OBJECT TEACHING OF GRAPH ALGORITHMS

Eva MILKOVÁ
University of Hradec Králové
Faculty of information and management
Víta Nejedlého 573, Hradec Králové 500 03
Czech Republic
e-mail:eva.milkova@uhk.cz
url: http://lide.vsp.cz/~fumilke1

ABSTRACT

The base concepts and theorems of the Graph Theory and related Graph Algorithms are taught in the context of the subject Discrete Mathematics at our university, the University of Hradec Králové, Czech Republic. The Graph Theory is a wonderful, practical discipline, often as little as puzzles. A good understanding of graph algorithms develops logical thinking in students, therefore we focus properly on these problems. When we explain algorithms we put emphasis on mutual relations between individual algorithms. When students make sense of the concepts tree and spanning tree we start to speak about the well-known optimisation problem, the minimum spanning tree problem. We show them three classical algorithms (Borůvka's, Jarník's and Kruskal's algorithms) and also for comparison one dual algorithm (dual Kruskal's algorithm). We describe all methods as an edge colouring process. On the base of Jarník's solution of the mentioned problem we continue our lectures with descriptions of other algorithms. We show the relationship of the Jarník's method to Dijkstra's algorithm for finding the shortest path. We speak about Breadth-First Search and Depth-First Search algorithms based on Jarník's algorithm. And on the base of these searching algorithms we discuss several other graph algorithms.

In this article we will present a theoretical background and at the conference we will introduce visual presentations in the Delphi environment, which we use in the lectures as a very nice complement for illustration of all above-mentioned algorithms.

KEYWORDS: Graph Theory, Graph Algorithms, Minimum Spanning Tree Problem, Dijkstra's algorithm for finding the shortest path, Breadth-First Search and Depth-First Search.

Introduction

The subject Discrete Mathematics taught at our university gives quite a large space for explanations of several graph algorithms. We are sure that a good understanding of graph algorithms greatly develops logical thinking of students. It is evident that our students first must be familiar with base concepts and theorems of Graph Theory. Then we can start to introduce interesting and practical algorithms on graphs.

We always explain individual algorithms with the help of mutual relations among them. On the one hand there are many algorithms solving one problem and on the other hand we can get algorithms solving other problems using various modifications of only one algorithm. For students it is easier to understand the problems and to remember the main idea of algorithms when they can see mutual relationships among described algorithms.

In the article, as in our lectures, the well-known Minimum Spanning Tree Problem will be first discussed more deeply. We meet our students with the history of the problem, introduce three classical algorithms and explain the basic differences among them. We also mention at least one dual algorithm solving the problem. Then on the base of Jarník's solution of the minimum spanning tree problem we illustrate the known Dijkstra's algorithm for finding the shortest path and Breadth-First-Search and Depth-First-Search algorithms. All methods are described as an edge colouring process. We are sure that exactly this way of description greatly increases understanding of algorithms.

Using description of algorithms as an edge colouring process enables object teaching not only with chalk and blackboard but also with help of new modern technology. We are very happy that our faculty has good, modern equipment and that there are several students there who are able and enthusiastically willing to prepare nice multimedia programs. At the conference we intend to introduce a multimedia program prepared in the Borland Delphi environment where several graph algorithms are visualized.

The minimum spanning tree problem

Some historical facts

Too important mathematicians, Vojtěch Jarník and Otakar Borůvka, were born in Czech Republic about one hundred years ago.

At the end of 1925 Otakar Borůvka met Jindřich Saxel, an employee of West Moravia power-stations, (Moravia is part of Czech Republic). It was during the electrification of south and west Moravia and Borůvka was asked for help in solving the problem Saxel was just working on. The challenge was to how and through which places to design the connection of several tens of municipalities in Moravia region so that the solution was as short and consequently as low-cost as possible. Otakar Borůvka not only correctly stated this problem but also solved it. His technical solution is mentioned in the article *Příspěvek k řešení otázky ekonomické stavby elektrovodních sítí* (Contribution to the solution of a problem of economic construction of electricity power networks) [1] and mathematical background he gave in the article *O jistém problému minimálním* (On a minimum problem) [2].

