# ARE STUDENTS ABLE TO TRANSFER MATHEMATICAL KNOWLEDGE?

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## ABSTRACT

The ability to use mathematics in other disciplines is generally expected of all science and engineering students. Anecdotal evidence suggests that many students lack this ability. While there is a substantial body of research dealing with the transfer of training, and the transfer of mathematical skills to problem solving in everyday life, there is very little relating to the transfer of mathematics to other scientific disciplines.

This paper reports on the development and trialing of an instrument which can be used to research the ability of students to transfer mathematical skills and knowledge to other disciplines. The instrument consists of mathematical problems set in various contexts. All the problems involve exponential and logarithmic functions, and are based on scenarios from physics, microbiology and computer science. In each case, any discipline-specific knowledge required to solve the problem is given, so that all the problems can be solved with mathematical knowledge only. The problems were initially written by a physicist, a microbiologist and a computer scientist. The instrument has been trialed with 47 first year science students at the University of Sydney. Performance on the instrument has been correlated against final high school marks, first year university results, and subjects studied. These results are presented.

The paper also discusses some of the interesting issues which arose from the collaboration of a mathematician with academics from three other scientific disciplines. For example, differences in the ways the physicist, the microbiologist and the computer scientist used mathematics were apparent. Also, their use of mathematics was often quite imprecise. Such issues have important implications for the teaching and learning of mathematics, both as a subject in its own right and within other disciplines.

# 1 Introduction

Science and engineering degrees typically require students to study mathematics as a subject in its own right, with the expectation that students will be able to use the skills and knowledge acquired from their mathematics courses in other disciplines. Typically, also, lecturers in engineering and scientific disciplines complain that students are unable to apply mathematics in context. Clearly, the question of whether or not students are able to "transfer" their mathematics is an important one.

Psychologists use the term "transfer of training" to refer to "knowledge, skills and attitudes able to be transferred from training sessions to the work context, and from one job or task to another" (Hesketh 1997), and there is a substantial body of research on the topic. There is also a considerable amount of research on the extent to which mathematical skills transfer to problem solving in real world situations (Buckingham 1997, Carraher et al 1985, Lemire 1988, Sun 1995), and various papers which assume that students have a problem transferring mathematics. Gill (1999a and 1999b), for example, has studied the problems students of physics and engineering have with mathematics. Jackman et al (2001) report on a project involving assessment tasks designed to improve the ability of students to apply mathematics in context. Woolnough (2000) similarly describes a program in physics "designed to help students build effective links between mathematical equations ..... and the real world". There is little work, however, which specifically addresses the question of whether or not university students are able to transfer mathematical skills and knowledge.

The aim of the project discussed in this paper was to investigate the extent to which students are able to transfer mathematical skills to those disciplines represented by the project team members. The project team consisted of a mathematician (the author), a physicist, a microbiologist and a computer scientist. An instrument to test transferability was developed. The instrument consists of mathematical problems set in various scenarios. In the following sections the development of the instrument is described, and some results from an initial trial are presented.

# 2 Developing the instrument

Our original intention was to develop the questions which comprise the instrument around a topic taught in first year mathematics, and used in first year physics and computer science, and in microbiology. It was a little surprising to find that at the University of Sydney there is apparently no such topic. The questions are therefore based on logarithms and exponential functions, topics which are taught at high school in New South Wales.

Some purely mathematical questions were written by the author, and the other members of the project team wrote questions set in the context of their particular discipline. Their brief was to write problems which contained enough discipline specific information so that the problems could be solved using mathematical knowledge only, without any previous knowledge of the particular discipline. This proved to be rather a difficult task. The first draft of the instrument included some explanations which were not entirely comprehensible to those of us who had not written the questions. The computer scientist wrote a question, based on Big-Oh notation, which was almost

totally incomprehensible to those unfamiliar with the notation. It is clearly difficult for academics not to make certain assumptions, relating to their discipline, when writing background information. Of further concern to the author was the imprecise way in which the other scientists tended to use mathematics. The original questions included variables defined incorrectly, and some rather imprecise descriptions of mathematical concepts. (For example, one question included the statement: "On a logarithmic scale, the number of photons approximate negative slopes.")

