

THE MEDIATIONAL EFFECTS OF TEXTS AND TECHNOLOGY IN TEACHER PREPARATION

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ABSTRACT

Teaching at school and at tertiary level may be characterised by three types of practice: direct instruction, mediation and facilitation. The mark of an expert teacher is revealed by his or her ability to move between these various modes of practice, in response to the immediate needs of the students within a classroom, lecture hall or learning environment. This wisdom in practice comes from a clear understanding of the nature of these teaching practices and an awareness of the potential of each to produce the desired responses from the recipient students. This paper suggests that this wisdom or awareness in practice may be developed through learning programmes in which novice and experienced teachers experience and reflect on different modes of teaching practice, centred on their subject discipline, within an interactive learning environment. This environment extends the traditional modes of learning (guidance by an expert) by including mediated learning with written texts and interactive technology. Furthermore reflective practice is also built in as a necessary component for learning. This reflection refers to the learning of the subject discipline and to the teaching of that discipline.

This paper reports on a case study in which prospective teachers were asked to investigate a geometric problem, to reflect on the methods used to construct a solution to the problem and to produce a written formal report based on these actions and reflections. Particular attention is paid to the participants' use of written texts and computer technology in their resolutions to the geometric problems and as a consequence to their recognition of these resources in the report.

The result of this investigation raises questions as to the effectiveness of mediational resources in supporting mathematical progress and stimulating creativity and independence in the classroom. It also suggests that the introduction of resources of research into the learning environment may temper intrusive or inappropriate intervention by the expert.

KEYWORDS: Geometry, mediation, resources, pedagogical reasoning, teacher education.

1. Introduction

This paper explores the awarenesses of a group of prospective mathematics teachers who were part of an interactive learning course aimed at stimulating the process of pedagogical reasoning (Shulman, 1987) in geometry and its teaching. The course can be seen as a multi-perspective programme designed for future teachers of mathematics that focuses simultaneously on mathematical practice, on the resources that promote mathematical practice and, through reflection, on pedagogical practice. As such the course fills the gap between traditional mathematics courses at a tertiary level and pre-service teacher education courses (Hockman, 2000).

In this course it was hoped that awarenesses—in and on—practice would emerge through the students' reflective discourses and through the creative teaching units they produced as a course requirement. These discourses were analysed to reveal:

- the resources used or not used in solving a mathematical problem (action),
- the awarenesses or lack of awareness of the roles played by the resources in solving the problem (comprehension),
- the degree of transformation of learning strategies into teaching strategies as evident in the written reports (transformation).

The problem episode, under consideration in this case study, formed part of a sequence of problems within the course. The sequence was designed as a process to evoke awareness-in-counsel in mathematics and awareness-in-discipline in mathematics teaching (Mason, 1998). The study examined the activities and discourses of the students as they, firstly, solved a geometrical problem, and secondly reflected on solving the problem in a formal written report.

2. Scaffolding and Fading

Collins, Brown and Newman (1989) and others introduce the concept of scaffolding and fading in teaching and learning. This concept is widely used to refer to the ebb and flow of teacherly support or mediation during teaching practice. The aim of the scaffolding is to induct novice practitioners into the practice of the discipline. Once the novice practitioner is able to complete the task, or shows awareness-in-action in the discipline (Mason, 1998), the teacherly support fades away. The result is the attainment of independent working strategies by the student. Hence scaffolding and fading usually refer to the direct interaction between student and teacher, or to social discourse within the classroom environment. In this case study the process of direct and indirect teacher intervention is augmented by a broadening of the social dialogue in the classroom to include all the participants; through the accessing of traditional mathematical experience in written texts and visual mediums; and through explorations. The resources incorporated in this augmentation are referred to as resources of research.

The processes of action, comprehension and transformation in pedagogical reasoning are set in motion through the social dialogue in the classroom. That is, the teacher directs attention away from him/herself by encouraging the use of the resources of research. Teacherly guidance or intervention fades as these resources support the students in their work. In turn the students begin

to use the resources of research spontaneously in place of expert guidance.

From Mason (1998) it appears that awareness–in–action in mathematics confirms the ability to do or to practice mathematics. Awareness–in–discipline (awareness of practice) in mathematics confirms the awareness of the process of doing mathematics and is a generalisation of practice. These processes include not only the heuristic strategies suggested by Pòlya (1945) and Schoenfeld (1985) and deductive skills of conjecture and proof, (Hersh, 1998), but also resources (source of ideas) that may promote the transfer of ideas from context to context. I suggest that these resources may be found in the category of research. That is, through reading and writing, exploration and social dialogue (de Villiers, 1996; Henderson, 1990; Wood, 1997).