There did not exist suitable mathematical terminology in this area of mathematics at that time and thus the proof of the correctness of the solution was rather complicated. Vojtěch Jarník, another Czech mathematician, was aware of the complexity and importance of this problem. He wrote the article named *O jistém problému minimálním* with subtitle (*From the letter to Mr.*

Boruvka) [3]. In this article Jarník offered another and easier method of creating the demanded construction.

Both Czech mathematicians preceded their fellow mathematicians by a quarter of a century. The enormous interest about this problem, which is considered to be one of the best-known optimisation problems, broke out with unusual vigour again in after 1950 and that time was connected with the application of computers. That time Borůvka's and Jarník's method was discovered independently several times more.

The third solution of the problem different from the previous ones invented Joseph B. Kruskal in 1956 in his work *On the shortest spanning tree of a graph and the travelling salesman problem* [4]. The following fragment from the letter of J. B. Kruskal brings near the situation related to the birth of this problem [5]:

"It happened at Princeton, in old Fine Hall, just outside the tea-room. I don't remember when, but it was probably a few months after June 1954. Someone handed me two pages of very flimsy paper stapled together. He told me it was "floating around the math department".

Two pages were typewritten, carbon copy, and in German. They plunged right in to mathematics, and described a result about graphs, a subject which appealed to me. I didn't understand it very well at first reading, just got the general idea. I never found out who did the typing or why.

At the end, the document described itself as the German-language abstract of a 1926 paper by Otakar Borůvka.

The abstract described a method for constructing the shortest spanning subtree of a graph whose edges have known lengths, and from this method trivially derived the corollary that the shortest spanning tree is unique if no two of the lengths are equal. For me, and it appears for almost everyone else, the interest of the paper was the method of construction, not the corollary.

In one way, the method of construction was very elegant. In another way, however, it was unnecessarily complicated. A goal, which has always been important to me, is to find simpler ways to describe complicated ideas, and that is all I tried to do here. I simplified the construction down to its essence, but it seems to me that the idea of Professor Borůvka's method is still in my version.

After reaching this simplification, I started wondering whether it was worth publication. Fortunately someone advised me to go ahead, and many years passed before another of my publications became as well-known as this simple one."

Also Kruskal's algorithm has been discovered independently several times. The survey of the works devoted to the minimum spanning tree problem until 1985 is given in the article by R. L. Graham a P. Hell: *On the History of the Minimum Spanning Tree Problem* [6] and this historical paper is followed up in articles [7], [8], [9].

In spite of the fact that the minimum spanning tree problem was solved it remained in the centre of attention of many specialists. Their effort has been to invent the quickest and most sophisticated algorithm of the Minimum Spanning Tree Problem not only for common graphs but also for special classes of graphs or solving problems of gaining minimum spanning tree that satisfies additional conditions.

Three classical solutions of the Minimum Spanning Tree Problem

Now let us look at the modern formulation of the problem and modern description of all above-mentioned best-known solutions (Borůvka's, Jarník's and Kruskal's algorithms).

The Minimum Spanning Tree Problem

Given a connected graph $G = (V, E)$ having n vertices and m edges. For each edge e let $w(e)$ be a real weight of the edge e. Our task is to find a minimum spanning tree of the graph G.

In Borůvka's algorithm we will in addition presume that any two different edges have different weights. This condition does not restrict the universality of the problem (for example we can list all edges and in the case that two edges are equal weights the first on our list we consider as the bigger one).

Borůvka's algorithm

Initially all edges of the graph G are uncoloured and let each vertex of the graph G be a blue tree (we suppose a blue forest which consists of n blue trees).

Repeat the following colouring step until there is only one blue tree, the minimum spanning tree:

COLORING STEP: For each blue tree T select the minimum-weight uncoloured edge incident to T (i.e. edge having one vertex in T and the other not). Then colour blue all selected edges.

