"ARE MATHEMATICS FOR OTHER DISCIPLINES DIFFERENT MATHEMATICS? "

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ABSTRACT

Teaching Mathematics for different disciplines raises the question of whether the depth, the topics and the intensity of courses are or should be something different according to each area of study. The main argument is that at a basic level there is no difference and, on the contrary, there are enough reasons to avoid the creation of first and second-class Mathematics. Unfortunately, the literature is full of examples of books inviting students of Economics to learn Mathematics for economists or students of Biological Sciences to read only Mathematics for biologists. Nevertheless, at a higher level, there are also good reasons to split the group of students into more specialized, more Applied Mathematics, mainly because of the demand for proper models and the more extensive use of Mathematics in other areas. For the mathematician this implies a challenge because to teach good courses it is necessary to get a broad insight into other disciplines.

Keywords: Mathematics for other disciplines, basic courses, advanced courses, teaching of by mathematicians, curriculum

1 Introduction

Going to the mathematical section of a bookshop one can find titles like: Mathematics for Economists, Economic Mathematics, Mathematics for the Life Sciences, Applied Calculus for Engineers, Linear Algebra with Applications to Economics, and so on. If one looks at the Statistics area, the menu is even broader. The question is whether there exists a different approach, a different Mathematics, to what universities worldwide teach in basic Mathematics or Statistics courses for different disciplines which corresponds to the titles and the contents of these books? What about the teachers – should they be mathematicians, or should each discipline receive what they need in Mathematics from its own people?

This paper will try to answer these questions. Section 2 is devoted to fix a position towards teaching basic Mathematics for other disciplines. Section 3 will answer the question if there exists a moment of switching to a more specialized, i.e. more Applied Mathematics. Finally, section 4 will give some ideas about curricula and give some conclusions.

2 Teaching basic Mathematics for other disciplines

Like a child who wants to learn football, there are steps to be learned like how to kick the ball, how to stop it, how to dribble, i.e. solid foundations that must be learned step by step. Of course, one can apply the method of learning by doing, but if one wants to use your knowledge professionally, with very few exceptions, sooner or later you would run into trouble. A solid foundation of mathematical thinking and techniques is needed to undertake what can be called an application to other disciplines. Mathematics is by definition a rigorous discipline and students of any academic area have to learn not the mechanics of Mathematics or what can be called a mechanistic Mathematics, where for this type of problem this recipe is applied and for that other type another will provide the solution. It is more important to understand concepts than mechanics; nevertheless, a dose of carpentry must also be trained. Therefore, a solid foundation has to be created and this implies that basic Mathematics is the same for all.

Naturally there can be disagreement as to what is understood under basic Mathematics, but I think that at least Calculus - Differential and Integral in one variable, including some Sequences and Series theory – and Linear Algebra are a common denominator. Is this short sequence different for Mathematics students? The answer is not unique. Some more demanding theorems even in Calculus and Linear Algebra should be discussed with this last group because this is the essence of Mathematics itself. Take, for example, the Intermediate Value Theorem: the proof that an odd polynomial expression has at least one real root is not simple. The proof that a convex function is continuous is even more difficult and perhaps the majority of students different from the pure mathematician will not enjoy them and will gain very much trying to follow such developments. Other aspects, even if they are also not easy to fully understand, have a more intuitive application. For example, knowing that as n increases

$$(1+\frac{x}{n})^{n} \rightarrow e^{x}$$

has to do with interest rates which are of common use and the proof shows how to pass from a discrete case to a continuous one. So at this level at least there is no good reason to split students of different disciplines into specialized courses. Calculus for Economists or Matrix Algebra for Engineers does not exist. Another reason to maintain students of different disciplines together is to avoid opinions that there are first class Mathematics and other courses, which are not so prominent. And this is exactly what happens if the students are separated "according to their disciplines".

