

SOON UNACCOUNTABLE*

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ABSTRACT

The Mathematics Across the Curriculum Project at Dartmouth College produced a number of new courses integrating mathematics with a humanistic discipline such as literature, art, or philosophy. These courses all were free of any prerequisite and attracted a wide variety of students. The mathematical topics were chosen for their relative modernity and sophistication, e.g. group theory, infinity, or the fourth dimension. How does one come up with math that can be offered in these interdisciplinary courses? How do you present it in a way that isn't trivial? What sort of understanding is it reasonable to expect students to carry away as a result of such a class? Why is it worth the trouble to educate this body of students in this particular way? In this talk we will consider these questions and get a glimpse into some unusual courses.

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*When I Heard the Learned Astronomer

When I heard the learned astronomer,
When the proofs, the figures, were ranged in columns before me,
When I was shown the charts and diagrams, to add, divide, and measure them,
When I sitting heard the astronomer where he lectured with much applause in the lecture-room,
How soon unaccountable I became tired and sick,
Till rising and gliding out I wandered off by myself,
In the mystical moist night-air, and from time to time,
Looked up in perfect silence at the stars.

Walt Whitman

Proofs and figures

Class begins and the students arrive, homework in hand. The work, consisting of block-printed mandalas and small handwritten tables, is not dropped on the front desk for the benefit of the instructor, but posted on the blackboards for all to see. The first ten minutes of this two-hour class is an art exhibit. The homework problem was to design two mandalas whose symmetry groups were of the same order but not isomorphic. The calculations accompanying each design are the group tables associated to its symmetry group. The designs are beautiful, but most solve the mathematical question the same way. One at a time the students comment on their work, acknowledging that their choice (“Mine are also a D_4 and a Z_8 ”) was usually the popular one. Once in a while the problem is solved incorrectly and the students themselves point that out. No comments are necessary from the instructor except for an indication when to move on to the next piece. All is going exactly as expected until J.P.’s mandala.

A few weeks earlier J.P. had brought in a sort of paper bracelet with symmetric fishes swimming on the inside and outside of it. He wanted to know if it qualified as a “mandala” for the purposes of an earlier assignment. That discussion (without firm resolution, by the way) together with the instructor’s assurance that there were lots more groups out there besides the dihedral and cyclic ones, led J.P. to a clever solution to this week’s problem. The paper bracelet he brought in on this day had six horizontally symmetrical motifs and could be flipped inside out to yield a total of twelve symmetries. J.P. believed that the resulting symmetry group was neither Z_{12} nor D_6 , but how could he be sure? The answer to his question involved a discussion of clock arithmetic, direct products of groups, and methods for telling two groups apart by counting the number of subgroups of a given order. All finally agreed that J.P. had made an object whose symmetry group is $Z_2 \times Z_6$.

One purpose of this paper is to argue that courses like the one described above, and experiences such as those of J.P., are essential to educating a population to quantitative literacy. Please notice that I am not saying that every student should take such a course or have such an experience, but students should have access to such experiences if they desire them. I would like to emphasize the point that educating a population to accomplish a certain thing is a completely different proposition from educating a large number of individuals to some dubious Platonic mathematical ideal. J.P. and his fellow classmates had a strong intellectual experience that taught them something about what it is to do mathematics, how mathematics informs the arts, and what modern mathematicians are sometimes thinking about. Such understanding can only come from a freely chosen course of study that speaks to individual interests. It cannot come from a forced march through statistics or calculus or any other course.

The students in J.P.’s course were not generally math or science majors. If anything, most were the so-called “math avoiders” that we sometimes dread to teach. Yet, those twenty or so students now have a deeply embedded knowledge of very elementary group theory. They say that they bore roommates and friends by constantly pointing out examples of D_4 in fabric and architecture. They talk about these groups as if they were personal acquaintances, as they are. We can’t predict what they will do with this knowledge later. But we can say that the ideas of group theory will be carried into vocations and situations that they would not otherwise have entered. This effect is an extremely pragmatic outcome of educating a population as such, rather

than as an aggregate of individuals held to a single standard. Intellectual diversity in the national population would be a blessed thing.

