DEVELOPMENT OF CALCULUS CONCEPTS THROUGH A COMPUTER BASED LEARNING ENVIRONMENT

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ABSTRACT

This paper investigates the students learning of calculus, particularly the use of the definition of derivative, in undergraduate calculus course in a computer based learning environment in which Interactive Set Language (ISETL) and Derive were used. ISETL was used to help students to construct mathematical concepts on a computer, followed by the discussion held in the classroom. Derive was used to do the manipulations and to draw graphs. The study was carried out with 59 first year undergraduate mathematics and mathematics education students. An essay type test measuring students' understanding of limit and derivative was developed and administered as a pre-test and post-test. Follow-up interviews were conducted with 11 randomly selected students. The analyses of written and verbal responses to the tasks given in the test revealed well increase in the development of derivative concept. The results also showed that computer, particularly ISETL, prevented students to acquire knowledge by rote learning.

Key words: Calculus, Computer, ISETL, Derivative, Errors

1. Introduction

This study is part of a comprehensive research concerning students' learning of calculus concepts. In this paper, descriptive and qualitative results concerning the effect of an instructional treatment, based on having students make various constructions on the computer using ISETL (Dautermann, 1992) and developing manipulative skills and visualization using DERIVE (1989), followed by classroom discussion of mathematics concepts corresponding to these computer tasks, on the learning of the use of the definition of derivative are reported. There was also a certain amount of paper - and – pencil work for the students to do, both in and out of class. The results of the statistical analysis are reported in detail and discussed more fully elsewhere (Ubuz & Kırkpınar, 2000).

Studies about derivative and ideas related to it (such as tangent lines) have emphasized students' misconceptions and common errors (Amit & Vinner, 1990; Artique, 1991; Orton, 1983; Ubuz, 1996, 2001). Ubuz (2001, p.129) reported that students' common misconceptions on derivative were as follows: " (a) derivative at a point gives the function of a derivative, (b) tangent equation is the derivative function, (c) derivative at a point is the tangent equation, and (d) derivative at a point is the value of the tangent equation at that point." Ubuz also stated that students seem to think different concepts as the same. The reasons appeared to be "(a) the lack of discrimination of concepts which occur in the same context or the confusion of a concept with another concept describing a different feature of the same situation, (b) the inappropriate extension of a specific case to a general case, and (c) the lack of understanding of graphical representation."(p.133). To improve students' conceptions of calculus, there have been studies (e.g. Breindenbach, Dubinsky, Hawks, & Nichols, 1992; Dubinsky & Schwingendorf, 1991; Dubinsky, 1997) concerning teaching and learning of mathematical concepts using ISETL since the development of the programming language SETL (Schwartz, Dewar, Dubinsky, & Schonberg, 1986). These studies have mainly focused on the constructions of mathematical knowledge in a theoretical perspective rather than students' misconceptions and common errors. A central idea of the constructivist theory is "that understandings are constructed by learners as they attempt to make sense of their experiences, each learner bringing to bear a web of prior understandings, unique with respect to content and organization" (Simon and Schifter, 1993, p.331). Within this theoretical perspective students' existing and acquired concept images were investigated.

The purpose of this study was to answer the following research questions: 1) Is there any improvement in learning calculus concepts through the computer-based learning environment?; 2) what particular errors or misconceptions are in evidence?; 3) what kind of patterns do errors and misconceptions form?; 4) are the patterns of the errors associated with the different tasks?; and 5) which of these endure over time?

2. Method

Subjects

The sample consists of 59 first year undergraduate students in four sections of Math 153 Calculus I course offered at Middle East Technical University. Students were pursuing a major either in mathematics or mathematics education. The sections were formed randomly and different teachers taught each section. Two of those teachers who taught section 1 and 2 were male and the rest were female.

Table 1 shows the numbers of students, who took the pre-test and the post-test on derivative. The students who took both the pre-test and the post-test were taken as the sample of the study.

