A CASE STUDY IN THE HISTORY OF CALCULUS REFORM

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ABSTRACT

A case study of calculus reform at the University of Wisconsin-Eau Claire is presented. Instruction of calculus at this institution has passed through four identifiable stages. Assessment of these stages are discussed and reasons for changing modes of instruction are explained. A conclusion is that teaching environments need to be designed to accommodate different teaching styles and learning styles.

1 Background

Assessing pedagogies for calculus instruction is a complex undertaking because of the diversity of students, faculty, departments and institutional missions involved. Case studies at the department level have proven to be an informative methodology for learning about mathematics instruction [15].

The University of Wisconsin-Eau Claire is an undergraduate university with about 11,000 students. About half of its freshmen graduated in the top quarter of their high school class, and about 62% are female. Formerly a teacher's college, the University still has large elementary and secondary teacher education programs. Students in these programs, together with students majoring in mathematics, computer science, physics, chemistry, and geology, constitute the core of the population taking calculus at UWEC.

The department of mathematics at UWEC is traditional in many ways, but it has an established reputation [9] as a department that encourages experimentation in teaching. Calculus classes at this institution typically have 15-35 students per section, and are always taught by tenured or tenure-track faculty.

2 The Stages of Evolution

Calculus instruction at UWEC has evolved through four stages, beginning with "traditional lecture." The second stage consisted of some courses being taught traditionally while others were "experimental." The third phase, which we will call "hybrid calculus," came from an effort at compromise, while we will call the fourth and current phase "post-hybrid."

2.1 Stage I, or the 4-0 model

Prior to 1995, calculus instruction at UWEC consisted of four one-hour lectures per week, and cooperative learning was not widely employed. This is evidenced by the first two questions on the department's student evaluation form:

- My professor's writing on the chalkboard or overhead is readable.
- My professor's voice is clear and understandable.

We will call this model of instruction "4-0," to emphasize the four hours of traditional lecture [8]. All sections used (and continue to use) a common textbook, and there has always been a common final exam. Other than this, prior to 1995 there was no tradition of collaboration or coordination between instructors.

2.2 Stage II, the introduction of the 2-2 model

In 1995, two of the authors attended a one-week workshop about the C^4L calculus reform project[1] [3, 11]. In Fall 1995 they team-taught two sections of Calculus I using the constructivist pedagogy of C^4L by meeting with students for two hours of lecture and then for two one-hour time blocks in a computer lab (hence the terminology "2-2").

[1] Calculus, Concepts, Computers and Cooperative Learning

The other sections continued to be taught using the 4-0 model. The environment of the computer lab was that of a "general access computer lab." This is a computer lab designed not for any pedagogical purpose, but rather for general student use primarily outside of class time. The software used initially in the 2-2 sections was ISETL, a simple mathematical programming language, while Derive was used for symbolic computation. Both softwares were replaced with Maple in 1997.

The constructivist pedagogy of C^4L is complex and well documented [4], so we shall here describe only the so-called ACE cycle. The acronym stands for Activities, Classroom, Exercises. The idea is that students first encounter ideas and construct concepts in the *activity* phase, which takes place in a cooperative learning environment using computer software. *Classroom* discussion of the concepts follows, and then solidification occurs in the *exercise* phase.

Soon all three calculus courses had 2-2 as well as 4-0 sections. Those students in the 2-2 sections who were able to rise to the challenge of a new pedagogy had a rewarding experience. A qualitative study [2] of a 2-2 section of Calculus I, during the summer of 1997, revealed that students liked the technology labs, felt they understood the concepts, and attributed their success largely to their cooperative groups. Small group discussions helped the students "see all parts of a question," as one group expressed it.

A second assessment [1] compared students in the 2-2 model and the 4-0 model, using faculty perception of success. The methodology for this second assessment was inspired by Wright [16], who assessed the effectiveness of pedagogies in chemistry classes at UW-Madison. The idea behind this methodology is reminiscent of the Turing Test for artificial intelligence[2]. Faculty from departments such as physics, chemistry and mathematics probed the content knowledge and attitudes of students in the case study by interviewing them, and the investigators concluded that there was no discernable difference between students from the two types of sections.

Four semesters into the program, it became apparent that students had very mixed reactions to learning mathematics via the ACE cycle. Anonymous student comments gathered throughout these two years illustrate the range of student reactions.

- Functions are not that hard to do but when you have to learn to tell the computer what to put in, it gets a little tough. I'm starting to like learning more about functions now that I have a better grasp on things. I probably would like a lot more of my homework if I had the hands on experience I had from this class.

- I must say that my ideas on the nature of mathematics have definitely changed this semester. $\cdots$ I have gotten a small taste of what making mathematics really is. Sure, you might be able to slide by with only knowing "finished mathematics." but what do most employers like to see in the people they hire? The ability to think and do things on their own.