To add, divide, and measure

A class of about fifteen first year students, one of the “first year writing seminars” at Dartmouth, is trying to figure out how to admit the guests who arrive at a rapid rate at Hilbert’s Hotel. Hilbert’s Hotel has infinitely many rooms, numbered by the natural numbers. Alas, the arriving guests are evacuating a similar motel chain—an infinite sequence of motels each containing an infinite number of guests, all enumerated by the natural numbers. They arrive on a sequence of very large buses, one for each motel. Can Hilbert’s Hotel contain them all? The students begin to argue. They are organized into three groups. One group solves the problem by dividing the rooms in half and using only half of them on the first bus. They then divide the remaining rooms in half and use half of those on the second bus, etcetera. The second group assigns a prime number to each bus and an integer to each passenger on a bus. The passenger is put into the room corresponding to the bus’s prime raised to the passenger’s integer, without overlap. The third group of students tries to make a probabilistic argument. If the rooms are assigned at random, they wonder, then aren’t the chances of using up all of them zero? So there should always be some room left, if assignments are made at random. The desk clerk can stop worrying. The instructor, however, looks unconvinced.

The students in the “How many angels?” course are a completely different population from the ones in the “Pattern” course of our first example. Although each of these courses had some math lovers present, in this class all of the students were very interested in math. Indeed, many of them go on to major in the subject and some of that class of sixteen are now in graduate school. We have found that when the required first year writing course centers on mathematics and the humanities, it attracts a room full of prospective majors. When the course touches on advanced subjects it gives the students a foretaste of the courses they will see as juniors and seniors. By its nature, it asks these mathematically inclined students to read and write about mathematics in a different way from standard courses. In this course mathematical literacy is inseparable from the usual sort of literacy, as all mathematical learning is demonstrated through written essays.

A mathematician and a philosopher designed this course jointly. Each helped the other select readings and design exercises that would illuminate both the mathematics of infinity and the philosophical debates that accompanied mathematical developments. The philosopher visited Dartmouth for a quarter to work with the mathematician. Each learned enough of the other’s subject to feel at ease with the material. Each of the professors taught a version of this course by themselves in their own department at their own institution. Neither could have done as well had they designed the course alone.

Much applause in the lecture room

One of the most popular interdisciplinary courses at Dartmouth, with over a hundred students in attendance, is a course on “Time”, taught by Professor D in the mathematics department and Professor B in comparative literature. The students have studied ancient methods of timekeeping

and alternative concepts of time from various cultures. It is late in the term and Professor D is lecturing on relativity and Einstein. The class listens and takes notes. After a while, Professor B interrupts Professor D. She informs him that he is not making any sense. He has made statements, but hasn't explained them to her satisfaction nor argued them convincingly. Then she turns to the students. She points out that if she said completely incomprehensible things about a piece of literature, they would never let her get away with it. They would question, disagree and push hard until they at least understood why she made her claims. Why haven't they questioned the professor of math on his unclear and ill-explained statements? Do they really understand everything he just said? The students admit they do not. Why, then, this reverence for the authority of the scientist? Why this unquestioning acceptance of nonsensical claims? Both professors agreed that the ensuing discussion made that particular class the best day of the term.

This particular course came about because the mathematicians most interested in doing work with the humanities were booked solid. Therefore others were enjoined to "make friends with a humanist" in order to see if there were any areas of commonality that could form the basis for an intellectually strong course. Within a few months of making such a strategy explicit we had two new collaborators in the humanities. In both cases, the mathematicians were flexible, rearranging their interests around the areas of expertise of the humanities faculty members with whom they worked. One goal was to keep the humanist feeling secure in what sometimes seemed to be a risky endeavor. The quite evident security displayed by Professor B in the above example owes its remarkable character to the fact that Professors B and D are a married couple. (You have no excuse for not talking to a humanist, we had told D.)

Up in perfect silence

Rarely is it necessary to consult astronomical charts before offering a class, but the "Renaissance Astronomy and the New Universe" course can only run in terms where a planet is visible in the night sky for most of the term. The laboratory exercise depends on such a planet, preferably Mars. If we are very lucky with scheduling, Mars will show us some retrograde motion before the term is out. Mars offers two advantages over other planets. First, it travels relatively quickly against the backdrop of the stars. Secondly, the "Mars problem" was the source of great dissatisfaction with all predictive models of astronomical behavior prior to Kepler. So, the course is planned around the appearance of Mars in the night sky, which for some reason seems to like to happen during winter quarter. The students are sent out to look for it and track its motion on clear nights when the temperature drops below zero Fahrenheit. They become remarkably quick at locating it.

