USING TECHNOLOGY TO INTEGRATE CONSTRUCTIVISM AND VISUALISATION IN MATHEMATICS EDUCATION

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ABSTRACT

This paper provides a discussion of the pros and cons of instructivism and constructivism in the mathematics classroom, and endeavours to show why the latter is a preferable methodology to the former when considering the effective use of technology to enhance visualisation.

The adoption of a constructivist approach to the teaching and learning of Mathematics has highlighted a shift from teacher dominance. Visually stimulating computer environments can allow students to become immersed in their own knowledge construction. However, it is not a trivial matter how to utilise this considerable technological capability most effectively for educational benefit, emphasising the importance of a teaching and learning methodology.

It is necessary to encourage more exploratory approaches to learning, where students can be the initiators and controllers of their own learning. There is much empirical evidence that this approach significantly improves the understanding of higher order concepts.

Knowledge is built up from personal experiences, and making these experiences more dynamic will assist in the development of cognitive structures. Computer-based attractive environments with visually compelling displays, together with facilities for interaction, can provide the setting for more dynamic, powerful experiences. These environments are filled with stimuli, which encourage rich constructions, by students. The integration of constructivism and visualisation can encourage the reformulation of conceptual structures and the development of higher order skills.

Having reviewed and examined the effectiveness of previous work by authors such as Tall, Dubinsky, von Glasersfeld, etc., and different constructivist perspectives, consideration is given to the best way to employ constructivism in teaching and learning with computer-based visualisation. The effectiveness of this approach is evaluated, and students' experiences are discussed in terms of the enhancement of mathematical skills via the constructive use of visual software.

1. Introduction

For completeness, this section provides an overview of instructivist and constructivist approaches to teaching and learning in the mathematics classroom, and endeavours to explain why the latter is a preferable methodology to the former as the use of technology in teaching increases.

Instructivism reflects the traditional hierarchical view of mathematical study, where instructive representations are finely tuned to a particular purpose (O'Reilly et al., 1997). Students who are subjected to this instructivist approach have to learn to discriminate between contexts in order to appreciate when one finely tuned representation is needed as opposed to another, which is clearly a non-trivial process.

The instructivist, or behaviourist, approach is to pre-plan a curriculum by breaking down a subject area (usually seen as a finite body of knowledge) into assumed component parts, and then sequencing these parts into a hierarchy ranging from simple to more complex (Fosnot, 1996). Instructivism assumes that listening to explanations from teachers will result in learning. Learners are viewed as passive, and educators spend their time developing a sequenced, well-structured curriculum and determining how they will assess, motivate, and evaluate the learner. The learner is expected to progress in a continuous, linear fashion as long as clear communication and appropriate reinforcement are provided.

Schifter sums up the instructivist way of thinking in the following - *The teacher shows the students procedures for getting right answers and then monitors them as they reproduce those procedures. To ask a question without having previously shown how to answer it is actually considered 'unfair'* (Schifter, 1996).

As a result of schools taking an instructivist approach to teaching, it was reported almost a decade ago that students could not apply their knowledge to unknown problem solving situations (Honebein et al., 1993). This is unfortunately still an issue that needs addressing today with teachers using technology. A different type of learning activity is required, i.e. constructivism. Here the concern is not mastery in a test of procedural skills, but rather the ability to function successfully in unknown problem solving situations. The focus here is to be able to take the knowledge gleaned from local tasks and apply it globally (Honebein et al., 1993). The learning activity has a purpose that goes beyond simply demonstrating mastery of the local tasks; the purpose for a learning activity is driven by the global underlying concepts. It is therefore not the ability to recall information that educators should be interested in, but instead the ability to apply knowledge and skills in different problem based environments. The constructivist approach, therefore, concentrates on a holistic view of learning mathematics, and focuses on deep understanding and strategies, rather than facts and rote memorisation (Honebein et al., 1993; Fosnot, 1996).

The fundamental principle of constructivism is that learning is very much a constructive activity that the students themselves have to carry out. From this point of view, then, the task of the educator is not to dispense knowledge but to provide students with opportunities and incentives to build it up (von Glasersfeld, 1995, 1996).