Jarník's algorithm

Initially all edges of the graph G are uncoloured. Choose any single vertex and suppose it to be a blue tree.

At each of (n - 1) steps colour blue the minimum-weight uncoloured edge having one vertex in the blue tree and the other not. (In case, there are more such edges, choose any of them.)

The algorithm finishes by gaining a blue spanning tree, the minimum spanning tree of the graph G.

Kruskal's algorithm

Initially all edges of the graph G are uncoloured. Order the edges in non-decreasing order by weight. Let each vertex of the graph G be a blue tree.

At each of m steps decide about colouring exactly one edge if it is coloured by blue colour or not. The edges are examined in a sequence defined by above-mentioned ordering. The chosen edge is coloured blue if and only if it doesn't form a circle with the other blue edges (i.e. in case that both vertices do not belong to the same blue tree).

The algorithm is finished when (n-1) edges are coloured blue. Blue edges form a minimum spanning tree of the graph G.

If we consider the weight of edge as its length then ***the basic difference between these three algorithms can be characterized as follows***:

Kruskal's algorithm connects the two nearest blue trees in one blue tree at each step in which one edge is coloured blue.

Jarník's algorithm at each step spreads the only blue tree, which contains the initial vertex by the nearest vertex.

In Borůvka's algorithm at each step the union of all the blue trees being the nearest one another is performed.

Students know that each spanning tree in a connected graph with n vertices can be found not only by including (n-1) edges that don't form a circle but also in the dual way, it means by consecutive removing edges from circles until there is no circle in a graph. Thus we introduce to our students to some dual algorithms for finding a minimum spanning tree too, as e.g. Kruskal's dual algorithm.

Kruskal's dual algorithm

Initially all edges of the graph G are uncoloured.

Order the edges in non-increasing order by weight. Let each vertex of the graph *G* be a blue tree.

At each of the *m* steps decide about colouring exactly one edge if it is coloured by red colour or not. The edges are examined according the above-mentioned order. The edge will be coloured red if and only if the edge belongs to some circle, which does not have red coloured edge.

The algorithm is finished when $(m\text{-}n+1)$ edges are coloured red. Remaining $(n\text{-}1)$ edges form a minimum spanning tree of the graph *G*.

Modifications of Jarník's Algorithm

From Jarník to Dijkstra

Given the graph *G* (figure 1).

Jarník's algorithm for gaining the minimum spanning tree supposing vertex *a* to be a blue tree can be illustrated as it is shown on figure 2. By each vertex there is a window with 6 parts corresponding to steps of algorithms. At each of 6 steps we write into the corresponding part of all windows, which belong to the vertex that doesn't lie in the blue tree, *the actual information describing the nearest distance between the vertex and the blue tree* (the sign ∞ means that the vertex isn't connected to the blue tree in the given step). Among all these vertices we find the nearest one and we spread the blue tree by this nearest vertex (we colour blue the corresponding edge). Finally we get the minimum spanning tree containing 6 blue edges.

In the similar way we can illustrate the known Dijkstra's algorithm for finding the shortest path from the given vertex *u* to the other vertices in a connected graph with non-negative weights of edges. The only difference is that in each step we write *the actual information describing the nearest distance between the vertex and the initial vertex u* (figure 3).

From Jarník to Breadth-First Search and Depth-First Search

Given a connected graph with all edges having the same weight (e.g. weight $w(e) = 1$ for each edge *e*) and let us trace the Jarník's algorithm for gaining the minimum spanning tree on this graph. We see that at each step *an arbitrary edge*, having one vertex in the blue tree and the other not, is coloured blue. Jarník's algorithm works on the given graph in the same way as on a graph without weighted edges. A consecutive adding of vertices (at each step we spread the blue tree by one vertex, the end-vertex of an exactly blue coloured edge) we can understand as a consecutive search of them. To get either Breadth-First Search or Depth-First Search algorithm (for consecutive searching of all vertices) we simply modify Jarník's algorithm in the following way.