The transformation of mediation marks a change from student teacher dependence on direct teacher mediation or scaffolding during the work process to a situation where such intervention has faded and has been replaced by independent work strategies using the available resources. That is, there is transformation in mediation starting from direct instruction by the coach, to indirect mediation through the resources, and then to direct and independent use of resources.

3. The Method

3.1 The participant sample

The case study involved 15 students in their third year of mathematics study. Most but not all the students were prospective mathematics teachers. The students worked in small groups of 3-4 throughout the course. These groups are named A, B, C and D. Teaching in the course ranged from standard lectures, to coaching in an interactive environment, to mediation in a computer laboratory. Thus the environment supported various types of teaching and learning, yet the central focus of learning was through an interactive environment or *reflective practicum* (Schön, 1987) using the resources of research.

3.2 The study design

This study focuses on a particular problem–solving exercise during the course. The exercises in the course were sequenced in a way to allow the resources of research to become operational through direct instruction, and then to be integrated as working strategies through the mediational prompting of the teacher educator. The final problem-solving exercise in this sequence set the scene for the resources to be assimilated and then utilised as independent learning and teaching strategies by the participants. The problem–solving exercise under consideration falls midway in the sequence of problems, aiming at the integration of the resources of research into the working strategies of the participating students.

3.3 The problem: construction of touching circles

Construct a heritage village: The village is to have three round huts, to accommodate the chief, his wives and his children. In addition two enclosures have to be built.

(i) A circular enclosure that must touch the outer section of each hut to separate the village from the fields

(ii) A circular enclosure that must touch each hut on the inner sides to demarcate the cooking and entertaining boma of the compound.

Apollonius' problem of tangent circles asks if it is possible to construct, with ruler and compasses, a circle tangent to three given, non-concentric, non-coaxial circles. Apollonius' problem has both algebraic and geometric solutions. While the algebraic solution involves solutions to simultaneous equations, a geometric solution uses tangency together with circle inversion.

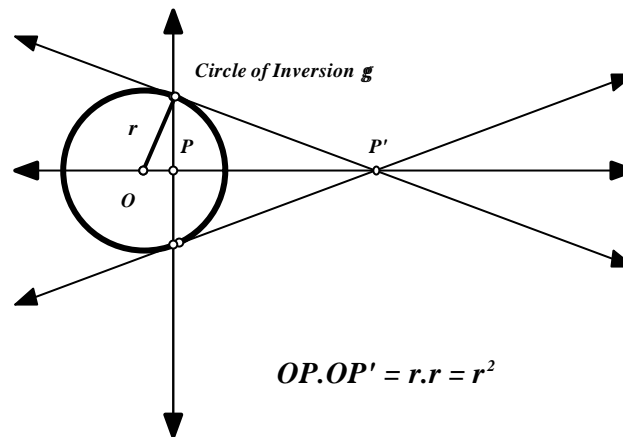


Figure 1: Circle of Inversion γ centre O and radius r , with inverse points P and P'

Although the problem is quite difficult to solve, its statement is easily understood. The exercise was completed in a workshop over a period of four hours. The students had access to computer technology, including *Geometer's Sketchpad*, course notes, source material (Courant, (1948); Coxeter H. S. M. (1967), Johnson, R. A. (1960), Cadwell, J. H. (1966), Eves, H. (1976)) dealing with Apollonius and his problem; compasses and graph paper.

3.4 Data Collection

Each of the groups submitted a written report at the end of the period. Rough sketches and working documents were collected. The students recorded their feelings about this experience in journals and I kept a record of my impressions of the episode. The topic of Apollonius' circles was referred to during the interviews with the students at the end of the course. These comments were recorded. The audio and video recordings were made during the work period. These recordings form the primary source of data in this episode with the other data complementing and supporting claims.

4. Reflections on Action

“ Although the problem was not as easy and intuitive as we thought at first, and an extra push and some material was required to get into the right track, the experience was good. It was good to be able to see how all our knowledge from the course thus far could be called into action. As expressed previously, the trouble I have had with maths content in the past is that it involves too many theorems, too much memory work and too little application and integration of all the work.