Lerman has described how Piaget's constructivist perspective is that the individual is responsible for his thinking and his knowledge, and is the central element in meaning-making, whereas Vygotsky attempted to develop a fully cultural psychology, placing communication and social life at the centre of meaning-making (Lerman, 1996a), where the individual can construct knowledge facilitated by a teacher or more able peers.

The zone of proximal development (Scardamalia and Bereiter, 1991; Lerman, 1996b) is the area in which the student can perform tasks successfully, but only with some assistance. The student therefore works in a constructivist manner, inside an instructional domain. Vygotsky defines the zone of proximal development as the gap between what a child can do on her or his own and what she or he can do with, for example, a teacher. The learning activity constitutes the zone of proximal development; it is actually the difference in activity between 'with or without' the teacher. The teacher is there to guide, and to share in evaluating their progress.

Strategic questioning, known as the Socratic method, is used to facilitate the construction of a target concept, working within the students' zone of proximal development (Rowlands et al., 1997). Rowlands et al. explain that this method of strategic questioning challenges (and hopefully removes) misconceptions, and facilitates the construction of knowledge. The key is to ask qualitative questions that lead the student to reach the target concept without it actually being given by the teacher. Consistent with the Vygotskian perspective, these questions provide hurdles to overcome in order to develop cognitive growth, yet which also serve as props or hints to facilitate the process. The teacher must use questions that challenge students to think according to the properties of the target concept. Rowlands et al. discuss how intuitive concepts stand at one end of the zone of proximal development, and the target concept stands at the other - strategic questions stand in between and facilitate the progression from the former to the latter.

2. Constructivism in Relation to Educational Technology

The constructivist use of technology allows the opportunity to change the nature of the material to be taught and learnt from routine-based to discovery-based activities. Knowledge, as discussed in the previous section, is built up from personal experiences, and making these experiences more dynamic will assist in the development of cognitive structures (see for example Tall, 2000, 2001). Computer-based environments with visually compelling displays, together with facilities for interaction, can provide the setting for more dynamic, powerful experiences. These environments are filled with stimuli, which encourage rich constructions, by students (Nelson, 2000). Graphic representations, coupled with social interactions, are seen as leading to the development of an individual's knowledge, and are seen as leading to the adaptation of concepts (von Glasersfeld, 1996).

The authors have observed, via classroom experiences, that 16-19 year old students find it difficult to answer questions about concepts that have been placed in contexts separate from their immediate concrete experiences. The constructivist use of the computer is a more powerful means of providing the student with vivid experiences in order to convert the concrete into the abstract more successfully (Dubinsky, 1991). This can in turn provide students with the appropriate mental structures that can be called upon to utilise conceptual knowledge in unknown situations (Honebein et al., 1993). The activities that are carried out in a computer environment provide meaningful experiences for learners that help them transfer skills and knowledge to other problem solving activities and subject domains.

While engaged in mathematical activity, students construct images (Wheatley and Brown, 1994). When they 're-present' their image at a later date, they are operating from the image that they originally constructed. The nature and quality of the image will influence the re-presentation, hence the importance of quality mathematical software for image generation. This act of re-presentation is a complex one. Piaget has shown that the image constructed may undergo change over time without any intervention - the original image-making process supported by appropriate

software is therefore vital. Activities that encourage the construction of images can greatly enhance mathematics learning. Students who naturally use images in their thinking easily make sense of novel mathematics tasks while students who are not good visualisers often do not (see for example Habre, 2001). It would be desirable to develop learning activities that promote the development of image-making skills for all students.

Powerful, multiple representation software can be used to encourage the learner to construct meaning for different representations and their interrelations. The relationship between representations lies at the heart of much mathematics (O'Reilly et al., 1997). Multiple representation software can demonstrate these links explicitly. Within such software, constructive changes in one representation trigger automatic changes in another. Thus, for example, a change in algebraic representation of a function should immediately promote a corresponding change in the graph. A learning tool cannot be used constructively, however, unless the students are genuinely in control.