Table 1: The sample of the study

Section	Pre-test	Post-test	Pre-test $\cap$ Post-test
1	21	25	17
2	15	17	13
3	26	18	15
4	26	15	14
Total	88	75	59

33 (%56) students of those 59 were majoring in mathematics and the rest 26 (%44) students in mathematics education. 57 (%97) of those students have not taken this Math 153 course before and 53 (%90) students have also not taken Math 100 course given prior to Math 153 course. Math 100 course is given to the students who are not able to do 35 mathematics questions out of 52 in the university entrance examination. In the sample, 20 (%34) students were female and 39 (%66) students were male.

Instrument

The test used for assessing students learning of derivative consisted of 6 questions, some of which having different tasks (altogether 32 tasks), on which students were to work individually to provide written responses. Demographic survey questions to gather personal information about each student were included at the beginning of the test. The test was given as a pre-test and post-test without prior warning. The pre-test was administered at the beginning of the semester and the post-test at the end of the semester. Each semester lasts 14 weeks. Each task in the questions were graded by one of the four categories: correct (3), partially correct (2), incorrect(1), and missing (0). The factor analysis carried out for the questions in the pre-test revealed that the test was two dimensional. The first factor was related to the graphical interpretation (GI) (questions 1, 2, 4, and 6) and the other was related to the use of the definition of derivative (DfD) (questions 3, and 5). As mentioned previously, the results related with the questions on the definition of derivative (see Appendix A) are the focus of this study.

Treatment

The study was conducted in a course (Math 153) designed to teach functions, limit, derivative of a function, graph sketching, problems of extrema, and basic theorems of differential calculus: intermediate, extreme, and mean value theorems. The instructional treatment consisted of mainly having students make various constructions on the computer using the programming language ISETL, followed by class discussion of concepts corresponding to these computer tasks. DERIVE was also used by the students for doing activities which are difficult to do by hand. For example, drawing the graph of ($\mathrm{Sin}\frac{1}{x}$). There were also exercises to be done with pencil and paper after the class. Handouts were given on how to use DERIVE and ISETL at the beginning of the course. The textbook used in the course was *Calculus, Concepts, and Computers* (Dubinsky, Schwingendorf, & Mathews, 1995). This course has been conducted for approximately last ten years as it is.

Classes met 6 class hours of a week for 50 minutes each. Two of these hours were at the computer laboratory. There were two 2-class hour sessions during the week and students had to attend only one of these sessions. Some weeks, classes met in the class instead of computer laboratory, and quiz was given each such week. In the lab, students worked individually, each with her or his own terminal. Assistants were available to answer questions, give help with syntax, and etc. There were three computer rooms available, each equipped with 20 computers.

The first week of the semester was used to form the groups of 4 students and to make the introduction for the course. Students who knew and agreed with each other, and had common free time included in the same group. Each week groups were required to complete one activity on the computer by submitting it on the disk, and to complete exercises done with pencil and paper. The group members sat together in the class, because often they had to answer the questions collectively. Every member of each group must be involved in these works as they were going to take their exams individually. Late submissions were not accepted since solutions to the assignments were discussed in class.

The main purpose of the lab sessions was to make sure that every student had at least attempted to perform certain computer tasks before coming to class. The idea was to present the students with the problems so that they could make useful mental constructions. Brief explanations of the activities together with their examples are given below:

I. Functions

1. Writing computer programs of the given different situations where the functions are given in the form of: piecewise , graph, (in)finite SMAP , table, tuple, and string. For example, see question 1 in the book called *Calculus, Concepts and Computers* (CCC) (Dubinsky et al., 1995, p.69). This question is an example of the type piecewisely defined function.
2. Interorizing the action by taking different values from the domain and evaluating them. This makes the students to think about what computer is doing when it makes those evaluations. For example, see the question 1 in the CCC.
3. Drawing the graph of given expressions to understand the function concept and to learn the graph reading.
4. Encapsulating the composition of functions by giving an ISETL code directly and then make students to give meaning to the code. For example, see question 3 in the CCC (p.80).

II. Limit

1. Understanding that the limit value exists regardless of the existence of the function value at that point. For example, question 2 in the CCC (p.132).
2. Interorizing the behaviour of a function near a specified point or at large values i.e. variable tends to infinity. For example, question 3 in the CCC (p.132).
3. Making the idea of the formal definition of the limit more concrete by writing a computer function for taking limit , right limit , left limit , limit at infinity and limit at minus infinity. For example, question 1 in the CCC (p.142).

