning* (MAA Notes Number 33, pp. 31-45). Washington, DC: Mathematical Association of America.

Freudenthal, H. (1991). *Revisiting mathematics education*. Dordrecht: Kluwer Academic Publishers.

Greeno, J. and the Middle-school Mathematics through Applications Project Group (1997). Participation as fundamental in learning mathematics. In Dossey, J., Swafford, J. Parmantie, M., & Dossey, A. (Eds.), *Proceedings of the nineteenth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 1-14). Columbus OH: ERIC Clearinghouse for Science, Mathematics, and Environmental Education.

Hudson, H. T. & McIntire, W. R. (1977). Correlation between mathematical skills and success in physics. *American Journal of Physics*, **45**, 4-27.

Huetinck, L. (1992). Laboratory connections: Understanding graphing through microcomputer-based laboratories**.** *Journal of Computers in Mathematics and Science Teaching*, **11**, 95-100.

Larkin, J. (1980). Skilled problem solving in physics: A hierarchical planning model. *Journal of Structural Learning*, **6**, 271-297.

Marrongelle, K. (2001). *Physics experiences and calculus: How students use physics to construct meaningful conceptualizations of calculus concepts in an interdisciplinary calculus/physics course*. Unpublished doctoral disseratation, University of New Hampshire, Durham.

McDermott, L.C. (1984). Research on conceptual understanding in mechanics. *Physics Today*, 24-32.

Nemirovsky, R. & Noble, T. (1997). Visualization and the place where we live. *Educational Studies in Mathematics*, **33**, 99-131.

Nemirovsky, R. & Rubin, A. (1992). *Students tendency to assume resemblances between a function and its derivative*. Unpublished manuscript, Cambridge, MA.

Nemivovsky, R., Tierney, C., & Ogonowski, M. (1992). *Children, additive change, and calculus*. Unpublished manuscript, Cambridge, MA.

Nemirovsky, R., Tiemey, C., & Wright, T. (1998). Body motion and graphing. *Cognition and Instruction,* **16**, 199-172.

Noddings, N. (1990). Constructivism in mathematics education. In Davis, R. B., Maher, C. A., and Noddings, N. (Eds.), *Constructivist Views on the Teaching and Learning of Mathematics* (Journal for Research in Mathematics Education Monograph 4, pp. 7-18). Reston, VA: National Council of Teachers of Mathematics.

Orton, A. (1983). Students understanding of differentiation. *Educational Studies in Mathematics*, **14**, 235-250.

Schoenfeld, A. (1985). *Mathematical problem solving*. Orlando, FL: Academic Press, Inc.

Strauss, A., & Corbin, J. (1998). *Basics of qualitative research: Techniques and procedures for developing grounded theory* (Second ed.). Thousand Oaks, CA: Sage Publications, Inc.

Thompson, P. (1994). The development of the concept of speed and its relationship to concepts of rate. In Harel, G, & Confrey, J. (Eds.), *The development of multiplicative reasoning in the learning of mathematics* (pp. 181-234). Albany, NY: State University of New York Press.

Vinner, S. (1989). The avoidance of visual considerations in calculus students. *Focus on Learning Problems in Mathematics*, **11**, 149-156.