HOT AND ABSTRACT:
Emotion and learning undergraduate mathematics

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ABSTRACT

Undergraduate mathematics students' affective responses to their studies have been collected from interviews, questionnaires and observations as part of a three-year longitudinal study of a cohort of mathematics students at two UK universities and from other opportunities from working with undergraduates and post-graduates.

The central point of this report is that emotion has a significant part to play in learning mathematics at this level. Far from mathematics being cold and abstract it is hot and abstract!

Affect has been classified into the three subdomains of belief, attitude and emotion (McLeod 1992). Attention here is on emotion, the least researched of these subdomains in undergraduate mathematics education. Reasons for the lack of attention in this area are attributed to the elusive task of tracking others' emotions as well as the abstract nature of mathematics with its concomitant 'cold' image.

This image belies the strong feelings expressed by or observed among mathematics students or recent graduates, and frustration is more prevalent than joy. Students mostly attribute their original choice of mathematics as a specialist subject to enjoyment. Enjoyment is highly correlated with skill. When these students become unable to understand the mathematics presented, frustration, fear or bitterness often arise. What role does the mathematics lecturer have in harnessing their emotion to pull them through to success? For emotional engagement, rather than just a good attitude or compatible beliefs, is the real key to desire to learn something which is abstract.

The report will be in three parts: firstly, a brief outline of some relevant literature will be given; then, secondly, a selection of data will be presented which will be interpreted to indicate the presence and importance of emotion in learning undergraduate mathematics; and finally, attention will be dawn to the role of mathematics lecturers.

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Introduction

In the UK young people generally choose a specialist subject to study at university or higher education around the age of 17 and, after recent expansion of higher education, more than 30% of this cohort participate in post-school education. However, the proportion choosing to study mathematics as a main subject at university is falling: from about 2.5% in 1986, to 1.5% in 1999 (Higher Education Statistics Agency, 1999). This paper, stimulated by this evidence of decreased participation, is part of on-going research into the particular challenges and rewards of being an undergraduate in mathematics. The focus in this report is on trying to track ways emotion impinges on learning mathematics as an undergraduate. The issue of 'emotion' was not an a priori target for the research just mentioned, but has arisen from a grounded theorising from data which includes interview transcripts, questionnaires and discussion with students (within and outwith the project).

What is special about mathematics? Don't young adults tend generally to feel an unpredictable mixture of being excited and threatened or insecure when they enter the university? After all there is considerable investment in them from society as well as from family and school whatever subject they are studying. What *is* special about mathematics is that beginning university mathematics is invariably presented as an abstract subject, without fuzziness or debateable results, which is assessed through individuals' timed exam performance. Such assessment arrangements are personal and adrenalin-producing yet the assessment's mathematics does not express any personal view. There is nothing to hide behind in mathematics: no experiment, no interpretation of evidence, no comparison of criticisms. The students are relatively more exposed – intellectually and emotionally – than in other subjects.

This paper firstly presents a brief review of literature on affect relevant to this study then offers a selection of data which indicates the significance of emotion for undergraduate mathematicians, after drawing out the centrality of emotion in learning this abstract subject, finally I briefly consider the significance of the role of mathematics lecturers/university teachers in the emotional and intellectual development of mathematics undergraduates.

From mathematics education literature

There has been considerable research on attitudes and beliefs in mathematics education (see Osborne *et. al.*, 1997, for a review), but Leron and Hazzan (1997) observe that there has been a "strong emphasis on cognitive aspects, and consequent neglect of affective and social factors" (ibid. p266). Indeed, McLeod (1992) having classified of affect into an ordered set: 'beliefs, attitudes and emotions' where beliefs were stable but less intense and emotions less stable but more intense, also remarked that emotions have not been a major area for research in mathematics education (ibid. p582) and attributes this lack of attention to the relative instability of emotion.