III. Derivative

1. Encapsulating the concept of derivative by the help of writing a computer program using the concepts difference quotient and the limit. For example, question 1 in the CCC (p.191).
2. Determining the extreme values of a function by graph reading. For example, question 7 in the CCC (p.219).

In the course there were 2 midterms and one final exam. These were in the form of solving problems or proving with paper and pencil without calculator or computer. Exams also contained short questions to be solved using the computer language ISETL. Grading was as listed: Assignments (activities and exercises) 10 %, Class work (participation in class, quizzes, and attendance) 20 %, 2 midterm exams 50 %, Final exam 40 %.

3. Students' Procedures and Conceptions

The analysis of students' written and verbal responses revealed significant information regarding the nature and characteristics of students' understanding of derivative.

The distribution of the scores for the 5 tasks according to four-point scale is reported in Table 2. The scoring criteria for each task are given in Appendix B. As mentioned previously, each task in the questions was graded by one of the four categories: correct (3), partially correct (2), incorrect (1), and missing (0).

Table 2: The distribution of the number of students according to the scoring criteria

	Pre-test				**Post-test**			
Questions	**0**	**1**	**2**	**3**	**0**	**1**	**2**	**3**
3a	-	2(3)	-	57(97)	2(3)	3(5)	-	54(92)
3b	1(2)	7(12)	-	51(86)	2(3)	12(20)	-	45(76)
3c	9(15)	7(12)	1(2)	42(71)	3(5)	9(15)	1(2)	46(78)
5a	5(8)	7(12)	24(41)	23(39)	1(2)	5(9)	9(15)	44(75)
5b	6(10)	12(20)	19(32)	22(38)	2(3)	7(12)	8(14)	42(71)

Students' attempts to find the value of a function in test tasks 3(a) and 3(c), and the derivative of a function in test tasks 3(b), 5(a) and 5(b) resulted in a variety of erroneous procedures being used. Appendix C contains the erroneous procedures used and the number of students who applied these specific procedures for the five tasks. For the purpose of discussion, the procedures in the table in Appendix C are numbered. It is evident that most of the erroneous procedures results from inappropriate graphical and numerical association or inappropriate visualization.

Although the erroneous procedures occurred on the post-test was not due to the erroneous procedures on the pre-test, the reasons behind these procedures were more or less the same. Also the same students in the pre-test and the post-test did not make these procedures. Two of the 12 errors on task 3(b), three of the 9 errors on task 3(c), one of the 5 errors on task 5(a) and two of the 7 errors on task 5(b) in the post-test made by the same students. Three of the 3 errors on task 3(a), ten of the 12 errors on task 3(b), five of the 9 errors on task 3(c), three of the 5 errors on task

5(a) and three of the 7 errors on task 5(b) in the post-test made by the students who had given the correct answer in the pre-test. The rest of the errors resulted from the omission answers.

Following the post-testing the interviews on test questions 3 and 5 led to the disclosure of various aspects of students' conceptions regarding the use of derivative and the definition of derivative. During the interview sessions students were encouraged to give reasons for procedures they had applied and to define the definition of derivative. The interviewees gave broader array of appropriate associations when explaining the concept of derivative. There was a considerable range among students in their explanations of derivative. Here are some typical responses from the students to the question, "What is a derivative?":

> *The slope of a tangent line drawn to a curve at any point*.(Student S)
> *Geometrically, the slope of a tangent line drawn to a curve at any point…the change in y over the change in x. The quotient I found the slope of a secant line. When* Δx *approaches to zero the secant line approaches to tangent line. As a result I can find the slope of the tangent line*.(Student U)

Responses from the students in the interviews also showed that students were able to distinguish the difference between the 'derivative at a point' and 'derivative of a function'. Student S made the following remark with respect to his application of erroneous procedure 3c.4 in the post-test (see Appendix C): "*.. first I found the slope as 4/5 rather than 2/5. By mistake I had written 4/5 in the equation of tangent line…*". It is evident that this student's carelessness had come into play here. Student U found the correct answer for task 3(c) in the post-test but wrote that the formula used was the mean value theorem. During interview he expressed his opinion: "*..I think I should have used approximation. But I have done it incorrectly..*".

