PROSPECTIVE PRIMARY TEACHERS' EXPERIENCES AS LEARNERS, DESIGNERS AND USERS OF OPEN MATHEMATICAL TASKS

Vassiliki SPILIOTOPOULOU
School of Technical Education of Patras (PATES),
Technical and Vocational Teacher Training Institute (SELETE),
Anthemiou 2 & Palama, 26442, Patras, Greece,
spiliot@otenet.gr

Despina POTARI
University of Patras, Department of Primary Education, 261 10 Patras, Greece,
potari@upatras.gr

ABSTRACT

This paper explores prospective primary teachers' views of open tasks. The study has been realized in the framework of developing student teachers' awareness of mathematics teaching and learning through a number of activities that aimed to relate theoretical perspectives to the mathematics teaching practice. Data was collected from students' portfolios, and those parts that refer to open tasks have been analyzed. In particular, students' ways of analyzing two different kinds of open problems, their approaches in designing and their experience from using open tasks in the classroom have been explored and aspects of their views have been identified. Overall, the study contributes to our understanding of the development of students' awareness concerning open tasks as developed through their involvement in different kinds of experiences.

Introduction

Teachers' education today is not a process of developing skills that the teachers can implement in their actual teaching, whereas most pre-service programs do encourage student teachers to get involved in the process of inquiry as learners but also as teachers during their teaching practice. The way that these two different contexts coexist with teachers' development has been discussed in a number of studies (Ebby, 2000; Georgiadou and Potari, 1999). This perspective is also shared in this study and more specifically the focus is on the development of prospective teachers' awareness of mathematics teaching (Mason, 1998). In this way, it is possible not only to see how teaching is itself a path of personal development, but also to discern the different cultural and cognitive phenomena constituting the teaching act. Mathematics teaching aims to develop pupils' knowledge, strategies and thinking tools. This cannot be done if teaching is as "telling", a well-established belief that student teachers carry with them from their school experience as pupils (McDiarmid, Ball and Anderson, 1989). Meaningful mathematics teaching means the use of a variety of teaching activities and thoughtful teaching interventions in contexts familiar to the children. One way of encouraging student teachers to develop such a view of teaching is through their involvement in open tasks as learners, as designers and as users. These three roles were embedded in all the activities of the initial training course for primary teachers discussed in this paper. This approach was in all the three parts of the course, the set of lectures, the lab-work and the teaching practice in classroom.

Within this context, student teachers' views of open tasks are studied. A debate on the meaning of open tasks is still going on (Ellerton & Clarkson, 1996; Silver, 1995). Moreover, pupils' and teachers' conceptions have recently attracted research interest (Pehkonen, 1995; Pehkonen, 1999). Nevertheless, little research has taken place in that area. In this study, we aimed to develop student teachers' meanings of open tasks through their involvement in solving, planning, using and reflecting on open situations. Through a variety of questions and activities, students experienced different aspects of such situations and we expected to encourage the development of their awareness. Their responses, their plans and their actions as described by them, were included in student teachers' portfolios and analysed.

Methodology

Student teachers' portfolios include extended data collected over a semester. Each week they had a lecture on issues of mathematics education, while every second week there was a lab meeting where the students themselves explored more concrete expressions of these issues and planned the activities for the school practice. Each lab was followed by a week of practice in school where the students implemented their plans. One hundred and twenty five student teachers participated in the course activities.