An illustration of the zone of proximal development is where the teacher takes on the role of facilitator in the construction of knowledge (rather than a giver of knowledge) by providing props and hints to develop students' cognitive framework. The teacher aids the learner in accomplishing the activity, not by doing the task for the learner or giving the learner the correct answers, but by providing guidance that require learners to formulate their own solution to the problem (Honebein et al., 1993). Strategic questioning is employed by asking probing questions which act as a catalyst to get students to reach the desired goal, without taking away the ownership of the task. In this manner, students can eventually arrive at a required level of understanding for themselves, which is not only advantageous in terms of the learning process, but also increases satisfaction and boosts confidence.

3. Examples of Constructive Mathematical Software and their Use

Teaching mathematics from a constructivist perspective involves the provision of activities designed to encourage and facilitate the constructive process. This can be achieved readily nowadays by employing visually compelling mathematical software such as Autograph (www.autograph-math.com), Cabri Geometry (www-cabri.imag.fr), or a Computer Algebra System such as Derive (www.derive.com), with which students can explore mathematics. These packages have various features which facilitate a constructive approach to learning mathematics. Autograph allows the user to 'grab and move' graphs, lines, and points on screen whilst observing changes in parameters, and vice versa. Cabri-Geometre encourages the user to drag points around the screen whilst observing the effects of such changes on geometric shapes. Derive, with its multiple representation capabilities, allows the user to switch easily between numeric, symbolic and visual representations of information. These examples of software that can enhance constructive learning can be used effectively to encourage 'what if ' situations for students to explore.

Strategic questions need to accompany the use of technology. For example, an instructive question concerning functions might be to find turning points, asymptotes, etc., and then, as an afterthought, to plot the graph. An example of a constructive question, however, could be to consider some function, f(x), and then determine what happens when a particular symbol or parameter within the expression is altered; the students would then be encouraged to explore and investigate. The constructivist philosophy thus invites students to find answers for themselves.

In order to establish any practical evidence of enhanced mathematical skills of students having experienced a constructive approach to learning, a research project was set up to assess the effectiveness of the constructive employment of computer-based visualisation. To develop students' conceptual understanding of the relationship between graphical and symbolic forms, a piece of bespoke mathematical software was written entitled 'Graphs of Functions: A Constructivist Approach'. The controlled study involved 16-19 year old students prior to entering undergraduate mathematics degree courses. The control group contained students who had been taught 'functions and graphs' by traditional instructivist methods, and the experimental group contained students who had learnt 'functions and graphs' via the interactive software (for further details of the experiment see Malabar, 2002).

Whilst using the software, the students were given a series of function graphs of polynomials, trigonometric functions, exponentials, etc., as well as combinations of these basic functions. The task was to determine, via constructive explorations, the correct symbolic form of the function. In this manner, students could build up their conceptual understanding of the links between algebraic and pictorial representations as a result of both successful and unsuccessful conjectures and evaluations. The teaching style adopted was the Socratic method of strategic questioning as described in Section 1. Working within the students' zone of proximal development, props and hints were used to challenge misconceptions and lead the student to the construction of the target concept.

The students in the experimental group felt that they owned the problem, which they felt compelled to resolve. This philosophy provided an organising role and a purpose for learning. When they were faced with contradictions to their own conjectures, it was up to them to find resolution. The activities were concerned with exploration and debate; there was not a finished body of knowledge to be accepted, accumulated, and reproduced. Instead of concentrating on technique and strategy, this approach helped the students to develop an attitude of inquiry toward the learning of mathematics.
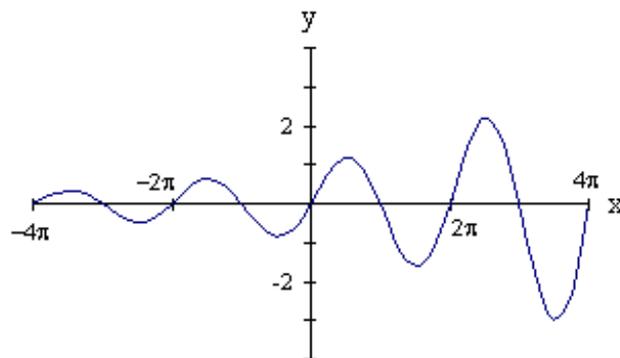
The constructive use of this software provided students with vivid experiences in order to convert the concrete into the abstract more successfully, and encouraged them to construct meaning for different representations and how they are related.