The principal sources of work on emotion in mathematics have focused generally on negative emotions like anxiety (gender studies, e.g. Fennema 1996) or panic (Buxton, 1981) though Celia Hoyles' (1982) survey of pupils' feelings about learning mathematics included some positive as well as negative replies. Laurie Buxton's seminal work showed through his case studies that generally successful adults could have a debilitating fear of mathematics, which could not be easily dispelled. He designed and ran 'mathematical therapy' for his subjects to enable them to re-establish the confidence essential for actually engaging in mathematical activity. Buxton's remark:

"reason is powered by emotion or, more often, hampered by it" (ibid.:3) is also relevant to undergraduates.

More recently, Jeff Evans has published his long term study on 'Adults Mathematical Thinking and Emotions' (confirming my thinking that mathematics produces 'hot' responses). Evans' study focussed mostly on anxiety, with adults of all ages and on quite a basic level of mathematics, nevertheless, the theorising he offers can be applied to the undergraduate context too. The main differences are the wider range of emotion which undergraduates exhibit (with pleasurable emotions more significant), the fact that they are all young adults (17-23 years old) and that the mathematics is 'abstract'. The core of Evans' theory is that "affect and emotion [are] inseparable from thinking, including mathematical thinking." (*ibid.* :228) and extends Buxton's work. Emotion does not necessarily 'interfere' with mathematical thinking – although panic-driven blocks do occur – rather "emotion [is seen] in terms of *charges of feeling attached to* ideas and thus related to the cognitive" (*ibid.*, italics in original: 230).

Evans' theorising furthermore employs the notion of 'practice'. This is a sociological term which signifies a set of customs, language, values, interests, tools, etc. held by a 'community' defined in turn by these customs, language, values, etc. Wenger (1998) gives a thorough working of this concept and relates a person's practice(s) to their identity (and so their feelings or emotions). Examples of 'practices' include those of being a nurse or a skate boarder or a yogi. With Evans' subjects, their identity was not essentially bound up with mathematics – as our undergraduates' identity inevitably is - so he is able to consider which practices were called upon by the subjects when they are engaged in mathematics. When considering mathematics undergraduates, the situation is more subtle, for one of the things which develops – or arguably should develop - over the period of undergraduate study, is a connection with the mathematical community and an increased sense of oneself as a mathematician. In the undergraduate context, students' investment in the practice of mathematics holds personal significance for them and is related to their other attitudes and beliefs about their course of study. Mathematics undergraduates' attitudes and beliefs have been studied by Kathryn Crawford and colleagues in Australia (e.g. 1994) showing that a fragmented notion of mathematics tends to correlate with a more superficial learning style.

This question of learning style also impinges in our discussion on emotion and learning mathematics. John Mason (1989), noting that the etymology of the word 'abstract' is to 'draw away', associates the "extremely brief moment" (*ibid.* p2) when the mathematician/student draws away from the particular to the general as the experience of abstraction. Students habituated into a superficial style of learning dare not 'draw back' - or 'abstract' – they focus on their need for rules to pass exams. A link between 'hot and abstract' can be found in this notion of Mason's that abstracting is a "delicate shift of attention" (*ibid.*) which brings together personal and mathematical processes: the undergraduate person wrestles with the subject matter of mathematics.

Now 'hot' is a metaphor for visceral energy, felt by a person; while 'abstract' in mathematics connotes ethereal, rational, person-independent generality. Why has there been, in western culture, a prevalent image of a universal or a generality as 'cold'? Could not the image of a universal have been 'like the sun' giving life and heat? Jere Confrey's answer to this question comes from her delving into prior usages of the word 'abstraction', (Confrey, 1995): the medieval priesthood associated abstraction with being "free from sin" and of course sin is of the body and hell fire! Confrey proposes to characterise abstraction by recognising: "1) a genuine dialectic between practical activity and sign use; 2) the value of multiple forms of representation; and 3) the role of

action in the act of abstracting." (*ibid*. p40). Her notion of abstraction thus ties in with both Evans' 'practice' (1) and Mason's 'doing' (3).

Undergraduates' emotion and their learning of Mathematics

As mentioned already, the results of this report are not obtained from an explicit search for emotional response, rather it is from a detailed reading of interview transcripts and field notes that these issues have emerged. As students were not generally prompted for an emotional response so, recognising that our emotions are often subliminal and not available for conscious reflection, their emotion often comes out in a change of tone or juxtaposition of ideas. The quotations chosen are representative or illustrative.