Our students have a modern education and they "know" that the earth is not the center of the universe, that the stars and planets do not rotate around it, but that our predecessors believed it so due to the appearance of the night sky. They consider the Copernican model to be an obvious truth. Most of them have never really looked at the sky. Many come from cities where you can't see the stars.

Textbooks evidently paint a picture of early astronomers as quaint old gentleman who were at the mercy of a lot of silly assumptions and lack of a telescope. The students learn otherwise as they proceed to make every possible error in measuring the location of Mars. They try to use

landmarks to judge where a star is, a mistake Ptolemy would never have made. After some discussion they abandon that method. Of course, they “knew” all along that the night sky moves all night. They learn to use the zodiac the same way our ancestors did. After a few months of observation they learn a few things.

First, there are some kinds of science in which data is absurdly hard to obtain. One of the things Europe imported during the renaissance was the star record kept by Arabic astronomers for a thousand years. As the students go out night after cloudy night, they begin to understand the priceless nature of such data. Sometimes they complain that the course should be held in a less cloudy location, but are reminded by the instructor where Copernicus and Tycho Brahe were living. It was not so different.

Second, the students eventually come to see the Ptolemaic system as the most natural explanation for celestial motion. They make a full turn and begin to wonder why anybody would have believed Copernicus at all. Then the course gets interesting, because this is the right question. Another thing astronomers obtained in Renaissance times was “Euclid’s Elements of Geometry”, one of the great Greek intellectual mathematical advances. The students read Copernicus (in English) to see how closely he imitated the rhetoric of Euclid. Copernicus had two things at his disposal—a mountain of valuable rare data and an irrefutable form of argument, the mathematical proof. How these factors interacted, which was more important to his argument, and how surrounding beliefs and historical forces contributed to the discussion; all become part of the answer to that question.

Charts and diagrams

All four of the courses described above were a success at some level and we are sure to offer all of them repeatedly at Dartmouth. We have several more successes, including a math and music course and a mathematical science fiction course, not described in this paper. Some courses attract science and math majors, most attract the so-called “math avoiders”. Some are very large, some quite small. Some require two instructors, others only one. Some require problem sets, others require papers, artwork, or musical composition. Extensive evaluation shows that the courses are a success by delineating for us what kind of things the students learned, whether they felt they learned a lot, and whether faculty felt the students were doing good work. However, for each course the details are different. What a potential math major gets out of the Infinity course cannot be compared directly to what an English major in the Time course takes away. Nonetheless, after many years of tracking these courses we are confident of their varied kinds of worth. The question we must ask is this: what accounts for success? I will offer you some design principles that have guided us in the hope that they will serve you as well as they have served us.

Before creating any courses it is important to achieve some consensus, preferably across departments, as to why you are doing so. What is the goal of all this work? It is fairly easy to get faculty to agree that calculus serves the sciences better than it serves students in other disciplines, but that observation alone does not tell us why we need other courses or what they should look like. It is difficult to get humanists in particular to articulate a mathematical goal for their majors.

It is worth the trouble to do so, however. Here are the sentiments a small group of humanists produced when we locked them in a room with us for several hours:

“A humanist needs to understand that mathematics is a human endeavor too.”

“There must be some overwhelmingly important cultural advances in math that everyone ought to know about.”

“There should be less of a split between the cultures of science and humanities.”

These ideas have been stated more eloquently elsewhere, but no matter. They were ours and so we could use them as a basis for action rather than persuasion.

Plan on offering a variety of courses because one size definitely does not fit all. As our evaluators have shown via survey instruments and interviews, students choose courses based on their current interests, not on their perceived preparation for the course. Unless you have made your entire career out of teaching required courses, you know yourself that this is true. I believe it safe to extend this observation to their actual learning. Mathematics connected with a subject that already interests a student ought to be learned more readily and retained longer than any topic visited during a forced march. Furthermore, every faculty member knows from personal experience the quality of educational experience that results from having a class full of people who have freely chosen to be there.

