# COLLEGE ALGEBRA IN CONTEXT:
## Redefining the College Algebra Experience


**Ronald J. HARSHBARGER**, Ph.D.
University of South Carolina Beaufort
1 College Center
Hilton Head SC 29928
ronharsh@hargray.com

**Lisa S. YOCCO**
Georgia Southern University
P.O. Box 8093
Statesboro, GA 30460
lisay@gsvms2.cc.gasou.edu


## ABSTRACT


We have developed a redefined college algebra course, which uses an informal approach, is application driven, is technology-based, and uses real data problems to motivate the skills and concepts of the course. Each major topic contains real data examples and problems, and extended application projects that can be solved by students working collaboratively. Students can take advantage of available technology to solve applied problems that are drawn from real life situations. The students use technology, including graphing calculators, Excel, and Derive, to observe patterns and reach conclusions inductively, to check answers of solved problems, to study function types, and to create models for use in the solution of problems.

The course provides the skills and concepts of college algebra in a setting that includes applications from business, economics, biology, and the social sciences. The course was designed to provide the required college algebra skills for students in the Business major, in the Hotel, Restaurant, Tourism Administration major, and for majors in the biological, marine, and social sciences. The real life applications included are the result of collaborations with faculty in Business, in Hotel, Restaurant, Tourism Administration, and in Biology Departments in three Universities.

Each mathematical topic of the course is introduced informally with a motivational example that presents a real life setting for that topic. The problem in this example is then solved as the skills needed for its solution are being developed or after the necessary skills have been developed. Some applications provide the models for the data and have students solve related problems, while others require students to develop the models before solving the problems. For some topics, students work in small groups to solve extended application problems and to provide a written report on the results and implications of their study. For other topics, students find appropriate real data in the literature or on the internet, develop a model that fits that data, and use the model to solve problems. Most of the examples and exercises in the course are applied problems.

# Introduction

Most of the students taking College Algebra are not math or science majors, and although they are quite intelligent, many are just not interested in mathematics. If we give them compelling applications, they will see that there is some reason for the mathematics to exist, they will be more interested, and they might even come to like math in this context. These people are as likely to be leaders in our communities and country as are people with science and engineering degrees, and so they need the reasoning skills and problem solving skills just as much as science majors do.

Those of us teaching college algebra realize that students taking this type of course will not likely become math majors or enter careers that are heavily dependent on mathematics. However, they will likely have a career that requires reading for comprehension, problem solving skills, and the ability to analyze and interpret. Thus we emphasize real problems rather than mathematical theory for its own sake.

As mathematicians, we sometimes think that the value we should impart is the logic involved in a rigid outline with proofs of theorems. But for students whose future is not in a mathematical field, there is a much more interesting, challenging and useful approach to mathematics. Thus we sought to offer a wide range of applications, to keep the interest of all students and show that algebra is useful and necessary to solve real problems. Thus our mission was to design a course where algebra skills were not the goals of the course but rather the tools for the attainment of more far-reaching goals.

# College algebra in context

With this mission in mind, we developed an algebra course based on real life applications from business, economics, biology, and the social sciences in a setting that connects mathematical content with the real world. The course can best be described as using a transitional approach, because it has most of the positive attributes of reform algebra without sacrificing the wide variety of algebra topics included in a traditional graphing approach to college algebra. Data analysis, modeling, and technology are woven into the course so that the approach is refreshing and interesting to the students. The course provides the algebraic skills and concepts for a core course, or for the future study of calculus, in an informal, less threatening, and more meaningful setting. The course was designed to provide the required algebra skills for students in the business and economics majors and in majors in the biological and social sciences. In fact, this course provides the algebraic background needed for success in all majors other than the physical sciences and engineering. It is designed so that students can solve meaningful applications and see how mathematics relates to their future.

We feel that keeping the skills and concepts in the context of applications gives the course a new perspective for students who were successful in high school algebra and for those who were not. Students respond better to this approach and see the necessity of learning algebra skills to solve these problems. The course attempts to relate algebra to everyday life with examples and exercises relating to real-life math problems. The quality and quantity of the examples and exercises is the strength of our approach to college algebra.