The problems were rewritten several times before all the researchers were satisfied. Some post-graduate and higher year undergraduate students were then asked to attempt the questions, and provide feedback with regard to clarity of the questions, perceived difficulty, and length of time taken to complete the questions. Five physics post-graduate students, one microbiology post-graduate student, one undergraduate microbiology student, one mathematics and computer science graduate, and one mathematics honours student agreed to do so. The feedback we received was extremely useful, and the questions were further refined in the light of the students' comments. We had expected that these students would be able to complete the questions without difficulty, and so were surprised to find that most of them were unable to successfully complete all the problems. The computer science questions proved particularly difficult for some of the students who had not studied computer science. The mathematics questions, which were of high school standard, were completed successfully only by the mathematics students. The instrument was revised further in response to this feedback.

The current version of the instrument consists of a physics problem based on exponential decay of the number of photons in a photon beam, a microbiology problem based on killing bacteria, a computer science problem based on Big-Oh notation and four straightforward mathematics questions. Where possible, the questions have a similar structure, allowing us to test the application of a particular skill in different contexts. We were able to achieve this with the physics and microbiology questions. The computer science question is quite different from the others, and may well be deleted from the instrument in future versions. The following extracts from the instrument illustrate some parallel questions.

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**Physics question**

Consider a beam of photons with identical energies all travelling in the same direction, head-on into a particular medium. The number of photons which survive as the beam passes through the medium decreases exponentially. The distance over which the number of photons is halved is called the half-thickness of the medium. Let $N$ be the number of photons which have survived at a distance $x$ into the medium, and let $g$ be the half-thickness.

1. If $N(x) = N_0 \times 2^{-kx}$, where $N_0$ is the initial number of photons, and $k$ is a positive constant, express $k$ in terms of $g$.

2. Suppose a medium is 10 mm thick, with a half-thickness of 0.5 mm, and that $10^{10}$ photons enter the medium head-on.

   Draw a graph of $\log N$ against $x$, with a scale marked on the axes.

**Microbiology question**

The bacterium *Staphylococcus aureus* ("golden staph") found in poultry stuffing is killed by heat. After a quantity of poultry stuffing has been heated to 62°C, the cell concentration of the golden staph bacteria decreases exponentially. The Decimal Reduction Time at 62°C, $D_{62}$, is the length of time required for the cell concentration to decrease to $1/10^{\text{th}}$ of its original value. Let $N$ be the cell concentration of the bacteria at time $t$ minutes after the stuffing has been heated to 62°C.

1. If $N(t) = N_0 \times 10^{-kt}$, where $N_0$ is the initial cell concentration and $k$ is a positive constant, express $k$ in terms of $D_{62}$.

2. For golden staph, the decimal reduction time at 62°C, $D_{62}$, is 8 minutes. Draw a graph of $\log N$ against $t$ if the initial concentration is $10^5$ cells/g.

**Mathematics question**

1. If $P = 5e^{kt}$ and $P = 10$ when $t = 3$, find $k$.

2. If $y = 4e^{-0.1x}$, draw a graph of $\ln y$ against $x$, for $0 \le x \le 10$.

# 3 Trial of the instrument

In Semester 2 2001, forty-seven first year students attempted the questions. The students were volunteers, and were paid a small amount for their participation. There were 30 science students, 16 engineering students and one arts student. Each student was given a version of the instrument with the physics, microbiology and computer science problems collated in random order, and the mathematics problems at the end. They were given 40 minutes to attempt the problems in the order in which they appeared, and then asked to attempt the mathematics questions. Each student's work has been marked, and scores for each of the questions recorded.

In the following table the students have been grouped according to the subjects they had studied in Semester 1 2001. Only those subjects of interest to us are included. For each group, the table gives an average score (out of 10) for the mathematics question, and an average score (out of 10) for the computer science, microbiology and physics questions. The latter is labelled "Transfer mark".

|  | *No. students* | *Math mark* | *Transfer mark* |
|---|---|---|---|
| Maths only | 1 | 6.0 | 1.5 |
| Maths + Chemistry + Computer science | 1 | 6.0 | 3.0 |
| Maths + Chemistry + Physics | 3 | 4.7 | 2.4 |
| Maths + Computer science | 7 | 5.6 | 2.7 |
| Maths + Computer science + Physics | 7 | 7.3 | 4.6 |
| Maths + Chemistry | 8 | 6.5 | 3.0 |
| Maths + Biology + Chemistry | 10 | 6.8 | 3.8 |
| Maths + Biology + Chemistry + Physics | 10 | 6.9 | 3.9 |

Given the size of the sample, and the fact that the students were self-selected, it would be unwise to attempt to draw too many conclusions from these results. For example, the highest-scoring group (Maths + Computer science + Physics) in the above table contained two of the three top-scoring students, but also two of the lowest-scoring. Nevertheless, some interesting observations can be made.