Breadth-First Search: At each step we choose from the uncoloured edges, having one vertex in the blue tree and the other not, such *an edge having the end-vertex being added to the blue tree as the first* of all in blue tree lied end-vertices belonging to the mentioned uncoloured edges.

Depth-First Search: At each step we choose from the uncoloured edges, having one vertex in the blue tree and the other not, such *an edge having the end-vertex being added to the blue tree as the last* of all in blue tree lied end-vertices belonging to the mentioned uncoloured edges.

Conclusion

In the article we have outlined one possible way of object teaching concerning graph algorithms. It is always very useful to present more solutions (if they exist) of the given problem to be more thoroughly understood. Moreover, it is also useful to use a modification of the already known algorithm by explanation of a solution of the given problem.

In conclusion let us mention that the description of all explained methods as an edge colouring process is really very welcomed and favoured by our students.

REFERENCES

[1] Borůvka, O.: Příspěvek k řešení otázky ekonomické stavby elektrovodních sítí. Elektrotechnický obzor,15, 1926, 153-154.

[2] Borůvka, O.: O jistém problému minimálním. Práce Mor. Přírodověd. Spol v Brně, 3, 1926, 37-58.

[3] Jarník, V.: O jistém problému minimálním. Práce Mor. Přírodověd. Spol. v Brně, 6, 1930, 57- 63.

[4] Kruskal, J. B.: On the shortest spanning tree of a graph and the travelling salesman problem. Proc. Amer. Math. Soc., 7, 1956, 48-50.

[5] Kruskal, J.B.: A reminiscence about shortest spanning subtrees. Archivum Mathematicum Brno, 33, 1997, 13-14.

[6] Graham, R. L., Hell, P.: On the History of the Minimum Spanning Tree Problem. Annals of the History of Computing 7, 1, 1985, 43-57.

[7] Nešetřil, J.: A few remarks on the history of MST-Problem. Archivum Mathematicum Brno, 33, 1997, 15-22.

[8] Nešetřil, J., Miková, E., Nešetřilová, H.: Otakar Borůvka on minimum spanning tree problem (Translation of both the 1926 papers, comments, history), Discrete Mathematics 233 (2001), pp. 3-36.

[9] Milková, E.: Problém minimální kostry. Gaudeamus, Hradec Králové, 2001.

[10] Vrbík, V., Michalík, P.: Examples of the utilization of the program EWB simulation in teaching the subjects Computer techniques and Control. Proceedings of UWB, Vol 4/2000, pp.191 - 195.

[11] Bauerová, M., Turčani, M.: Tvorba multimediálnej didaktickej pomocky pre podporu výučby predmetu Molekularna genetika. Technológia vzdelávania, Slovdidac Nitra 2000, pp. 5-8.