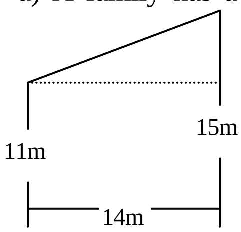
Description of the activities

Student teachers' portfolios consisted of worksheets that supported the students' involvement in different types of activities, their reflections on experiences from planning learning activities and from their classroom implementation. The worksheets, through a number of questions, asked students to focus on specific aspects of learning and teaching mathematics. A number of these worksheets referred to the meaning and use of open tasks. A sequence of questions had been developed concerning: the solution to an open problem; the identification of its characteristics; the comparison of it with another open task; the planning of their own open task (without the mathematical content being specified) to be used in the classroom; the use of this open task in the classroom; and the planning of an open task on a given mathematical content (this is not included in the analysis). This series of activities can be separated into three phases. In the first, student teachers are involved in the process of facing open problems and can be considered as *learners*. In the second, they use their previous experience in planning an open task for teaching and can be considered as *designers*. In the third, they implement the designed open task in the classroom and they evaluate this experience, so they can be seen as *users*.

The first phase

The questions on the worksheet concerning this phase are the following:

a) A family has a garden the shape of which resembles the one that is shown in the figure below. The family would like to plant tomatoes in it. Could you help this family to find ways of planting if, as the grandfather said, each plant should have around it a space of 35 cm. Try to solve this problem and write down one solution in the given space.



b) What are the characteristics of this problem? Which are the mathematical concepts involved? What could working on such a problem offer to someone?

c) In what ways does the following problem differ from the one you have already considered?

"A class of pupils would like to organize an excursion to Zakynthos. They are trying to find financial support by selling a magazine, which is edited by the class. Could you think of actions that need to be undertaken for the organization of this excursion? What kind of reasoning should they develop? Can you suggest anything that may help them?"

The second phase

In this phase of planning, the students-teachers were faced with the following tasks in order to encourage them to consider certain issues of their teaching more specifically:

a) Next time you visit a school classroom you want to organize a teaching approach, which can be considered 'open'. It would be helpful if you think of a situation from everyday life or the cultural environment or from a subject other than mathematics. Think about and describe this situation. Give two arguments for your choice.

b) Plan and describe the classroom organization for your planned teaching. What will your own teaching actions be and what will your pupils' involvement be?

c) Think of ways to evaluate the whole teaching approach. Write them down.

The third phase

This worksheet focuses on students' reflections of using open tasks in the classroom. More specifically the questions are described below:

a) You have implemented an open situation in a school classroom. What were your feelings during that experience and how do you feel now?

b) Think of the nature of your experiences at two levels: the cognitive level and the level of classroom management, and describe them.

c) Do you think that the situation was successful? In what respect?

d) If you taught this again, what would you do differently?

Data analysis

Here, we analyze part of the data referring to open problems. More specifically, the focus is on students' conceptions about the character of an open problem. The data analysed in this paper is students' responses to questions (b) and (c) of the task given in the first phase, to question (a) from the second phase and to questions (a) and (b) from the third phase. Initially, we examined each question separately. We analysed the answer given in terms of units that expressed a certain view. We studied these units and looked for different dimensions that underlined these views. A kind of categorization emerged, based on the construction of systemic networks (Bliss, Monk & Ogborn, 1983) that are presented and discussed below.

Student teachers' views of open problems as learners

Student teachers' characterizations of the problems emerged explicitly in their descriptions of the characteristics of the problem that they faced in the first task as solvers, and implicitly through their writing about its importance or by comparing the two types of problems. The categorization of their conceptions about the character of an open problem is presented in the systemic network of figure 1.

The student teachers characterized the problem both in terms of the problem itself and of the solver. In some descriptions these two dimensions seemed to coexist: "It is an open problem, it has a lot of solutions and so it is difficult for the pupils to solve." The solver was considered in two ways, one referring to the actions, either mental or physical, that he/she had to undertake and the other referring to the implications of the process of solving an open problem for the solver as individual or as a member of a group.

The problem itself was evaluated as open in terms of the kind of data given: "it is open (the second problem) as it does not give numbers"; its complexity: "the basic characteristic is the complexity of the demands which need a particular way of thinking to be understood"; its phrasing: "The phrasing is interpreted by the reader in different ways"; its applicability: "It has applications in everyday life"; the number and the kind of solutions: "It has more than one solution", "There is not a fixed way for solving it"; the degree of openness: "The second problem is more open than the first because it can be further extended according to the data that we give each time".