It is also worth noting that allowing students to sort themselves by their own interests is something mathematics departments have traditionally avoided in structuring mathematics curricula, relying instead on a battery of tests of mathematical preparation and sorting students entirely by level. When courses are arranged in a single linear pattern of prerequisites for each other, you have automatically constructed a “sieve”. The probability is high that the student will eventually encounter material that isn’t interesting enough to prompt him or her to take the sequel, so the student opts out of the only sequence available. No amount of good teaching or technology can change that: it is forced upon teachers and students alike by the basic structure of the system. When there are multiple points of entry to a variety of kinds of mathematics at wide ranging levels, then at least the potential exists for the system to avoid being a sieve. The potential application of this simple observation to the problem of attracting women and minorities into these classes is obvious, as we say in mathematics all too often.

Work the system because a course that satisfies no curricular need will not be well subscribed. In other words, take an inventory of the types of requirements a student must satisfy. Is there a quantitative requirement? Some of your new math and humanities courses should satisfy it. Is there a writing requirement? Some could be constructed explicitly to satisfy that. At Dartmouth, there is an interdisciplinary requirement, and those courses staffed by two faculty members from different departments can be made to satisfy it. We also have requirements such as technology and applied science and western culture, each of which is satisfied by one of our courses. This part of the strategy is essential to making sure new courses are well attended from the outset and can therefore justify their existence to the administration as well as the department offering the course.

Additionally, the kind of student who enrolls in a course is closely connected with the type of credit offered. A math and humanities course carrying quantitative credit is likely to be taken by “math avoiders” who see it as a palatable way to satisfy that requirement. A similar course satisfying the writing requirement will, instead, be heavily attended by potential math majors.

Plan for variance. Some of the traditional one-track sequencing of math courses can be explained as an attempt to lower variance among students, the better to target content and pedagogy to the correct level. To a large extent it succeeds so that we can have a fairly good idea of the background of someone entering second semester calculus, for example. On the other hand, a prerequisite-free course such as “Pattern”, the first example in this paper, has a huge variety of students in it. The variation in math background goes from high school algebra and geometry right through fairly advanced calculus. Furthermore, there is a large art component to the class, and the variance for that subject is far greater than for math. Some are planning to major in art and others have never taken a class in art of any kind, nor art history. And as we all know, when you put those two factors together the variances sum.

The instructor is left with two options. Either one can proceed to deliver lectures and other activities, assuming everyone knows nothing, or else one must rethink pedagogy to capitalize on the variance among students. In other words, the instructor has the option of learning to use the variance in student background to advantage. Carefully thought out assignment of students to groups, coupled with well-chosen activities, can turn students into helpful tutors for one another. The interdisciplinary subject matter lends itself well to this because different students have different areas of strength and can thus be leaders at different moments. Discussions can be very rich.

Sometimes the subject matter lends itself well to the first strategy. In the math and science fiction course (not described here) much of the mathematics has to do with the fourth dimension. It is completely safe for the instructor to assume that nobody in the class knows anything about this topic, and to proceed accordingly. In the best situation, there would be opportunities for both of these approaches in any given course.

Some conclusions. In imitation of the courses it describes, this paper is trying to make several parallel points simultaneously. First of all, interdisciplinary courses in math and humanities are completely viable if built to respond to both faculty and student interest. Students are not allergic to intellectual work in mathematics if they find the topic inherently interesting. Second, a road to quantitative literacy that is broad enough for everyone to travel must have a multiplicity of entry points and natural connections to other subjects. Any one course proposing to solve the Q.L. problem is doomed to failure, because it cannot possibly respond to the needs and interests of an entire student population. Third, diversity of knowledge is a good thing in any population. The population can hold more knowledge that way.

Finally, it is unfortunately necessary to point out that there are some goals of a college education that we do not often acknowledge and that I have rarely heard spoken aloud. We spend a lot of energy discussing preparation for citizenship and the job market, yet life consists of more than our duties. My suspicion is that the courses described above are successful because they also respond to another human need. We crave delight and pleasure, whether in getting a new understanding of how part of the world works or from the joy of fruitful intellectual activity. Surely if we can think broadly enough to put “quantitative” and “literacy” into the same phrase, we might also draw “mathematics”, “delight” and “pleasure” in with the same breath of fresh air.