We have made an effort to have real applications for every algebra skill and concept introduced in the text. The goal is to provide students with a real sense of the relevance of algebra to the real world. To this end, we concentrate on real problems as opposed to contrived applications.

We have sought a good balance of skill-building and application exercises, but we think the purpose of this course is applying algebra to the real world. We use Skills Check exercises to warm students up for the exercises that follow. This gives the students a chance to get familiar with the concepts and skills and to gain confidence.

After the skills checks, the exercises are all applied (real or realistic), most with source references. We ease students into word problems by giving similar problems with equations given. We also give exercises that break problem solution into steps. Later exercises involve critical thinking, complete solution, and interpretation. Where reasonable, problems involving integer cases are used at the beginning of the exercise sets, with real data problems requiring technology later in the set.

Those skills that are prerequisite for the course are included in an Algebra Toolbox, which provides a "just-in-time" review for the chapter in which they are needed. These topics can be left for the students to review, or can be included in the day's lecture. We prefer this to teaching or ignoring prerequisite topics in a Chapter 0 or an appendix. The Toolbox for each chapter includes the intermediate topics needed for that specific chapter.

The level of our course is appropriate for students taking a terminal course or for students going on to a non-science calculus course that uses some technology. In particular, it is appropriate preparation for a business calculus course. A slightly different approach to the topics we have prepared could also be used in a modeling course. The materials we have developed could be used in a course aimed towards business preparation or one aimed towards modeling.

In some cases, we have students develop their own models from the data, after deciding which function best models the data. Models are created from real data and problems are then solved using the models. We have students consider first, second, third, and fourth differences, and constant rates of change to determine which model may be most appropriate for a set of real data. We have included a large number of problems that ask the student to interpret, analyze, and make predictions from real data models.

Mathematical concepts can be introduced informally with technology rather than with more formal methods. The students can use graphing calculators to observe patterns and to reach conclusions inductively, to check answers of solved problems, to study function types, and to solve equations graphically and numerically. Technology can be used to develop equations that model real data, and the equations can be used to reach conclusions about the data and to solve problems about the data.

We emphasize graphing calculator use, but as an aid, not as a substitute for analytical methods. We frequently solve problems with both analytical methods and technology. The calculator or other graphing utility is integrated seamlessly into the discussion, but students are required to use analytical as well as technological approaches to problem solving. And when applicable, students are shown why the analytical method is easier than graphical or numerical methods of solving a problem. In some cases technology and analytical methods are combined to solve problems. That is, there are cases where technology is used to assist in analytical solutions, and cases where analytic methods are used to assist with graphing (for example, finding the vertex of a parabola to help set the window of a graphing utility).

Collaboration is encouraged throughout the course. Students are encouraged to work in teams just as they might in the workplace after graduation. The Extended applications are especially designed for collaboration.

A few of the examples of applications used in the course follow.

## Real Data Applications

**EXAMPLE 1: SALARIES OF U. S. COLLEGE PROFESSORS**

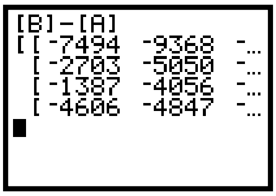
Students are asked to find data from a number of resources and to reach conclusions about salaries of male and female college faculty teaching at different levels in different types of institutions. Students were asked to discuss the relationship between male and female professors' salaries, using information from the table below.