Concerning the solver's actions the student teachers often referred to the need to find extra information: "The children have to organize (in the 2nd problem) the procedure needed. For example, how many children are in the classroom? How much does the ticket cost? How much does the magazine cost?". They also appreciated the role of open problems in the development of thinking: "it needs critical thinking". Finally, the use of everyday or mathematical knowledge and more often the combination of the two were also included in their arguments: "It leads someone to thinking both on mathematical and practical levels"

The implications for the solver were identified mainly from student teachers' responses about the importance of the first given problem. These referred to the development of thinking, to the understanding of mathematical concepts, to the development of children's abilities to find relationships between mathematical concepts, to solve problems, to apply mathematics and finally to develop an awareness of what a problem is. Some characteristic responses are the following:

> *"This problem offers the deepening of mathematical thinking, the recalling and the use of multiple mathematical concepts and the relation between them."*

> *"He will understand better the mathematical concepts, clarify their operations and this will result in his handling them in the best way."*

> *"Someone realizes that these kinds of problems have a lot of solutions according to each one's thinking."*

> *"He will practice his thinking, he will learn to think in a different way, to find multiple solutions."*

Moreover, they acknowledged the importance of dealing with open problems through teamwork as if encourages cooperation and communication.

Student teachers' views of open problems as designers

In the second phase we analysed the problems that the student teachers designed and the classification that emerged appears in the systemic network of figure 2. Two main dimensions have been identified: the kind of problem and the arguments for its choice. The first is further analysed in terms of the openness of the problem, the mathematical content used and the context in which the problem was situated. Regarding the openness, some student teachers designed a closed problem, which they considered "open" because it involved more than one arithmetic operation or had a reference to everyday life. An example of such a problem is the following:

> *"A poultry-farmer has 15 hens. Each hen lays 4 eggs per week. Each egg costs 80 drachmas. How much does he earn every month, every semester, every year? If he spends 2000 drachmas per month to feed a hen, what will his net income be?"*

Another group of students conceived an open problem as a teaching situation where the children had to use physical materials to understand certain mathematical concepts. An example of this was a situation where the children were asked to share a chocolate bar among a number of other children in order to develop the meaning of fractional units. In a similar situation, the children were asked to make an open exploration through drawings in order to discover properties of triangles.

A large number of problems that the student teachers designed could be characterized as open. These have been classified either in terms of the structure of the problem or of the process of solving it. So, three groups of problems have been constructed: problems without numerical data, logical problems where the development of strategies was emphasized and problems including both closed and open questions. We give below some examples:

> *"We know that in a weekday a pupil goes to school, attends English classes, studies, eats, sleeps etc. We want to calculate whether the hours that he plays daily (during the weekdays) are more or less than the hours that he plays during the weekend when he does not go to school."* (non-numerical data given)

> *"At the toll stations of Patras, the buses and the lorries pay 1000 drachmas, the cars 600 drachmas and the motorbikes 400 drachmas. From 8.00 am to 12 am yesterday, the cash*

taken was 200.000 drachmas. How many buses, lorries, motorbikes and cars could have passed?" (logical-combinatory)

In terms of the solution process, the problems could have a number of solutions, as in the problem of the toll station described above, a specific method of solution, as in the problem with the hours of play. There was also a group of problems, which had a specific solution but could be solved in different ways.

Most problems were arithmetical involving the use of the four operations while very few were geometrical. In terms of the context only a few problems were purely mathematical. Some of the problems like the one regarding the toll station referred to a realistic situation. The school type context included situations, which were 'dressed' in context:

"We have a basket with one orange that has on it the number 2, two apples that each have the number 4, three pears that each have the number 6, and four strawberries that each have the number 8. If we have a basket and we want to fill it with fruits that give us the following sums 10,12,6,8, which and how many fruits we will choose"?