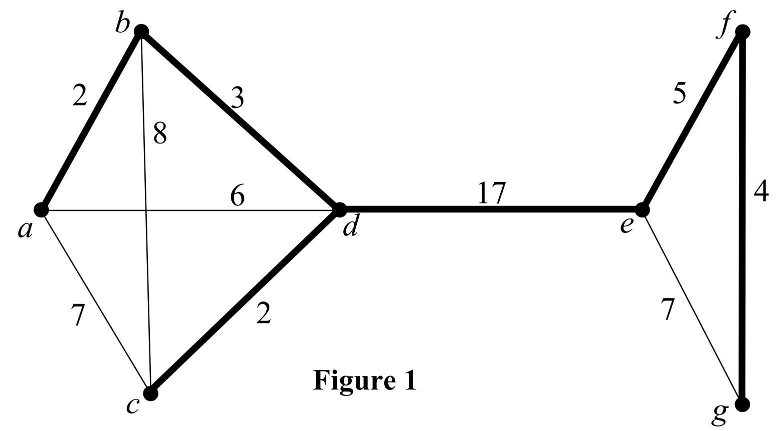

Figure 1

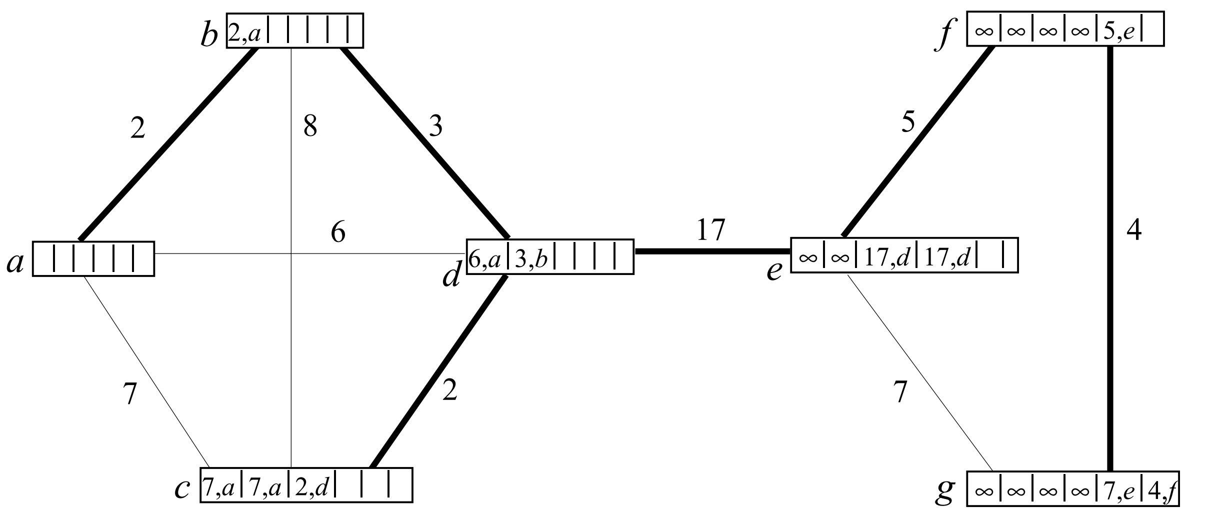

Figure 2

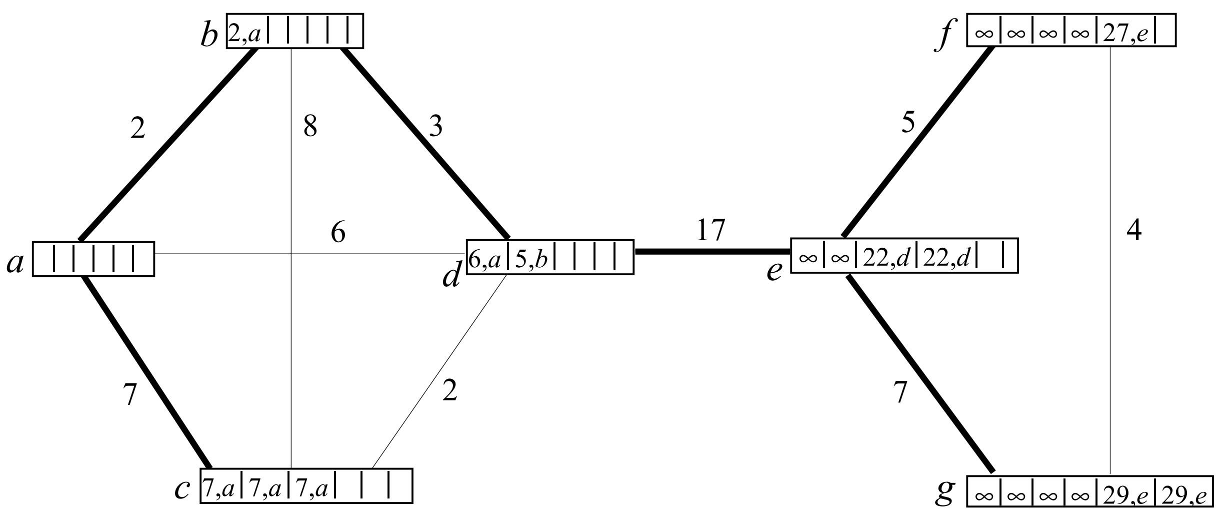

Figure 3