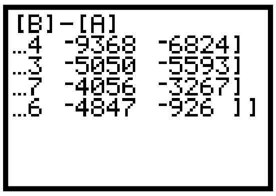
**Salaries of College Professors, 1998-99**
**Source: American Association of University Professors**

| | MEN | | | WOMEN | | |
|---|---|---|---|---|---|---|
| | Type of Institution | | | Type of Institution | | |
| **TEACHING LEVEL** | **Public** | **Private/ Independent** | **Church-related** | **Public** | **Private/ Independent** | **Church-related** |
| **Doctoral level** | | | | | | |
| Professor | $80,379 | $99,979 | $84,796 | $72,885 | $90,611 | $77,972 |
| Associate | 57,653 | 65,843 | 60,059 | 54,322 | 61,956 | 56,180 |
| Assistant | 48,647 | 57,296 | 50,009 | 45,203 | 52,521 | 46,427 |
| **Master's level** | | | | | | |
| Professor | 64,414 | 70,643 | 66,151 | 61,711 | 65,593 | 60,588 |
| Associate | 51,812 | 54,260 | 52,634 | 49,615 | 51,273 | 48,189 |
| Assistant | 42,673 | 44,511 | 42,317 | 41,189 | 43,002 | 40,312 |
| **General 4-year** | | | | | | |
| Professor | 58,432 | 68,145 | 52,945 | 57,045 | 64,089 | 49,678 |
| Associate | 48,643 | 51,044 | 43,412 | 46,808 | 49,202 | 41,791 |
| Assistant | 40,625 | 41,551 | 36,534 | 39,245 | 40,634 | 36,017 |
| **2-year** | | | | 52,461 | 40,252 | 35,496 |
| Professor | 57,067 | 45,099 | 36,422 | 44,835 | 35,513 | 34,609 |
| Associate | 48,321 | 40,515 | 36,359 | 39,561 | 34,219 | 29,774 |
| Assistant | 41,515 | 35,715 | 30,342 | | | |

The students created two matrices, containing the salaries of male and female Professors, respectively, in each type of school and at each level of instruction.

Subtracting the matrix of male professor salaries from the matrix female professor salaries shows that female salaries are lower than male salaries in every category. Looking at other comparisons gives the same result. The consistency of this shortfall led students to conclude that gender bias was present in educational institutions.

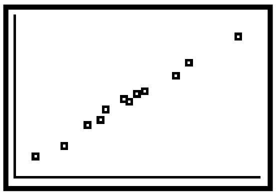
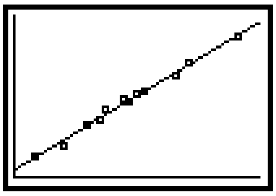





This group of students also attempted to find the relationship between salaries of the male and female professors.

The scatter plot of the data shows that there is a linear relationship between the male and female salaries, and linear regression gives the function that gives female professors' salaries as a function of male professors' salaries.

$$y = 0.8823x + 3020.51$$

Students in this class who were also taking Sociology 101 discussed gender bias in that class using the conclusions of this study.

**EXAMPLE 2: ROOM PRICE AND OCCUPANCY RATE**

Students who are majors in Hotel, Restaurant, and Tourism Administration who were taking both a Tourism course and College Algebra were asked to collect data that investigates the relationship between room pricing and occupancy rate at resort hotels on Hilton Head Island during the off-season. We knew that if occupancy was related to daily room price and if the resort hotels had accurately made a connection between room price and occupancy, they could set the room price so that occupancy would remain nearly constant in the off-season.

Students in the HRTA course collected the data in Table 1 below, which gives the room price and occupancy rate for 10 resort hotels during the off-seasons.

**TABLE 1**

| Daily Room Price | Occupancy Rate (%) | Daily Room Price | Occupancy Rate (%) |
|---|---|---|---|
| 110 | 67 | 119 | 65 |
| 120 | 47 | 79 | 50 |
| 100 | 60 | 89 | 52 |
| 120 | 40 | 69 | 75 |
| 115 | 38 | 39 | 65 |
| 75 | 55 | 39 | 45 |
| 65 | 50 | 45 | 50 |
| 70 | 52 | 62 | 47 |
| 44 | 39 | 67 | 47 |
| 38 | 21 | 36 | 27 |

As the table shows, the relationship between room price and occupancy is not as linear as might be expected; there is wide variation in the distribution, especially for rooms costing more than \$100 per day. Students in the HRTA course did not reach any useful conclusions about pricing from looking at the table and scatter plot, so they sent the data to the College Algebra class.