Student teachers' arguments for choosing the problems

Arguments can be categorized in terms of their reference to the problem itself, in terms of the implications for the children and in terms of the broadening of teacher's knowledge. So, student teachers argued for their choice of open problem referring to its context or to its other characteristics. The familiarity of the context to the children, its relation to everyday life and to current social issues were arguments concerning context given by the student teachers. Some of the other characteristics mentioned were problem's "exploratory nature" and the possible extensions that the problem offered.

In terms of the implications for the children regarding their involvement in solving open problems, reasons mentioned were children's positive feelings, the development of their thinking and the appropriateness of the problem to children's cognitive needs. An example of the first category is: " the children feel happy as each solution they give can be acceptable", while for the appropriateness of the problem to children's cognitive needs arguments were like: "it is relevant to children's prior knowledge", "it is appropriate to the class age". Most reasons referred to the development of mathematical thinking skills: "the children are practising their thinking skills", "they relate different parameters", "they recognize the appropriate mathematical concept", "they are practising existing knowledge". Arguments referring to the development of other types of thinking, like "it can develop awareness of the nature of a problem" and "it can broaden children's conceptions about mathematics" were also given.

Finally, the broadening of a teacher's knowledge was also given as an argument. Some examples were, "it gives us evidence about children's thinking", "it evaluates children's prior mathematical knowledge".

Student teachers' reflections after applying open problems in the classroom

Student teachers' reflections as expressed in the third phase have been analysed and the identified aspects are presented in the systemic network of Figure 3.

The experience of using their own open problem in a real classroom created to most teachers positive feelings like, in their own words, satisfaction, certainty, and new challenge. Some of them, however, had negative feelings after the completion of this task and expressed anxiety, fear, dissatisfaction and uncertainty. There were also some students who showed mixed feelings. A few of them stated neutral feelings. A characteristic example follows: "My feelings were neither pleasant, nor unpleasant, neither during that experience, nor now. It was a normal situation. It was something different, but nothing so sensational."

Reasons offered for their feelings were related to the appropriateness of the problem, to the kind of previous experience, to the success of the whole process and to students' attitudes. The following extract is an example of the last case:

> *"During my experience in the classroom I felt both nice and uneasy. This happened because of the children, who at the beginning were quiet, cooperative and eager to solve the problem, while at the end they shouted, wandered all over the classroom, which means that the situation couldn't be controlled. I also have mixed feelings now, good feelings because the children solved the problem fast and in many ways, and bad because at the end we had a problem in managing the classroom."*

In the descriptions of their experiences, two dimensions have been identified: implications for the teachers and implications for the students. The first dimension implies a development in teacher students' awareness concerning their children and also concerning themselves. So, an open problem can offer to the student teachers opportunities to obtain knowledge of their pupils' ways of thinking, cognitive problems and deficiencies. It also gives them opportunities to refresh previous knowledge in the classroom. The use of open problems can also reveal capabilities that low and high achievers have or do not have in problem solving. Such experiences are described in the following example:

> *"There were some students who answered all the questions with ease, while others met difficulties. However, even the second group, with our help and guidance managed to give the correct answer in the end".*

Student teachers, in terms of themselves, think that they had a new experience, broadened the meaning of the open problem, developed awareness concerning the fulfillment of their teaching goals, or of the goals of the course. They also felt more capable in differentiating the initial problem in order to become appropriate for classroom use and more mature in managing the classroom organization more effectively. The last ideas are expressed in the following extract: " We should take care to create a more open problem… and also I think we should be less directive in handling the classroom"(The two problems they used were closed ones, referring to children's everyday life).

The second dimension concerns the benefits of the open problem for the students. Student teachers think that the use of the open problem helped children to cooperate, encouraged classroom dialogues and supported the spirit of teamwork. They also noticed that such situations encouraged the children to express their ideas and develop their thinking, offered all children opportunities to act mathematically, aroused their interest and finally supported the development of children's problem solving skills. A characteristic example is the following: "I believe that the situation we have chosen aroused children's interest. All the children have tried to give an answer, while some have managed to find more than one solution".