Firstly, almost all the students had university entrance scores, and final high school mathematics marks, very much higher than average. The mathematics questions in the instrument are of high school standard. Performance on these questions was, therefore, surprisingly bad. The fact that performance on the "transfer" questions was even worse supports the widely-held view that students have difficulty applying mathematics in context.

Secondly, there is some evidence from the results to suggest (as one would expect) that students are better at applying mathematics in context when they are familiar with the context. For example, students who had studied physics in semester 1 scored an average of 4.2 (out of 10) on the physics question, while those who had not studied physics scored an average of 3.1. Similarly, biology students scored an average of 3.4 on the microbiology question, while those not studying biology scored an average of 2.5. The difference was less marked on the computer science question, with computer science students scoring an average of 4.6, and those without computer science an average of 4.1. On the other hand, the top-scoring students performed equally well on all the questions, regardless of subjects studied in first semester, and some students seemed better able to apply their mathematics in the context of a discipline they were *not* studying.

At the time of writing this paper, further analysis of the students' work on the instrument is planned. We have, for example, identified five mathematical skills and seven pieces of mathematical knowledge needed to successfully complete the mathematics questions. Students' responses on all the questions will be analysed in relation to those. We hope to be able to construct an algorithm for calculating a "transferability score" for individual students.

We regarded this trial as a test of the instrument as well as of the students. In this respect some questions, such as whether or not the students could at least attempt all the questions within the allotted time, were easy to answer. (Only half of them were able to do so, despite the fact that the questions had been seriously pruned in response to feedback from the postgraduate students.) Other questions, such as whether or not the instrument reliably tests the ability to transfer, are more difficult to answer. On the assumption that the ability to transfer mathematical skills and knowledge is important for academic success in other scientific disciplines, we compared students' results on the instrument with their university entrance score (UAI), as well as with their WAM (a weighted average of first year university results). The correlation coefficient between "Math mark" and UAI was 0.54, and between "Math mark" and WAM was 0.62. Between "Transfer mark" and UAI the coefficient was 0.47, and between "Transfer mark" and WAM it was 0.57. Despite the non-random nature of the sample, these results are significant enough to lead us to believe that the instrument is a useful test of transferability.

# 4   Conclusions

There is little argument amongst academics that students do indeed have trouble transferring mathematics. The results of our trial bear this out. With few exceptions, students performed better on the pure mathematics questions than on the "transfer" questions involving the same skills and knowledge. Some reasons for this are obvious. Applying mathematics in context generally involves translating a problem expressed in words into a mathematical statement. Many students have such poor language skills that the problem may well be insurmountable. Nevertheless, it is in the interests of academics in all scientific disciplines to attempt to find ways in which to help students overcome their difficulties in relation to transfer.

Communication between academics is an obvious starting point. The collaboration of academics from four different scientific disciplines on this project has been most instructive. We have discovered that our use of mathematics is often different, in ways which are unlikely to be helpful to students. For example, mathematicians all but ignore exponentials and logarithms to bases other than $e$, whereas in physics and biology the use of base 10 is more common, and computer scientists generally use base 2. We have also learned that we have different ideas of what is mathematically correct. Physicists at the University of Sydney, for example, believe that it is incorrect to take logarithms of both sides of an equation involving variables which represent quantities with units attached. (So a Sydney University physicist would claim that the equation $q = q_0 e^{-t/RC}$, where $q$, $q_0$ are in Farads and $t$, $RC$ are in seconds, is not equivalent to the equation $\ln q = \ln q_0 - t/RC$.) Further, it would appear that mathematicians expect mathematics to be used much more precisely than other scientists are accustomed to doing. While the use of mathematics in an imprecise way may not hinder scientists in their everyday work, it may be confusing for students. There is much food for thought with respect to the implications for teaching raised by all these differences.

Finally, it should not be forgotten that the acquisition of mathematical skills and knowledge is a pre-requisite for the ability to transfer them. Not surprisingly, the students in our study who performed strongly on the mathematics questions were, in general, much more successful on the transfer questions than were those students whose performance on the mathematics questions was weak. "The power of mathematics as a tool... is that if the working of the tool is understood then it becomes possible to apply it in novel situations" (Gill, 1999a). In teaching mathematics to science and engineering students we should certainly keep in mind the transfer problem. However, our first priority is to ensure that students acquire a good understanding of the tool.

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