The exercise also called into action skills. Having to work in a limited time to digest information, solve a problem, express the solution, co-ordinate a group to prepare a document entailing all concepts, and ensure that each person understands each stage of the solving process. It was probably the most worthwhile life experience maths has ever created” (PH: 27/8/98).

The placing of the problem in the novel context of the construction of a heritage village appeared to have caught the imagination of many of the students. Many of the students had never seen mathematics used in a meaningful way or thought about the real contribution they might make as mathematicians in the future. The experience appeared to have heightened their awareness of the potential each one had to make a difference in life. The ‘real’ application of work that they were engaged in made their endeavours all the more worthwhile. While the application of the ideas of tangency and inversion to this new context in some cases needed considerable support from me, this intervention in no way diminished the feelings of satisfaction recorded by the students.

The difficulty of the task forced the students to engage with the printed texts. The initial reaction to these texts differed from group to group. Group A, who had previously used text material as a strategy to solve problems, quickly adapted to using the texts. They quickly picked up on the references to inversion and were able to integrate this new idea with their knowledge of tangency and inversion formed during the course. I certainly was called on during this period to confirm these connections and later to suggest ways of making the problem slightly easier but I did not dominate the activities.

In contrast to the action of group A, the rest of the class (groups B, C and D) took a long while to come to terms with the text materials. They felt that the text should have provided a clear and complete solution to the problem, not just suggestions and oblique remarks. They also felt that using the text was a negative reflection of them as it highlighted the fact that they were not able to solve the problem themselves.

Action and exploration dominated the whole problem-solving episode. These activities ranged from rough drawing, to paper, compass and ruler constructions, to the use of *Geometer’s Sketchpad* to validate and check hypotheses. Group B only rejected a suggested solution after exploring it with *Geometer’s Sketchpad* and Group A attempted to use the program to create a more accurate diagram and plan for their construction. Groups C and D used only compass and ruler constructions for explorations.

The circumstance of working in a *reflective practicum* within a limited time frame to complete a difficult problem forced the students to become dependent on each other. This social interaction dominated the transcripts and was a central issue in the remarks made by the students after the problem-solving episode. All the students were aware of the role that they played in solving the problem, valued the teamwork and understood the nature of my interventions. More importantly they also realised that a great deal of effort was needed to communicate their ideas succinctly and logically. The quality of ‘group work’ and of ‘sharing the load’ contributed to the students growing awareness of the nature of social discourse and its contribution to problem resolution.

Rigorous proof dominates mathematics studies at a tertiary level and has also had a negative effect on attitudes to mathematics at school level. The problem that the students investigated

during this episode could have been stated as follows: “*Use the principle of inversion to prove that it is always possible to construct a circle tangential to 3 non-coaxial circles*”. This statement would have hidden the rich history of the problem, de-contextualised the situation, and robbed the students of seeing how mathematics finds its way into real life situations. Furthermore the very request of ‘proving’ would have had a deflating and negative impact on the behaviour of the students. The students’ idea of what constituted a rigorous geometrical proof had something to do with the ‘given – required to prove – proof’ format that is inculcated as part of a standard geometry course at school, and this experience did not encourage the possibility of different solutions to one problem. In the case of Apollonius’ tangency problem, the students proved the theorem by using their existing knowledge, drawing inspiration and ideas from traditional mathematical experience, exploring various options, and working together as a unit within their groups. Each of the students participated in the writing of these proofs, perhaps the first they had ever attempted to construct, as opposed to copy and learn. They could validate or back-up each statement with a reason or a reference. In this way they were initiated into the community of practice of working mathematicians.

Teacher intervention remained a significant feature of the social discourse within the reflective practicum. However, as the period progressed changes occurred in the relationship between myself, as coach, and the students. I noted various factors with respect to my interventions, which influenced the end result of the problem-solving activity.

On the one hand, my interventions of direct instruction and mediation did not appear to be completely intrusive and the students not only completed the exercise but also thoroughly enjoyed the experience. They integrated their texts, experience, and explorations and worked as a group to complete the problem. In addition they felt empowered by the experience recognising that they could use the tools of research to supplement teacher mediation.

On the other hand, it became apparent that the authority of the teacher could be replaced and challenged by the students when empowered by the tools of research. This was particularly evident when a solution was proposed that deviated from the train of thought that I, as the coach, had envisioned. My conception of the problem was founded on my own solution. This particular method of solving the problem coloured my interventions in the class. I believed the students needed to be explicitly aware of the degenerate cases of the problem and hence I spent a substantial portion of time drawing the students into recalling and listing these cases. This effort was certainly beneficial to the problem-solving activity of group A. This group of students produced a solution that mirrored mine and used the approach of expanding the given circles to touch at the centre of inversion. The degree to which I supported their progress was limited and this scaffolding was successfully transferred to the independent agents research.