Concluding Remarks

From the analysis above, a number of different aspects in student teachers' thinking of open tasks have emerged. Most of the categories of the networks seem to exemplify the main categories that Pehkonen (1999) formed by analyzing teachers' responses to the question "What are open tasks in mathematics?". In our study, student teachers in expressing their ideas about what an open problem is, they seemed to appreciate not only features of the structure of the problem itself but also how these features are related to the solver. However, the majority of the student teachers concentrated on the phenomenological characteristics of the problem and neglected deeper aspects of the problem such as the thinking processes required for solving the task. In the construction phase almost all the student teachers argued for their proposed open tasks but they expressed a surface understanding of the role of these tasks in learning and teaching mathematics. Another issue that emerged was the student teachers' difficulties in designing an open problem. A number of student teachers proposed closed problems as open. Moreover, some of the open tasks that produced had a similar content or structure to those that they had experienced in the first phase. It seems that their experience with the open tasks in the course was not adequate to help them design a form of open problems that would meet their expectations. The phase of the implementation of the designed open tasks in the classroom and the student teachers' reflections revealed the difficulties that were met in a real situation. These were due to student teachers' lack of teaching experience and in particular of the use of open tasks. However, the student teachers seemed to have developed a degree of awareness of the meaning of open tasks and of their function in the classroom. In particular, they realized that with such teaching situations they had more evidence about pupil's learning, they recognized the limitations of the open tasks designed and used and they became aware of the differentiation of classroom management needed while using open tasks.

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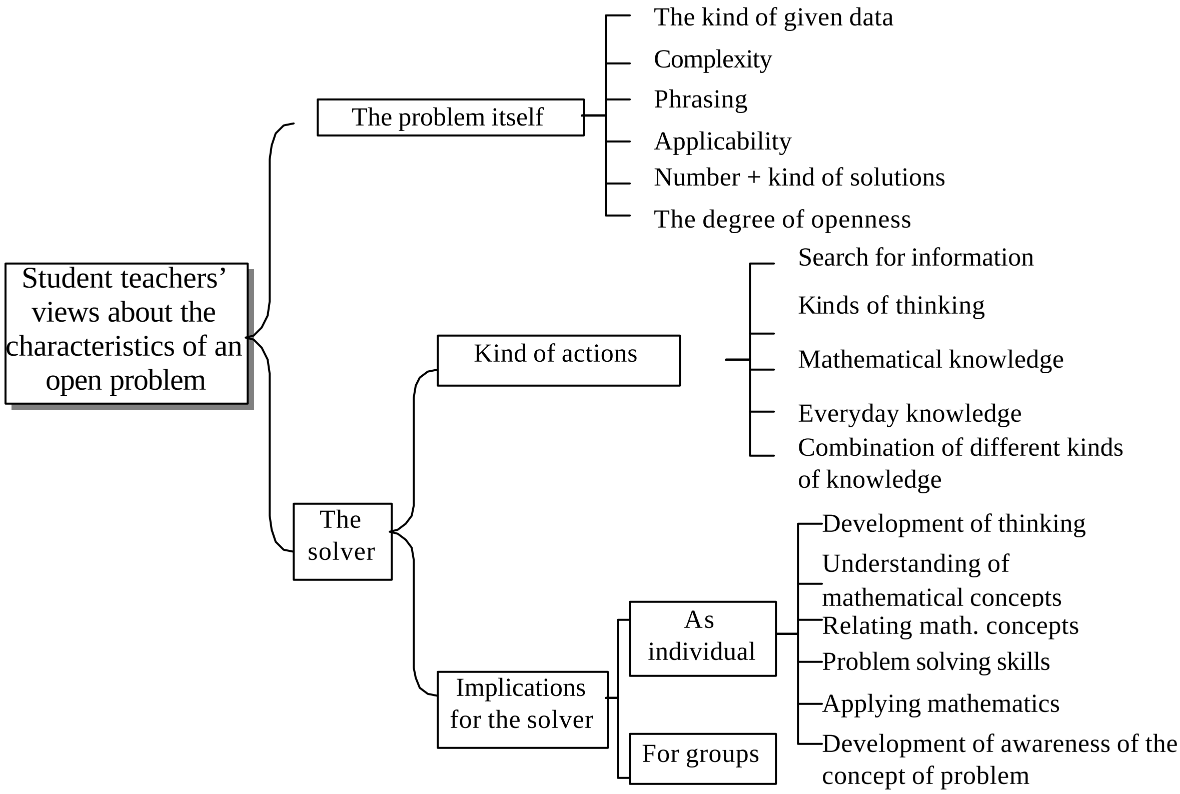