Observing the shape of the graph over the interval under consideration led the students to use a quadratic function to model the data. The resulting model was

$y = -0.007327x^2 + 1.2865x + 0.6871$, where $x$ is the room price and $y$ is the occupancy rate.

They found the price (value of $x$) that gives the maximum value for the occupancy rate by finding the $x$-coordinate of the vertex of this parabola.

$$x = \frac{-b}{2a} = \frac{-1.2865}{2(-0.007327)} = 87.79$$

The maximum occupancy for this group of hotels is 57%, when the room price is \$87.79 if this model is accurate for the group.

The conclusions from the college algebra course were given to the HRTA class for their consideration. They eventually raised the question of whether maximizing occupancy would necessarily maximize revenue or profit. So they suggested that students in the algebra class determine the price, if it existed, that would maximize revenue. The occupancy rate was then converted into an occupancy and multiplied by the corresponding price to get points relating room price and revenue. When this data resulted in a cubic model, it was decided that the maximum could be found graphically or that it could be found in a calculus course that many of the students were taking next semester. They graphically found that the revenue was maximized if the price was \$114.25.

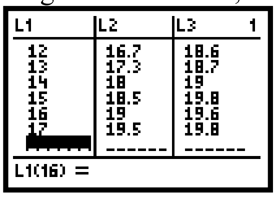
**EXAMPLE 3: U.S. KNOWLEDGE WORKERS**

WORKING WOMEN (January, 1997) states that the ratio of male to female knowledge workers-engineers, scientists, technicians, professionals, and senior managers-was 3 to one in 1983. The following table, which gives number (in millions) of male and female knowledge workers from 1983 to 1997, shows how that ratio is changing.

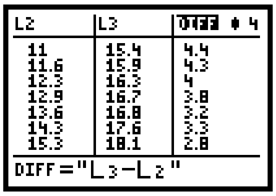
| **Year** | **Female Knowledge Workers(millions)** | **Male Knowledge Workers (millions)** |
|---|---|---|
| 1983 | 11.0 | 15.4 |
| 1984 | 11.6 | 15.9 |
| 1985 | 12.3 | 16.3 |
| 1986 | 12.9 | 16.7 |
| 1987 | 13.6 | 16.8 |
| 1988 | 14.3 | 17.6 |
| 1989 | 15.3 | 18.1 |
| 1990 | 15.9 | 18.6 |
| 1991 | 16.1 | 18.4 |
| 1992 | 16.7 | 18.6 |
| 1993 | 17.3 | 18.7 |
| 1994 | 18.0 | 19.0 |
| 1995 | 18.5 | 19.8 |
| 1996 | 19.0 | 19.6 |
| 1997 | 19.5 | 19.8 |

Source*: Working Woman,* January 1997

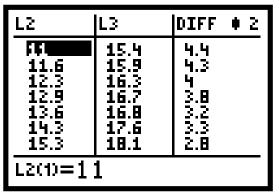
To compare how the growth in the number of female knowledge workers compares with that of male knowledge workers, the students entered the data above in lists, with the number of years from 1980 in L1, the number of female knowledge workers in L2, and the number of male workers in L3.

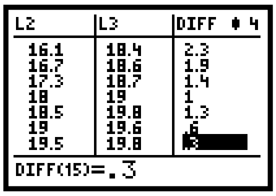



The difference "male minus female" is found by using the formula "L3 – L2" after DIFF=. Note that the calculator operates as a spreadsheet if the formula is in quotes and that the values will not change if the formula is not in quotes.



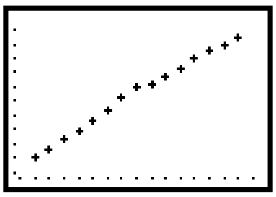

The largest difference occurs in 1983 and the differences are, for the most part, getting smaller. Because the differences are getting smaller as 1997 approaches, it appears that the number of female workers will soon equal the number of male workers.



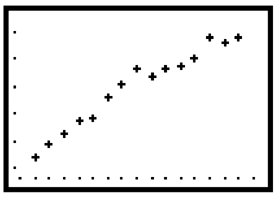




To find the year in which the number of female knowledge workers equals the number of male knowledge workers, the students found functions that model the number of female knowledge workers and the number of male knowledge workers and used INTERSECT to find when the numbers are equal.