Success, pleasure and belonging

We found that students are attracted to mathematics principally because of their prior success in the subject or because of their pleasure in engaging in mathematical activity, and these are linked. The following are extracts from interviews after one semester at the university:

Lucy: … I've always liked doing maths. I've just really enjoyed it, I could do it … I don't know, it's more enjoyment than anything else.

Stephen: Knew when you got out on good maths degree you'd usually be earning quite a lot. …Well I wouldn't have done it if I didn't, you know, enjoy maths in some way.

Here we see that doing and enjoying go hand in hand, even when there are other motivations. While we expect initial euphoria to be tempered by the reality of course demands, we can see from the following extracts from Robert's first interview how 'up and down' he is in himself at the end of his first semester and how dependent this mood is on his ability to do the maths:

Robert: [of a lecturer] He just went through it really quickly. I don't know there was just - there was no time in between writing it down and listening to him, and trying to understand. … I dunno cos you don't know what to do, you think, well what's the use cos you can't do anything. I'll just have to go and read it on my own and see, so.

Later on Robert says of the applied maths module:

it was a bit of a struggle and then towards the end of the term, it began to make sense and that was good.

Interviewer: Can you give me any specific examples?

Robert: I dunno. I can't think of any, I just remember enjoying it because I could grasp it, you know.

Despite this brief pleasure, still further on in the interview he says:

Robert: … I just sort of, you know getting myself in a rut of not doing enough work and then doing badly and getting rubbish marks, I was thinking I shouldn't really be here, but I dunno, I just sort of, my mum, I said to my mum that I wasn't really enjoying it …

Robert is on edge emotionally: he is frustrated that he doesn't understand the lectures, feels he ought to be self motivated and get the books out, is dependent on a feeling of understanding for some pleasure, and seeks nurture and advice from his mother. Another student who expresses vividly his feelings about 'doing it' and pleasure is Raj:

Raj: I've not had the, satisfaction of getting it and I miss that cos I'm falling so far behind and you know in like the A levels and in other maths when you can do it like you can kind of things get like satisfaction and so you just like get stuff like that click, clicks into place quite easily, and cos I wasn't getting that and I wasn't making an effort and I knew and I was like depressed with all that an all kind of things …I wasn't enjoying maths and kind of thing, I'd always liked maths and was happy doing it and cos I know what makes me happy and I have to be on top of my work for me to be happy kind of thing…

Raj is one of the few students in his university's cohort who is of Asian (Indian sub-continent) ethnic background and whose parents are in working class occupations. Raj attributes some of his problem as due to a disjunction between communities. Earlier in the interview he says:

Raj: Really have to start work. It was probably the worst start I could have hoped for, but to be honest I couldn't realise, what it was going to be like cos I really hadn't a clue what was involved in going to university, hadn't a clue when I applied and then I came here and it was like the biggest eye opener I could imagine, just being here, nobody told me, nobody knew and I never knew what to ask, kind of thing cos I never really thought much about it and nobody I knew, knew much cos people at my school don't go to university, my mum and dad's never been so nobody could tell me. I can look at what I've done and that was a mistake, and now I know I've got to catch up,…

These two students illustrate how identities are bound up in different ways with their feelings about their position at the university and their ability to learn and do mathematics. Success and pleasure and belonging are closely linked.

