THE ROLE OF THE TEACHER IN A TECHNOLOGY BASED CALCULUS CLASS

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ABSTRACT

Although the focus of this paper is on the method I use in teaching a multivariable calculus course, based on Calculus and Mathematica (C&M) of Davis, Porta and Uhl, at the University of South Carolina Aiken, the issues I discuss and the teaching techniques I describe may be relevant to many other technology based math courses. In particular, I discuss how the objectives of the course are set and why I favor C&M over the traditional approach, how I try to alleviate problems related to student weaknesses in computational skills, lack of interest or commitment, and lack of familiarity with Mathematica, and how I evaluate the success of the course.

The method I use differs substantially from the method suggested by the authors of C&M. In the presentation I will discuss, in some detail, the following four basic elements of my approach. (a) Introduction. Each chapter is introduced by a lecture that is very close to the C&M approach, using both hand calculations and Mathematica code. The students also receive a handout containing a complete list of the new concepts and some brief explanations. (b) Tutoring. The students are required to complete, at least part of each homework assignment, in class, usually in teams of two students. This allows the students to discuss problems among themselves and to ask for the instructor's help when needed. (c) Feedback. The students are required to study the answers to the homework problems they miss. (d) Constructive testing. The tests are used as diagnostic tools of student deficiencies. Students who fail a test are required to study further and retest.

This method is quite demanding on the instructor's time. It seems to be quite beneficial for the students, however, because it helps reduce considerably their frustration, excite their interest, and improve their conceptual understanding of calculus.

1. Introduction

In 1994, our department, the Department of Mathematical Sciences at the University of South Carolina Aiken, at the recommendation of Dr. Robert Phillips, then a senior member of our faculty, adopted the use of *Calculus and Mathematica* (C&M) (Davis, Porta and Uhl 1994) in all of our calculus classes. Very quickly, however, this approach stirred up an intense controversy among our faculty that continues to this day. While some of us are extremely pleased with C&M, others dislike either the approach C&M takes or the way it is received by their students. Some have abandoned C&M and are using other texts.

Six years ago I started using C&M in my vector calculus class. Compared to the traditional approach, I find that C&M can be much more beneficial to the students. Its greatest strength is that C&M can help the students develop an intuitive understanding of vector fields, of the operators grad, div and curl, of line and surface integrals, and of the meaning and usefulness of Green's, the divergence and Stokes's theorems. In this respect, the traditional approach is not very successful. Though the traditional approach is supposed to be a rather formal approach, based at least to some extent on theorem proving, in reality the emphasis is almost exclusively on symbolic manipulations. Most exercises are on such manipulations and few, if any, require a deeper conceptual understanding. Furthermore, many important topics, such as line integrals, Green's theorem and surface integrals, are given such a brief treatment that the students are unable to develop any useful understanding of them, while quite often the divergence and Stokes's theorems are completely left out.

I believe that the intuitive understanding of the concepts and theorems of vector calculus that the C&M students can develop will help them considerably in their future studies. Students majoring in mathematics will be better prepared to understand and appreciate the rigorous and complete theorem proving development of the theory when they are introduced to it. Science or engineering majors will be able to develop early a more complete understanding of abstract physical theories. I have the rather unusual opportunity to compare the results produced by C&M and by the traditional approach, because I also teach the calculus-based physics classes. As I discuss elsewhere (Kapranidis 1998) I find that, compared to the students of the traditional calculus courses, the C&M students are considerably better prepared to understand the theory of physics and in particular the theory of electricity and magnetism. For these reasons, the main objective I set for my vector calculus class is to help the students develop an intuitive understanding of calculus.

Despite its potential, the C&M approach does not always produce the desired results. Soon after I started using C&M in my class, I realized that the teaching method recommended by the authors of C&M did not work well for many of my students. The authors recommend that no introductory lectures should be given and that the students should not read the book before they experience a new concept on the computer screen using the C&M electronic text. This approach works well for some students, but many other students find the way the C&M text develops new calculus concepts very difficult to follow.

Certainly, part of this problem can be attributed to the students' lack of interest or discipline, weaknesses in their math background, unfamiliarity with Mathematica and the limited amount of time they allow for their study. It is not, however, an exclusive problem of the students who are unprepared, uninterested or have poor study habits. It also affects many of the rather typical students of the American colleges, who have adequate preparation and ability, and make a genuine effort to learn.

In my opinion, the difficulties experienced by these students are strongly related to the nature of calculus and of the learning process itself. I believe that these problems seem to be more noticeable when the C&M approach is used, because the emphasis of this approach is on the concepts of calculus rather than on symbolic manipulations that are easier to master. Further, I believe that, to alleviate these problems and still take advantage of the C&M approach, the teacher must assume a much more active role in the students' learning process than that required of a teacher in a traditional calculus class.

After experimenting with different ideas, I developed a special teaching method that I use in my vector calculus class. The basic text I use is C&M. I also use some support materials that I have developed. My approach has substantial differences from the approach suggested by the authors of C&M. I feel, however, that it takes care of some of the problems encountered by many teachers who use C&M in their calculus classes. This approach is not particular to vector calculus, and can be adapted and used in any technology based math class in which the emphasis is on mathematical concepts.