Females

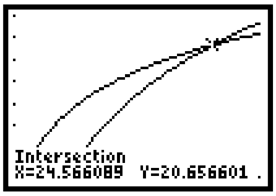


Males



$$F(x) = 4.26958 + 5.11876 \ln x \qquad M(x) = 12.11515 + 2.67076 \ln x$$

The graphs of y = F(x) and y = M(x) intersect at 24.57. This indicates that the number of female knowledge workers will pass the number of male knowledge workers in 2004.

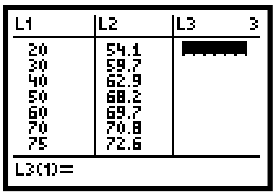



**EXAMPLE 4: EXPECTED LIFE SPAN**

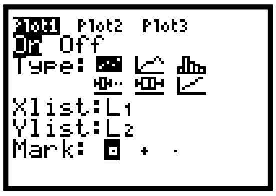
The following table shows the expected life span at birth of people born in certain years in the United States. The following steps compare different models for this data.

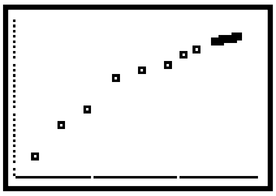
| *Birth Year* | *Life Span (in Years)* | *Birth Year* | *Life Span (in Years)* |
|---|---|---|---|
| 1920 | 54.1 | 1988 | 74.9 |
| 1930 | 59.7 | 1989 | 75.1 |
| 1940 | 62.9 | 1990 | 75.4 |
| 1950 | 68.2 | 1991 | 75.5 |
| 1960 | 69.7 | 1992 | 75.5 |
| 1970 | 70.8 | 1993 | 75.5 |
| 1975 | 72.6 | 1994 | 75.7 |
| 1980 | 73.7 | 1995 | 75.8 |
| 1987 | 75.0 | 1996 | 76.1 |

Data in the Year column can be realigned to represent the number of years since 1900, and this data can be stored in $L_1$. The life span data can be stored in $L_2$.




A scatter plot of the data is shown below.

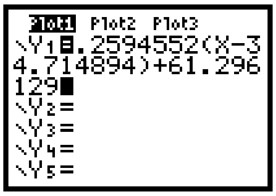


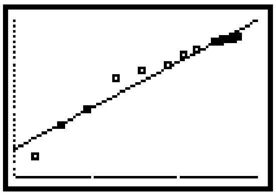



A piece of spaghetti can be used to estimate a line that is the best fit. The free-moving cursor can be used to find two points "under" this line. Example: (34.714894, 61.296129) and (84.195745, 74.134194) are two points on the visual fit line. Finding the slope of this line is used to write its equation.

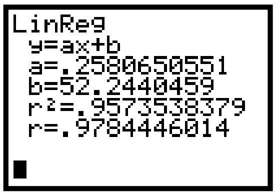
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{74.134194 - 61.296129}{84.195745 - 34.714894} \approx 0.2594552$$

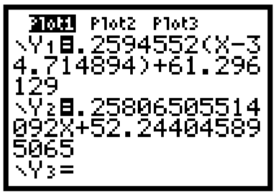
y = 0.2594552 (x – 34.714894) + 61.296129)

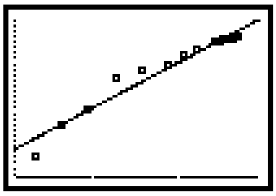


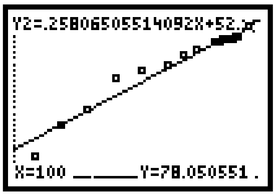
The built-in linear regression feature of the calculator can be used to model the linear function that is the best fit for the data.