4. Evaluation

In order to help evaluate the effectiveness of our constructivist approach in terms of students' skills, we can refer to a taxonomy known as the MATH taxonomy (Mathematical Assessment Task Hierarchy). The MATH taxonomy (Smith et al., 1996) describes a hierarchy of skills ranging from lower order skills, such as factual knowledge and the ability to follow procedures, to higher order skills such as the ability to interpret, conjecture and evaluate, as in the table below:

Group A	Group B	Group C
Factual Knowledge	Information transfer	Justifying and Interpreting
Comprehension	Application in new Situations	Implications, conjectures and comparisons
Routine use of Procedures		Evaluation

It was conjectured that the constructive process had enabled students to develop more Group C skills, whereas students undergoing the instructivist treatment were mainly limited in skills to those of Group A. The evidence suggested that this was indeed the case, but moreover there was evidence to suggest that *linkages between the skill groups were more pronounced, creating a more holistic view of mathematics*. This is best summarised by considering a typical posed question (although only one example, it is indicative of the findings in general. A detailed statistical analysis of the above experiment can be found in Malabar, 2002):

The above graph is a graphical representation of which of the following functions ?

A $y = \sin(x) + e^{0.1x}$ **B** $y = \sin(x) + e^{-0.1x}$ **C** $y = \sin(x)\, e^{0.1x}$

D $y = \sin(x)\, e^{-0.1x}$ **E** all of the above

The above question assesses whether or not the students have been able to take the knowledge gleaned from local tasks and apply it globally. When faced with a graph, which was the result of a combination of functions, the group who were subjected to an instructivist approach struggled to find the correct solution, whereas the constructivist group used their knowledge relating to other families of graphs to arrive at the correct function. The group that learnt constructively had a more holistic view of the topic and were therefore not fazed by the nature of the task, i.e. to employ their conceptual knowledge of combining familiar, specific functions (and the effect on the graph) to an unfamiliar (but similar) situation. The instructivist group's sequential style, however, hindered their progress as they could not see any other way around their limited, linear methods.

The constructivist group had done some work with the bespoke software concerning combining different functions, and so this could clearly have helped in solving the above problem. They were more successful as they had the ability to combine functions and understand the effect this would have on the graph, irrespective of the specific functions studied. Their whole approach to learning equipped them with better strategies for problem solving. The richness of global thinking proved beneficial as they could check their answers by more than one approach.

The instructivist group had not studied combinations of functions explicitly, and were struggling to match this question to any prior experience. They did not have a 'recipe' or 'template' to solve such problems, and therefore had a very limited solution strategy. The problem could be solved in an instructivist manner, e.g. to methodically eliminate possible answers by considering values of x where the graph cuts the x-axis, then considering the substitution of different values of x into e^x and e^{-x}, etc., but the instructivist group did not seem to have the necessary problem solving skills to tackle it, even in an instructivist way.

It would appear that the constructivist group had a greater mathematical skills set with more flexibility in moving between the different skills when applying them. The instructivist group tended to see things only that had been explicitly taught, as the goals were specified by the teacher and success was determined by the teacher. As a consequence, students often operate mindlessly in this type of environment, simply following rules without any critical evaluation, and hence without a clear understanding of the reason for the rules (Honebein et al., 1993).

This example illustrates that understanding needs to be independent of the specific examples used. For example, the bespoke teaching software looked at investigations specific to certain functions, but the newly acquired conceptual structures could be applied to *any* function. It is through the learning of concepts separate from the immediate and the concrete that cognitive structures are built (Vygotsky, 1962).

5. Discussion

This paper has produced evidence of some positive and practical findings for the benefits of a constructivist approach to teaching with technology and the use of visualisation, and there is some evidence that a constructivist approach to learning can broaden a student's skills base. However, as a result of this and other experiments, important questions have surfaced that require further research:

- Can any **generic** conclusions be derived?
  - ➢ Are the outcomes limited to certain age groups? e.g. is an instructivist approach necessary before a constructivist approach takes over?
  - ➢ Are the outcomes limited to particular subject domains? e.g. will a constructivist approach to teaching develop better ideas of formal proof?

- Do traditional **assessment** methods favour an instructivist approach and hence limit constructivist activities?
  - ➢ Which methods of assessment effectively document genuine learning?
  - ➢ Should technology be used in examinations to measure abilities in conceptual understanding?