Figure 1. *Systemic network representing student teachers' views of open problems as learners*

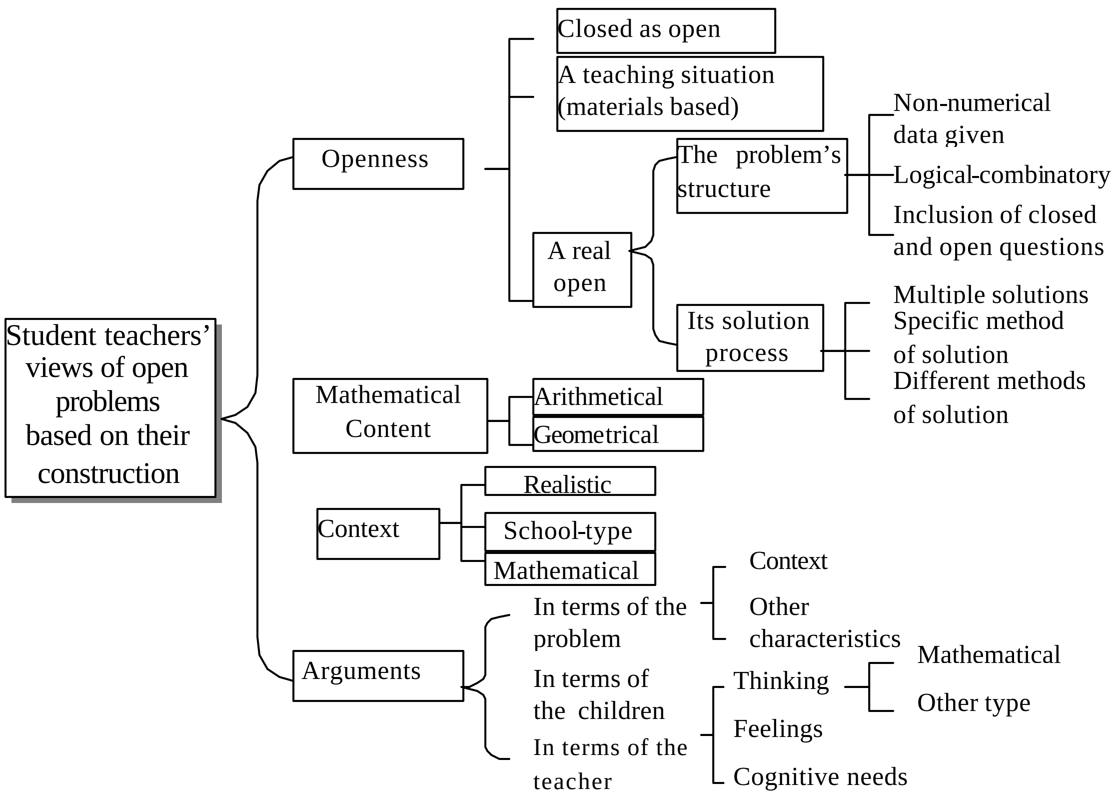

Figure 2. *Systemic network representing student teachers' views of open problems as designers*