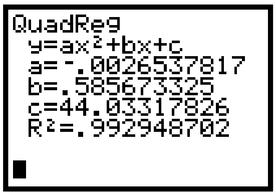



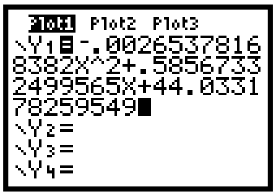
Assuming that the model applies in the year 2000, the life span of people born in the year 2000 can be predicted. If we evaluate the function at x = 100, we find the life span to be 78.

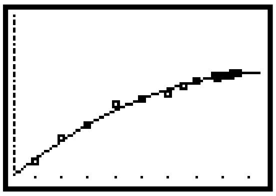


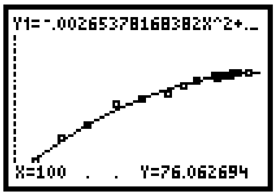

A **quadratic function** can also be used to model the life span data.

The quadratic function that is the best fit for the data is $y = -0.002654x^2 + 0.58567x + 44.03318$. The scatter plot and the graph of the quadratic regression equation are shown below.



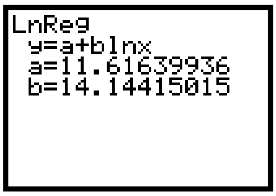



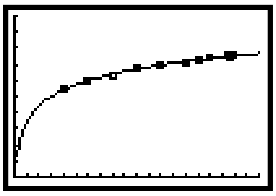



This model can be used to predict the life span of people born in the year 2000. Evaluating the function at x = 100, we find the life span to be 76.

A **logarithmic function** can also be used to model the life span data.

Using the natural logarithmic regression gives the equation y = 11.6164 + 14.441 ln x.

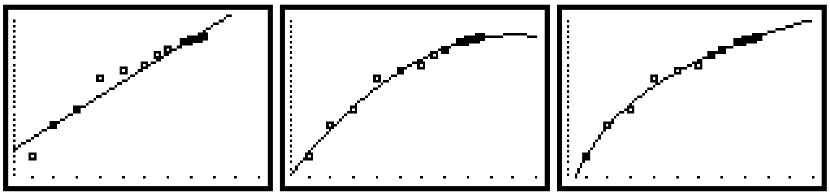


This model appears better for values of x after 20, that is, after 1920. (Distances from plotted points to each model can be checked to see which model is better.)

Evaluating this logarithmic function at x = 100 gives the predicted life span of people born in the year 2000 to be 78.1.

Recent data indicates that the expected life span for people born in the U.S. in 2000 is 76.7, so it appears that the quadratic model is the best predictor of life expectancy for the near future. The graphs below show the linear regression line, the graph of the quadratic regression equation, and the graph of the logarithmic equation, respectively, for the years through 2020.



**EXAMPLE 5: TAXATION**

Modeling doesn't just mean using a computer or calculator to get a regression curve from data points. It also means creating equations from available information. Consider the following real problem that requires either the creation of linear models or iteration methods for its solution.

Federal income tax allows a deduction for any state income tax paid during the year. In addition, the state of Alabama allows a deduction from its state income tax for any federal income tax paid during the year. The federal corporate income tax rate is equivalent to a flat rate of 34% for taxable income between \$335,000 and \$10,000,000, and the Alabama rate is 5% of the taxable state income.

Suppose both the Alabama and federal taxable income for a corporation is \$1,000,000 before either tax is paid. Because each tax is deductible on the other return, the taxable income will differ for the state and federal taxes. One procedure often used by tax accountants to find the tax due in this and similar situations are called iteration and are described by the first five steps below. A second method is the direct method, which requires us to create mathematical models from the given information.

**Iteration Method:**

1. We first make an estimate of the federal taxes due by assuming that no state tax is due. The estimate of the federal taxes is 0.34(1,000,000) = \$340,000.

2. Based on this federal tax, we can then estimate that the taxable state income is \$1,000,000 - \$340,000 = \$666,000 giving a state tax 0.05(660,000) = \$33,000.

3. Deducting this estimated state tax from the federal taxable income gives \$967,000 as the adjusted federal taxable income. The federal tax on this income is 0.34(967,000) = \$328,780. The state taxable income is now \$1,000,000 – \$328,780 = \$671,220, with the state tax of 0.05(671,220) = \$33,561.