2. The Teaching Method

All of our calculus classes meet six hours a week, though they are four-credit-hour classes. In this respect, calculus classes are similar to our typical classes that have a lab component, such as physics or chemistry classes. The Mathematica based calculus classes meet in a classroom currently equipped with 20 Windows based PC's in which Mathematica 4.01 is installed. One of these computers is equipped with an LCD projector.

The course is organized in the following way. The material is divided into four units. Specifically, the first three units consist of three chapters each, while the last unit consists of two chapters of the C&M text. We complete one chapter per week. For each chapter, the students have to do a homework assignment consisting of several problems. The assignments are collected and graded promptly. At the end of each unit, the students take a test. At the end of the semester, the students also take a comprehensive final exam. The final grades are calculated as follows: Homework: 40%, four tests: 40%, final exam: 20%.

My teaching method is characterized by four basic elements for which I use the terms (a) introduction, (b) tutoring, (c) feedback, and (d) constructive testing.

a. Introduction. This is probably the main element that makes my approach substantially different than the approach suggested by the authors of C&M. Contrary to their recommendation, for each new chapter I give a thorough introductory lecture. These lectures are precisely structured. In the first part of the lecture the students are given a general orientation. That is, I define the objectives of the chapter, the particular context of the subject, and the connections of the new concepts to the previous knowledge of the students. In the second part, the new concepts are introduced. Though I always stay very close to the C&M text in terms of the subject matter, I introduce the new concepts in a way that is completely independent of Mathematica, and then I work my way down to the particulars of the C&M text. Finally, in the third part of the lecture, the actual C&M text is projected onto a screen and we discuss and analyze it in detail. During this time, the C&M approach is compared and contrasted with the previous introduction of the same concepts. Along the way, we also discuss the Mathematica code and we often use the power of Mathematica to explore cases and possibilities that are not covered in the C&M text or to explore in depth questions that the students may come up with. Each C&M chapter is divided in three sections named "Basics", "Tutorials" and "Give It a Try". In the introductory lecture for each

chapter I try to cover the “Basics” as completely as possible, and also some selected parts of the “Tutorials”.

To help the students focus better, both during the lecture and later in their study, for every new chapter I prepare a handout with a summary of the new concepts and the definitions, theorems and formulae the students are expected to learn.

b. Tutoring. After the introductory lecture the students, usually in groups of two, are required to first study the “Basics” and “Tutorials” parts of the chapter, and then work on their homework assignment, which consists almost exclusively of problems from the “Give It a Try” section of the C&M text. Occasionally the assignment may include problems requiring pencil and paper symbolic manipulations. On the average, they have about one hour of class time for the “Basics” and the “Tutorials” and two hours for the homework assignment. Though this is not enough time to complete the assignment, it gives them adequate opportunity to ask questions, receive personal tutoring when it is needed, and at least make sure that they have a good idea on how to approach the problems.

c. Feedback. Each group must turn in one completed homework assignment per chapter for grading. The assignments are graded promptly and returned to the groups. At this time the solutions to the problems in the assignment are provided, and the groups are required to study the solutions, especially of any problems they missed. The students are also advised, on a personal basis, on what deficiencies their errors may indicate and further studying is recommended. Occasionally, the groups are allowed to resubmit corrected solutions and recover some of the missed points.

d. Constructive testing. Testing is used not only as a means for grading the students but, most importantly, as yet another opportunity for them to learn. The whole testing process is organized in the following way.

Soon after a unit is completed, we have a review session to help the students prepare for the test. The students are given a new handout containing a summary of the concepts in that unit and examples of problems similar (but not identical) to the problems on the test. The tests are pencil and paper tests and do not involve the use of Mathematica.

Each test is graded right away and each student is personally given an evaluation of their performance. When deficiencies are detected, the students are given one week to further study and retest. Typically, if they score less than 70% they must retake a complete test, otherwise they have to take only a partial test, that is focused on the types of questions they had difficulties with on the first test.

At the end of the semester, we also have a two hour session in which we review all of the material in the course, in preparation for the final exam. The final exam is comprehensive. No retesting is allowed for the final exam.

3. Discussion and Conclusions

The method I use in my vector calculus class is the result of my efforts to address some serious problems that quickly became apparent when I first started using C&M in my class. All these problems seemed to stem out of the fact that the students had great difficulty in understanding the new concepts by studying the C&M text on their own. Feeling the pressure from the fast approaching homework deadlines, they would start working on the homework problems before they had a clear sense of what exactly they were supposed to do. Their way of dealing with this problem was to look over the examples in the text and to try to imitate and adapt them, by trial and

error, so as to produce “solutions” to the homework problems. These “solutions” were most often totally meaningless, while their effort was routinely frustrated by errors in their Mathematica code.