Mathematical security

While people the world over may have different conceptions of mathematics, there is a wide consensus that early university mathematics becomes more symbolic, employs more algebra and proves its results more than mathematics experienced at school. The respect for young children's own methods of calculating which we see reflected in projects like the National Numeracy Strategy (DfEE, 1999) does not seem to have an analogy at undergraduate level. At university it seems there are less negotiable and individual ways to get results, despite the considerable debate about standards of proof (see, for example, Thurston, 1995). However, it precisely the non-negotiable aspects of mathematics which some undergraduates find enhances their feelings of security about what they are studying. The following extract from a student who spent a year at studying law expresses this sense of security:

Janusz: I feel more in control of maths in a way, I know what I have to learn, I know when I've got it right and I think I understand the structure of the course and what bits go together. I never felt that with Law, it just all felt so immense and it was like I was always doing little bits of things and I never had any idea where it would lead or how big the thing was that it was part of. I just never felt I had any control and I don't know if it was just me or if that is what Law is like, I don't know but maths is different and I'm enjoying it much more. It feels smaller somehow, or perhaps it's just how it is at the moment but I do know that maths as a subject is immense but maybe it's the way it's taught, that makes it seem as if you can manage it, do it and get it right. I don't know but it's alright so far.

'Getting it right' confirms and satisfies. Other students also remarked about the insecurity of doing an arts subject where they believed essay marks were more subject to the assessor's interpretation than their mathematics problems would be.

Role Mathematicians

Mathematics does not speak for itself at this level of undergraduate study. New abstract material requires mental accommodation – rather than assimilation of related ideas (Skemp, op. cit.). Invariably, undergraduates have to struggle in learning such new mathematical content and the ability to struggle involves energy and desire for knowing. Where does this energy come from? Fear sometimes. Sometimes the energy comes from association with practices which are favourable (e.g., talking about mathematics, working mathematical problems, etc.) and the associated projection of meaning along a chain of ideas driven by a positive emotional call (e.g. respectively: I like him and he likes maths, I can complete this problem and feel satisfaction, etc.) (Evans, op. cit.). In particular, the lecturers are important to the students. Students show considerable insight when they praise or criticise their lecturers' efforts. For example, Oliver compares his Dynamics lecturer – who uses paper aeroplanes - with others he has had:

> He puts some, what's the word, if you, kind of enthusiasm for the subject, some kind of er, he just enjoys it … He's interested about what we're doing and it comes across. Which is good, a very good thing. I think some of the lecturers can be a bit bored with it, and if they're bored with it how are we supposed to be interested, d'you know what I mean. Cos some of it's hard and you have to get the enthusiasm and interest to try and understand it but when they seem bored you can think, this isn't, this just isn't worth the effort and you might just make the effort for the exams or something but you don't really enjoy it cos it all just comes to be a big effort to pass the exams or something.

In another related paper, (Rodd, 2002), I claim that one reason that students go to lectures is that there is a chance of having their imaginations stimulated by abstract mathematics presented by an inspiring lecturer. This resonates with Oliver's observation that when mathematics is 'hard' an extra surge of energy is required to 'go for' the understanding and this energy is sought from the lecturers, the 'guardians' of the required knowledge with whom the young adult neophytes may associate and may see as role models. (Clearly, there are gender issues here given the dearth of female university mathematicians). However, looking at Robert's early perceptions we can see that the rejection he feels from the lecturers may contribute to his feelings of alienation (see above) about the course:

Robert: Real teachers explain things so you can understand and real teachers help you. Like the teachers I had at school, they helped you but here you've just got to help yourself 'cos they're too busy doing their lecture and writing on the board, they don't even look at you. That's something I've learned and I never thought it would be like that. Some are good but some, doing the lecture is all that seems to matter to them. They come in, some of them just come in and start, like they can't wait to get going and if you've not got your pen out and that, you've missed the start. They don't speak to you or anything, they just start like they've never been away, like it's just a continuation of where they left off. That's hard to get used to. I don't like it, I don't know why they do it.

Are mathematics lecturers teachers? This undergraduate seems to think they should be. In the UK academics are given a university post by virtue of their successful mathematical research, yet their job includes teaching undergraduates, many of whom go through periods of being quite unsure about being a mathematics undergraduate.

In summary

While emotions are illusive and difficult to track, there is enough information from our undergraduates to tell us that indeed their feelings are very important and intimately bound with their learning mathematics at the university. This paper has not emphasised negative emotions like anxiety– though we have evidence of these feelings existing – but has considered the importance of pleasure, and consequences of its lack, as well as the satisfaction that mathematical completion can bring and the energising role of an enthusiastic lecturer.

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