4. Repeating step (3) gives

| Fed Taxable Income | Federal Tax | State Taxable Income | State Tax |
|---|---|---|---|
| 966,439.00 | 328,589.26 | 671,410.74 | 33,570.54 |
| 966,429.46 | 328,586.02 | 671,413.98 | 33,570.70 |
| 966,429.30 | 328,585.96 | 671,414.04 | 33,570.70 |

5. The state tax remains unchanged at \$33,570.70 in the last iteration above, so this amount will not change again. And thus the federal tax will remain unchanged at \$328,585.96.

**Direct Solution**

To find the tax due each government directly, we can create two linear equations that describe this tax situation and solve the system with graphical methods or matrix methods.

1. Let x = the federal tax owed and y = the state tax owed. Then the federal tax is given by

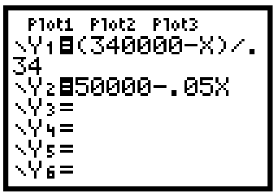
$$x = 0.34(1{,}000{,}000 - y)$$

and the state tax is given by

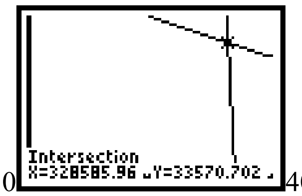
$$y = 0.05(1{,}000{,}000 - x)$$

Solving both of these equations for y permits us to graph them on the same axes and to find the simultaneous solution graphically. The equations are

$$y = \frac{340{,}000 - x}{0.34} \quad \text{and} \quad y = 50{,}000 - 0.05x.$$




40,000

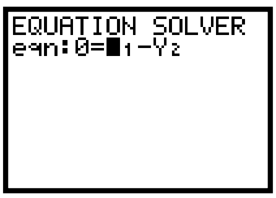


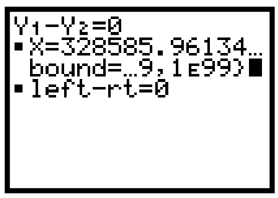

0 400,000

0

The intersect feature gives federal tax of $328,585.96 and state tax $33,570.70.

This solution can also be found by using SOLVER.

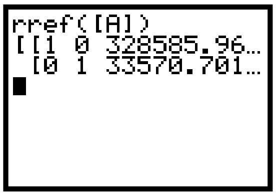


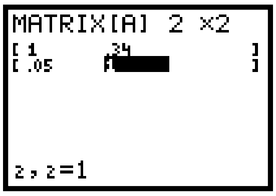

These equations can also be written in general form and solved simultaneously with matrices. The system is

$$\begin{cases} x + 0.34y = 340{,}000 \\ 0.05x + y = 50{,}000 \end{cases}$$

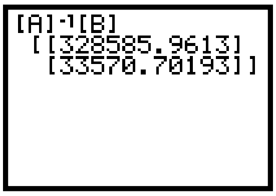
Using row reduction of the augmented matrix gives the same solution.


Using inverse matrices also gives the same solution.








# Conclusion

Students enrolled in this course initially thought that the course would be very difficult because most of their work was with "word problems." However, they adjusted quickly, and in general recognized that the "real problems" were much more interesting than the skills check problems. Although not every algebra topic was included in the course, the level of performance on algebra skills tests was not significantly different than with traditional tests, and students had increased confidence in their math and problem solving abilities.

**REFERENCES**

- Harshbarger, Ronald J. and Yocco, Lisa S., *College Algebra in Context: Applications and Models*, Boston: Addison Wesley.
- "Salaries of U.S. College Professors 1998-1999," American Association of University Professors, *World Almanac and Book of Facts,* 2000.
- "U.S. Knowledge Workers," Working Women, January, 1997.
- "Expected Lifespan," *World Almanac and Book of Facts*, 2002.
- Johnson, Kenneth H., "A Simplified Calculation of Mutually Dependent Federal and State Tax Liabilities," *The Journal of Taxation*, December, 1994.