Using an introductory lecture for each chapter seems to be an effective way to address this problem. The benefits of the introductory lectures are two-fold. First, the students get quickly oriented and focused on the subject. This increases the rate at which they learn and reduces substantially their studying time. Second, by using the actual C&M text in the lecture I have the opportunity to decipher the Mathematica code and help the students learn how to use the software correctly. This works quite well, though there are some students who can never get completely over some previous bad experiences with Mathematica. In general, however, as the semester progresses the ability of the students to use Mathematica correctly rapidly improves and soon Mathematica is not a problem any more. This is also true for students who have no previous exposure to Mathematica.

Students are not expected to understand the new concepts completely by the end of an introductory lecture. What is important however is that the students understand enough so that they feel well oriented and able to continue to study on their own.

The most intense learning happens during the time when the students working in groups study the C&M text and do the homework problems. For this reason, having at least part of these activities take place in class gives me the opportunity to provide some individual tutoring or to give to the whole class general instructions on how to approach some of the more challenging problems. I also have the opportunity to assess the level of understanding of the students by discussing with them the approaches they take in solving the homework problems. This allows me to develop a better sense of the strengths and weaknesses of my students, gives me the opportunity to suggest to them ways by which they may overcome any deficiencies they have, and helps my overall teaching to become more focused and effective. At the same time, the personal attention the students receive further decreases their level of anxiety and frustration and improves their learning.

Another very important element of my approach is homework feedback. I find this element to be especially important when the C&M approach is used, because the solutions of the C&M homework problems often require verbal explanations, descriptions or justifications rather than predominantly symbolic manipulations. Students are not accustomed to this type of problems, and initially the overall performance of the class is not very good. Requiring the students to study the correct solutions to the homework problems, especially those that they miss, helps them improve their understanding of the material, their ability to make logically consistent arguments, and the way in which they express themselves. It is very important that this feedback takes place as soon as possible, before the students forget their own work. Thus, the homework should be graded promptly and returned to the students as soon as possible.

The effect of homework feedback is quite remarkable. The class average of the first homework assignment is usually low, in the 70-75% range. Typically, by the forth assignment the class performance becomes quite good and remains high for the rest of the semester. During this time class average for the homework is in general above 90%, while quite a few groups achieve 100%.

Finally, testing is used in a constructive way in an effort to achieve three important goals. The first goal is to give the opportunity to the students not only to review the material of a substantial part of the course but, most importantly, to see how the elements we study in different chapters fit together to form a larger picture. This is accomplished in the two hour review session that we have before each of the four tests.

The second goal is to use tests as diagnostic tools of the deficiencies that the students may have. For this reason, the questions in a test should be carefully chosen and, if possible, they must cover all important topics studied in a unit.

The third goal is to use the tests as an incentive for the students to further study and improve their understanding in areas where they were found to have some problems. To achieve this, the students who miss some questions in a test are given the opportunity to further study and retest, for a substantial fraction of the points they missed the first time. This approach has another important result. It reduces test anxiety because the students know that they will have a chance to improve their initial result.

The method I use in my vector calculus class requires from the teacher considerable amounts of time outside the classroom for preparing handouts, detailed homework keys, multiple tests and also for retesting. Furthermore, teaching is more intense, especially during the time when personal tutoring takes place. The class must also be very well organized. If the time available for a unit is exceeded, other units will not be allowed enough time for all the required activities to take place. However, when the schedule of the class is closely followed, there is adequate time for all the units and for reviewing and testing. If the class is too large, having a well qualified teaching assistant during the tutoring phase may be necessary. Our department provides graders for all calculus classes. This is very useful because it makes it possible to have the homework assignments graded promptly. I choose to grade the tests personally, however, because this allows me to develop a better feeling of the level of class performance and helps me adjust my teaching.

Despite the high demands that my approach makes on the teacher's time, I believe it is worthwhile because it reduces considerably the level of frustration of the students, and improves their learning by making it possible for them to take fuller advantage of the C&M approach. For the last three years that I have used my method in the way I describe here, the drop rate because of problems that students had either with the C&M approach or Mathematica has been practically reduced to zero. When I first started using C&M, about 20% of the students who started the class would either drop it or fail it. Some students still fail the class. These are always very weak students who are certainly not prepared to take vector calculus in any format. There is also a percentage of the students, probably as high as 10%, who manage to pass the class though their skills are not as high as I would like them to be. I find this somewhat disturbing, but it is by no means a problem unique to this class. The percentage of the students who are very successful with this method and make an A in the class typically is in the range of 30-40%, while I believe that all the students develop a deeper understanding of calculus concepts compared to the students in the traditional classes.

I would certainly like to stress here that C&M deserves full credit for the teaching methodology I use in my class. My approach simply provides the support many students need to enable themselves to follow C&M and benefit from it. Though I have developed this method specifically for my vector calculus class and I have used it exclusively with C&M as the basic text, I feel that it could be applicable in other classes. I believe though, that it is not suitable for traditional classes. Without the C&M electronic text the class time could not be used as efficiently and there would not be time for all the other activities to take place in class. Thus, the method can only be used in technology based classes for which some text and software analogous to C&M are available.

REFERENCES

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