The interviews on test items 5(a) and 5(b) showed that even some students made some erroneous procedures in using quotient formula to find the derivative at a point of a piecewise function in the post-test they gave the correct explanations during the interview. Student E made the following remark with respect to her application of erroneous procedure 5a.2 in the post-test (see Appendix C): "*as x=3 is greater than –1, I should have used* $2x^3$*…. At for x=-1 I should have looked the right and left limit of the quotient formula and they should be equal to each other…the function must be continuous*." Students C, S and M who gave the correct answer by using quotient formula and student U found the correct answer by differentiating for task 5(a) in the post-test gave also the correct explanation in the interview as student E. Student M gave the incorrect answer, " $\lim_{x\to -1^+} 2x^3 = -2 \qquad \lim_{x\to -1^-} -x^2+4x+3=-2$ ", for task 5(b) in the post-test but responded correctly in the interview.

4. Conclusion

The general analysis of students' performance, which participated in the study, pointed to a growth of formation and development of derivative concept from the significative increase in the number of correct answers in Pre and Post tests.

The main conclusion supported by the analyses is that the learning process in the computer context with the ISETL becomes very efficient as students work on the computer prior to the class. Student M, for example, drew attention to the point that: " *my point of view has changed from the pre-test to the post-test. At the beginning I was doing without thinking. Now I feel that I am thinking or I force myself to think. While doing homework on the computer, I become obliged to think definitions in some degree*."

The students overwhelmingly reacted positively to the idea of using computers in a calculus class. A recognized drawback is that there is not enough time for both calculus and computers. In most cases, though, a compromise is thought possible. A significant number of students would like to expand the time spent on computers and their applications. It was observed that the use of computers served not only to facilitate and deepen the understanding of certain concepts but also produced changes in students' attitudes toward the subject.

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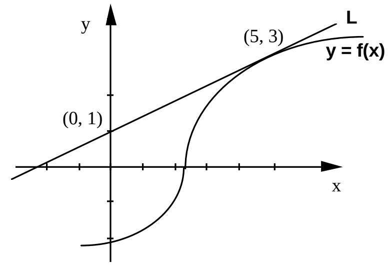
Appendix A

Test Questions

3. Line L is a tangent to the graph of $y = f(x)$ at the point (5, 3).
a) Find the value of *f(x)* at x = 5.
b) Find the derivative of *f(x)* at x = 5.
c) What is the value of the function *f(x)* at x=5.08?
(Be as accurate as possible)



5. Let *f* be a function given by f(x) = $\begin{cases} -x^2 + 4x + 3 & if\ x \leq -1 \\ 2x^3 & if\ x \rangle -1 \end{cases}$

Use the difference quotient to find the slope of the line tangent to the graph of f at

(a) x = 3 (b) x = -1

Appendix B

The Scoring Criteria for the Tasks together with the Examples from the Students' Answers

Questions		**SCORES**	
	3	**2**	**1**
	Totally correct answer	**Partially correct answer**	**Totally incorrect answer**
3			
(a)	Correct value of f(x) at x = 5 (e.g. "{3}")	N/A	(e.g. "{5}")
(b)	Correct value for the derivative of f(x) at x =5 (e.g. "{2/5}")	N/A	(e.g. "{0}")
(c)	Correct approximate value for f(x) at x = 5.08 using quotient formula (e.g. "{3.032}")	Estimated approximate value. (e.g. "It can be near to 3, but I can not say a number")	(e.g. " f(5.08) must be a bit smaller than 3.")
5			
(a)	Correct answer for the slope of the tangent line to the graph of f at x = 3 using the quotient formula (e.g. **"{54}")**	Finding the correct answer by using differentiation rather than the difference quotient formula	**(e.g. "{-2}")**
(b)	Correct answer for the slope of the tangent line to the graph of f at x= -1 using the quotient formula (e.g. **"{6}")**	Finding the correct solution without using the difference quotient formula	**(e.g. "{0}")**