- How do we take into account psychological and motivational factors when using a constructivist approach?
  - ➢ Is learning via a constructivist approach more 'fun'?, and does it lead to increased motivation for *all* students?

REFERENCES

Dubinsky, E. (1991). "Reflective Abstraction in Advanced Mathematical Thinking", *In Advanced Mathematical Thinking*, ed. D.O. Tall, Kluwer, pp 95-126.

Fosnot, C.T. (1996). "Constructivism: A Psychological Theory of Learning", *Constructivism: Theory, Perspectives, and Practice, Chapter 2*, ed. C.T. Fosnot, Teachers College, Columbia University.

Habre, S. (2001). "Visualization Enhanced by Technology in the Learning of Multivariate Calculus", *The International Journal of Computer Algebra in Mathematics Education*, Vol. 8, No. 2, pp 115-129.

Honebein, P.C., Duffy, T.M., Fishman, B.J. (1993). "Constructivism and the Design of Learning Environments: Context and Authentic Activities for Learning", *Designing Environments for Constructive Learning*, Springer-Verlag Berlin, pp 87-108.

Lerman, S. (1996a). "Intersubjectivity in Mathematics Learning: A Challenge to the Radical Constructivist Paradigm?", *Journal for Research in Mathematics Education*, Vol. 27, No. 2, pp133-150.

Lerman, S. (1996b). "Some Problems in Research on Mathematics Teaching and Learning from a Socio-Cultural Approach", *Proceedings of the Day Conference of the British Society for Research into Learning Mathematics*, Institute of Education, London, November, pp 33-36.

Malabar, I. (2002). "The Use of Computer Technology to Enhance Visualisation Skills in Mathematics Education", *Ph.D. thesis in preparation*.

Nelson, R.B. (2000). "Proofs without words II: more exercises in visual thinking", *Mathematical Association of America*.

O'Reilly, D., Pratt, D., Winbourne, P. (1997). "Constructive and Instructive Representation", *Journal of Information Technology for Teacher Education*, Vol. 6, No. 1, pp 73-92.

Rowlands, S., Graham, T., Berry, J. (1997). "The Socratic Method of Strategic Questioning to Facilitate the Construction of the Target-Concept within the Students' Zone of Proximal Development", *Proceedings of the Day Conference of the British Society for Research into Learning Mathematics*, Bristol, November, pp 50-55.

Scardamalia, M., Bereiter, C. (1991). "Higher levels of agency for children in knowledge building: A challenge for the design of new knowledge media", *The Journal of the Learning Sciences*, Vol. 1, No. 1, pp 37-68.

Schifter, D. (1996). "A Constructivist Perspective on Teaching and Learning Mathematics", *Constructivism: Theory, Perspectives, and Practice, Chapter 5*, ed. C.T. Fosnot, Teachers College, Columbia University.

Smith, G., Wood, L., Coupland, M., Stephenson, B., Crawford, K., Ball, G. (1996). "Constructing Mathematical Examinations to Assess a Range of Knowledge and Skills", *International Journal of Mathematics Education in Science and Technology*, Vol. 27, No. 1, pp 65-77.

Tall, D.O. (2000). "Technology and Versatile Thinking in Mathematical Development", *In Michael O.J. Thomas (Ed.), Proceedings of TIME 2000*, Auckland, New Zealand, pp 33-50.

Tall, D.O. (2001). "Cognitive Development in Advanced Mathematics Using Technology", *Mathematics Education Research Journal*, Vol. 12, No. 3, pp 196-218.

von Glasersfeld, E. (1995). "Radical Constructivism: A Way of Knowing and Learning", The Falmer Press, London.

von Glasersfeld, E. (1996). "Aspects of Constructivism", *(Introduction to) Constructivism: Theory, Perspectives, and Practice,* ed. C.T. Fosnot, Teachers College, Columbia University.

Vygotsky, L. (1962). "Thought and Language", (First published 1934), Massachusetts Institute of Technology Press, Cambridge, Massachusetts.

Wheatley, G.H., Brown, D. (1994). "The Construction and Re-presentation of Images in Mathematical Activity", *Proceedings of PME*, Vol.18, Pt. 1, pp 81.