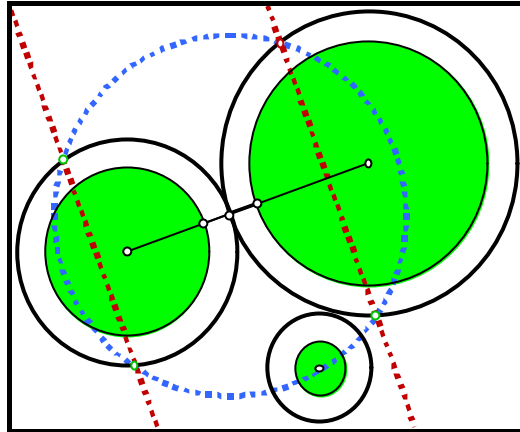


Figure 2: Inversion circle centred on the point of tangency of expanded circles and passing through the centre of the third circle.

The situation was not as clear-cut in group B. They chose to shrink the circles down to the case of a point and two circles. Although this certainly is a degenerate case of Apollonius' problems, its solution does not depend on solving this degenerate case, but rather on the construction of mutual parallel lines to disjoint circles. Using this approach made the time spent on discussing the degenerate cases redundant and may even have been obstructive and confusing. This mediation may have unintentionally shifted the attention of the students, and aborted a reflective moment that may yet have borne fruits.

As a result my support and mediation in the efforts of group B extended over a long period. Initially we appeared to talk at cross-purposes. The students worked hard to make meaning of my vague and cryptic suggestions. However as the period progressed there was a change in this dynamic. Firstly the students did not accept my rejection of their constructions as a solution to the problem, and moved to the computer to validate or refute their suggestion. As a result the refutation was through their explorations. Secondly, they then initiated a solution to the problem that I had not completely explored and was an innovation of the solutions suggested in the texts. In this case I had to work hard to make meaning of their suggestions. This reversal in roles, due to a quality of un-preparedness and un-seeness on my part, allowed for a community of practice to emerge between group B and myself. This period of interaction was very different to the uncertainty the students expressed during the previous interventions during the session. In this latter period, I believe my role shifted from instructor/mediator to joining in the general social discourse of the group discussions. These discussions were mediated by the texts and the explorations that the students themselves had made. The reports made by the students and their journal reflections confirmed these conclusions.

In the case of groups C and D I was much more forthright in my suggestions, and gave clearer directions to the students. As a result their progress was smooth, but closely monitored by me. In these latter cases, unlike the situation in group A and group B, I believe very little comprehension and transformation occurred.

5. Concluding Statement

This case study investigated the integration of resources of research into the working strategies of the participating students. The evidence shows that groups A and B adapted to using the social environment, the text materials and the available technology appropriately and creatively. They showed an awareness of the advantages of extending their resource base and thoroughly enjoyed their newfound independence. I claim that in these students mediational resources supported mathematical progress most effectively.

Group D showed slower progress, adapting well to the text material but still reticent to explore using the technology. They preferred to continue to use the compass and straightedge methods to construct their solution. Yet they made a point of noting that technology is not always available and should be used only with circumspection. I believe that these students used the resources of research in a meaningful way, showing awareness of the potential of extending their source of ideas to include research.

Group C however continued to rely on the guidance of the coach. They waited to be told how to proceed. In this case I believe that the resources of research were not fully integrated into the work strategies and remained at the operational level throughout the exercise.

The problem-solving exercise had been chosen for the specific reason that there were many ways to find the solution. Yet I was surprised at how my own conception of the problem coloured my conduct in the classroom. Close examination of the transcripts made me aware that, where possible, the expert must enter into the discourse of the students. The expert must attempt to remain open-minded in response to suggestions made by novices and aware of the dangers of prematurely aborting their reflective moments.

In conclusion, the case study confirmed the integrated use of, social dialogue, texts and exploratory devices as work strategies in many of the participants. These resources enhanced the independence of the participants in the learning experience. There was also a growing awareness of the potential of research to stimulate learning. Finally there was an indication that teacher intervention can be invasive. It is proposed that balanced co-operation and mutual support during classroom activity may be created by allowing the students to moderate their own ideas and progress, in their own time, with the resources of research. Future research may investigate this further.

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