Table 1 in the Appendix shows the marks obtained by a group of students at Universidad de los Andes in Bogotá – Colombia, during five semesters, second of 1997 – second of 1999, whose disciplines are Mathematics, Physics, Economics, Engineering, Business Administration and Biology. They have to take a compulsory Differential Calculus course in the first or second semester of study. It also includes other students, mainly of Law, Psychology and other Social Sciences as well as Architecture who decided freely to take this course instead of others given only for them. The course was given in small sections (at most 30-40 students) and there is no discrimination inside each section according to the field of study chosen. Students can retire their inscription at the middle of the semester and have to repeat in the next semester. The results show that their performances are not high but even, with the exception of the Business Administration and Biological Sciences students who have in general lower marks and also are the most numerous groups with respect to retirements.

Is there an explanation for these results? The most common answer is that these two groups of students are not so dedicated to Mathematics and feel that it is not so important as other courses of their area of study. I disagree strongly with this and am convinced that the main reason for the lower performance has to do with the use and teaching of Mathematics by those teachers in their disciplines who do not apply them in their courses. We arrive so to one of the most important aspects of our discussion: the use of Mathematics by other disciplines. The main argument is that the non-use of mathematical concepts and techniques by some disciplines creates a dichotomy in the students that results in an attitude of indifference if not of total rejection towards Mathematics. So if we accept that even basic Mathematics can and should be taken by other disciplines in mixed courses, the complement of showing in other courses of their own area the applications is a necessary condition. Most courses of Engineering, Economics and other disciplines are using Mathematics each day to model their own theories and these models are studied intensively. Therefore there exists an interaction between Mathematics and these sciences; there is a demand which requires a suitable supply.

Something similar happens with basic Statistics courses. It is possible to give Statistics with little more than elementary Algebra. Nevertheless, Statistics should be preceded by Probability concepts, in particular, the notion of randomness, which allows understanding that in real life few things are deterministic. But Probability ignoring Calculus and Statistics ignoring Matrix Algebra will be poor. Social scientists have expressed in several occasions their need for support research on empirical evidence and it is not enough to hire a statistician who presents some results. Many colleagues, graduated 10 or 15 years ago, feel a vacuum and even are willing to take lessons in Statistics. Resuming, in this area of study there is also a need to establish a solid foundation to be able afterwards to understand more advanced methods.

In basic courses, including Statistics, mathematicians have a lot to say. New techniques like the use of computers and software packages and modern calculators are at order and so these courses can and should be given by mathematicians. But that's not all. Like in the past mathematician had to know some notion of Physics to teach Calculus, today it is necessary to have some notion of Economics and even Biology to teach basic Mathematics for these areas.

3 Teaching advanced Mathematics for other disciplines

Economics, Biology and even journals of Social Sciences disciplines like Political Sciences, Anthropology, Psychology and others are full of research papers, which use Mathematics and/or sophisticated statistical techniques. As soon as one goes into the level of Multivariate Calculus, differences begin to appear. For Mathematics, Physics and Engineering more important than going deeply into Optimization problems are concepts of partial derivates, line integrals, Stoke's and Green's theorems. In contrast, and I will take the case of Economics in more detail, maximum and minimum with and without restriction are extremely relevant. A great part of consumer and production theory is based on Lagrangian and Kuhn-Tucker theorems. More advanced theories like Optimal Control Theory and Dynamic Optimization (see for example Escobar 2001 or Takayama 1996 or Seierstad & Sydsaeter 1987) need strong foundations, both in optimization as in topics like Differential Equations. So here there is a reason, a good reason, to split students according to their area of interest. This doesn't mean that economists cannot take elective courses in areas not directly related to Economics. Nor it is forbidden for engineers, for example industrial engineers, or pure sciences students, to study Convex Analysis exhaustively. But to be consistent with what we mention in Section 2, other disciplines make each day more extensive use of mathematics in their field of study and research and therefore a more specialized mathematics is at order. Should mathematician give this kind of courses? Two reasons, at least, provide a positive answer. First, there are new areas of Applied Mathematics where great contributions have been made in the recent past and new fields appear; for example, the use of Numerical Analysis in economic research or the relatively new developments in Mathematical Finances. Second, following J. Marschak "The fact that an internally coherent and determined theory be or not be formulated in mathematical terms, doesn't change its logical essence; but it is easier to verify its coherence and its determination if it is stated in mathematical terms" (Frechet 1946). Our interpretation is that mathematicians can and are beginning to make incursions into other fields of knowledge.