Appendix C

Classification and Distribution of Errors for Each Task

Error	Illustrative Example of Students' Responses	Description	Pre	Post	Both
3b.1	f'(5)=3/5	Tangent line is taken as passing through zero	**0**	**1**	**0**
3b.2	F'(5)=2.5	The slope formula $y=(y_2-y_1)/(x_2-x1)$ is taken as $y=(x_2-x_1)/(y_2-y_1)$	**0**	**2**	**0**
3b.3	f'(5)= $\frac{3-0}{5-(-2)}=\frac{3}{7}$ "tanx = ½"	Assuming that the graph is passing through (-2, 0).	**4**	**6**	**1**
3b.2& 3b.3	f'(5)=7/3		**0**	**1**	**0**
3b.4	f'(5)=3	The value of the function at a point was taken as derivative at this point	**1**	**2**	**0**
3b.5	Unclassified		**2**		
3c.1	F(5.08) must be a bit smaller than 3	Not aware of that the function is increasing	**3**	**0**	**0**
3c.2	F(5.08)=5	The value of x is taken as the value of the function at x=5.08	**0**	**3**	**0**
3c.3	F(5.08)≅2/5	The value of the derivative at a point is taken as the value of the function	**3**	**1**	**0**
3c.4	L = 3/5(5.08)+1= 4.015	Not aware of that the value should be quite close to 3	**1**	**4**	**0**
3c.5	F(5.08)=5.16/5	Unclassified	**0**	**1**	**0**
5a.1	$\frac{f(3+h)-f(3)}{h}=\frac{f(4)-f(3)}{1}$ $\frac{2\times 64-2\times 27}{1}=74$	Using quotient formula but taking the big h value	**1**	**0**	**0**
5a.2	$\lim_{h\to 0}\frac{-(x+h)^2+4(x+h)+3-(-x^2+4x+3)}{h}$ $\lim_{h\to 0}-2x-h+4=-2x+4\Rightarrow m=-2$	Incorrect function	**0**	**4**	**0**
5a.3	X=2 f(x)=7 X=-4 f(x)=3 M=(7-3)/(2-4)=-2 Slope of the line is equal to f′(3)=-2	The slope of the derivative function is taken as the derivative at a point	**1**	**0**	**0**
5a.4	M=tanθ=y/x At x=3 ⇒ y = 18 , tanθ = 18/3=6	Assuming that the tangent line passing through (0,0)	**1**	**0**	**0**
5a.5	Dq= f(x+h)-f(h)/h F(x)-f(x+h)/h	Incorrect quotient formula	**2**	**1**	**1**

Appendix C (Continued)

Error	Illustrative Example of Students' Response	Description	Pre	Post	Both
5b.1	$\frac{f(-1+h)-f(-1)}{h}=\frac{f(0)-f(-1)}{1}$ $\frac{0-(-2)}{1}=2$	Using quotient formula but taking big h value	**1**	**0**	**0**
5b.2	$\lim_{h\to 0^-}\frac{2(-1+h)^3+2}{h}$	Incorrect function	**1**	**1**	**0**
5b.3	We cannot draw a tangent line to the graph of f at x=-1 since it is not continuous at x=-1	As the function defined in parts according to the domain of the function being greater or less than –1, students thought that the function is not continous	**3**	**1**	**0**
5b.4	Since f does not have the same slope for neighbourhoods of –1 we have to be careful to choose close values X_1 =-1 $f(x_1)$=-2 X_2=0 $f(x_2)$=3 $f'(-1)\cong 5$	The slope of the derivative function is taken as the derivative at a point	**1**	**0**	**0**
5b.5	M=tanθ=y/x At x=-1 y=1-4+3=0 $\Rightarrow$ m=0	Assuming that tangent line passing through (0, 0)	**1**	**0**	**0**
5b.6	Dq= f(x+h)-f(h)/h f(x)-f(x+h)/h	Incorrect quotient formula	**2**	**1**	**1**
5b.7	Unclassified		**3**	**4**	**0**

Note: Both refers to pre and post tests together.

In task 3(a) while two students in the pre-test gave incorrect answers such as 5.8 and 3.1, three students in the post-test gave incorrect answers such as 4, 5/2, and 5.