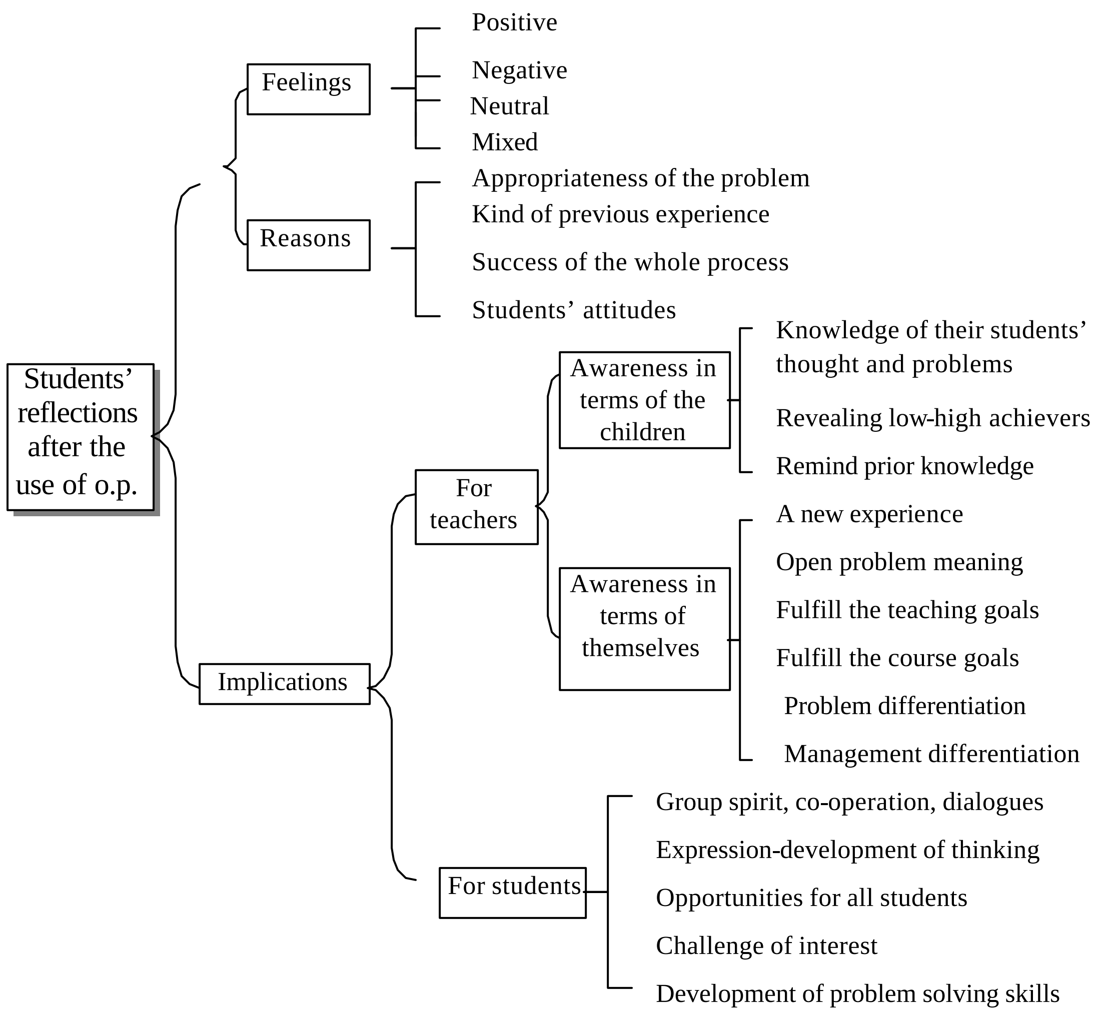

Figure 3 *Systemic network with categories emerging from student teachers' reflections after applying open problems in the classroom*