With respect to Statistics and for those mathematicians who work in this area, their support to other areas is enormous. Sampling is done practically every moment be it in Biology, Psychology, Economics or Business Administration. Social Sciences historically adverse to Mathematics are using them and specially Statistics. It is not surprising to hear students at the end of their first degree where perhaps a thesis or monograph must be written to dispose of a "lot of measurements" but not knowing what to do with them. And such final manuscripts are demanded by their teachers to support empirically underlying theories. Decisions with political, environmental and economic consequences are taken on that basis and it must be said that in many cases a lack of rigor is present. So in this sense statisticians acquire a responsibility with respect to society. The consequences are the same as above:

it is necessary for the statistician to know about the field he is trying to apply to be able to support students in their first research and to give better and more supported courses.

The above arguments have consequences and the most important is that to teach mathematical courses at certain levels demands from our part to delve deep into other areas like the case of Economics mentioned. So all our formation and analytical thinking must be complemented with a broad insight into the parallel discipline. But another consequence that follows is that these advanced courses are also given by mathematicians now for separate groups according to the area of study.

4 Curriculum and conclusions

For first degree if we take a period of about 4 – 5 years, the big problem is how to accommodate the Mathematics courses into this time frame. The three basic courses, Differential Calculus, Integration and Series and Linear Algebra can be absolved during the first year. As argued, there is no need for a split and what methodology should be applied, whether massive courses or small sections or a combination of the two, is not a subject of this paper. It is important because of the use of technology to support learning, but in any case it should not take more time. In this first group of students, Mathematics, Physics, Engineering, Biological Sciences, Economics and Business Administration can be put together. For the Social Sciences during the first year a fundamental sequence of courses in Calculus, Linear Algebra and Probability are sufficient to build upon these a one year course in Statistics which should include Sampling theory. We have therefore identified two big groups; each of them are together at least during the first year. For the second year, and depending of the available space, the first group can be divided into two subgroups. One that goes into Multivariate Calculus with more emphasis in Physics followed necessarily by a Differential Equation course. The other one goes more in the direction of multivariate Optimization. On the other hand, both subgroups can meet once more in a Probability course and a Statistics course. In this meeting the Biologist should not be absent.

Perhaps it sounds easy and simplified but it gathers the ideas expressed in this paper whose main conclusions are that it is not necessary to split students according to their field of study in the basic mathematical courses. The second important conclusion is that for the mathematician to give more advanced courses for other areas it is essential to get a broad insight in the respective discipline. And last but not least, the same applies for the statistician. Some fields of knowledge like Architecture or Medical Sciences were not involved in our analysis. Here is a broad field of study where some concern has been expressed but where proposals are scarcely beginning to be handled. I am also not sure that the perspective adopted in this paper is excessively tailored to conditions imposed in my country and university, but I hope that at least our reflections will serve other colleagues in other countries.

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Table 1 – Marks according to area of study in Differential Calculus 1997 –1999

	Mark frequencies						
	D	C	B	A	Total	Num. Obs.	Weighted mark mean
Engineering	4,4%	29,9%	56,4%	9,3%	100%	2344	3,1245
Economics	6,4%	18,8%	65,7%	9,1%	100%	405	3,2692
Mathematics/ Physics	2,5%	34,6%	50,6%	12,3%	100%	81	3,2026
Biological Sciences	24,6%	36,6%	36,9%	1,9%	100%	415	2,7827
Business Administration	11,1%	41,6%	45,9%	1,4%	100%	370	2,7568
Others	27,9%	24,6%	39,5%	8,0%	100%	276	3,1498
Totals	4,4%	29,9%	56,4%	9,3%	100%	3891	

Note:
(i) Marks go from 1,5 to 5,0
(ii) Maximum mark is 5,0; Minimum to gain the course is 3,0
(iii) C=1,5 - 2,5 ; B=3,0 - 4,0 ; A=4,5 - 5,0
(iv) D=retired before end of semester