- My own ideas of math have changed this semester. I had always found math to be quite easy, but I had some difficulty this semester. $\cdots$ Now I have had to learn to think and reason on my own to solve problems. I think, even if I have found it somewhat difficult, that I appreciate math much more and I am also more intrigued by math.

[2] An interrogator poses questions to a computer and a human via "teletype" so that the interrogator cannot visually distinguish between the two. The task of the interrogator is to distinguish between the two candidates by simply asking questions. If the machine can consistently "fool" the interrogator into thinking that it is the human, then we agree that the machine is intelligent, or that it can actually think.

- The computer stuff would work better if we went over the concepts in class before we did the lab.
- I am against working with computers in math. I think you should learn by the book.
- I didn't like the computers. I guess it's just difficult for me to relate calculus and ISETL. I didn't learn much from the computers.

To address students concerns, instructors modified the 2-2 sections. The ACE cycle was replaced the LAB cycle: *Launch*, *Activities*, *Build*. The idea for this learning cycle was that students would be "launched" into the exploration of new concepts by introductory lectures. The *build* phase was the same as the *exercise* phase of the ACE cycle, where students do numerous homework problems to solidify concepts. Also, the software was switched to Maple, which combines the "mathematical programming language" and "symbolic computation" aspects of C^4L into one software package that has a glitzier interface than does ISETL.

Another challenge to this reform program came from the mathematics instructors, many of whom were skeptical or hostile to the new pedagogy. There was clearly no hope that the department would agree to have the 2-2 format in all calculus sections. Having two distinct delivery systems for calculus had its shortcomings. The many students who switched systems midway through the sequence were fearful of success in their new environment. Instructors of post-calculus courses expressed dismay at the different approaches to mathematics exhibited by the two groups of students. Students in the 2-2 group were more fluent in the use of computers in mathematical investigations, as evidenced by the large proportion of the talks given by 2-2 students in an annual mathematics department day-long event, yet some traditional instructors complained that the 2-2 students were short on paper-and-pencil skills, while the 2-2 instructors felt the need for more contact time with the students.

2.3 Stage III or Hybrid Calculus and the 3-2 model

In order to solve these problems, faculty in the department decided to form a mixture of 4-0 and 2-2, and call it "hybrid calculus." In this compromise, students enrolled for a lecture, scheduled to meet for three one-hour blocks per week, and for a lab, which was scheduled to meet for one two-hour block per week.[3] This gave students a menu of lecture times and lab times to choose from, and provided the following benefits:

- All students would receive a common experience.
- Faculty who did not feel comfortable with "lab" contact could contribute by lecturing, and faculty who enjoyed lab contact could teach the labs.
- Since exams were conducted during labs, the students were given a two-hour block of time for tests. This practice had the benefit of freeing up additional lecture time, which was decreased from four to three hours per week under this system.

Such a structure clearly necessitated that all lecturers coordinate dates for presentation of material. In a given lab, there were typically students from several different lecture

[3] Thus there were $3 + 2 = 5$ contact hours for a 4 credit course.

sections, and so for lab activities to be meaningful, there needed to be some coordination among lecture sections. Moreover, there needed to be some agreement on what component of a student's grade would be based on performance in lab.

This common 3-2 model was implemented on a trial basis for one academic year, and then the department began discussions on whether or not such a model should be adopted. After an acrimonious debate the department adopted by a narrow margin the 3-2 template with the following laboratory goals.

- Reinforce, through exploration and experimentation, mathematical concepts presented in the lecture.
- Create a mathematically rich environment for encountering, representing, experimenting with, and reasoning about mathematical concepts of the calculus.
- Provide an environment that supports and promotes higher order analytical thinking skills.
- Become familiar with and be able to apply technology in order to
  - enhance the environment for understanding the mathematical concepts of calculus.
  - prepare students to solve those complex problems which can best be understood and solved using technology.
  - prepare students to use technologies which will be needed in other courses in our mathematics curriculum.

Many instructors believed that little or no technology was appropriate for Calculus I. There was some concern expressed about faculty work load, since one of the five contact hours was not credited by the administration. But the fundamental concern of many faculty on the traditional side was their discomfort with having to coordinate grades, adhere to a time-line for coverage of material, and to collaborate and coordinate with other faculty involved in the course.

Two independent assessments of student attitudes toward lab sections were conducted, and a simple assessment of skills was also conducted. The assessments of student attitudes both indicated that students were more pleased with the 3-2 model than with the 2-2 model.[4] The assessment of skills was based on a comparison of performance on multiple-choice final exam questions in classes taught by a traditional lecturer who taught lecture sections in the 3-2 model. These results showed that student in the 3-2 sections performed significantly better than those in the 4-0 groups. See Table 1. Note that there was no pretesting of the students, which limits the conclusions that can be drawn from the results in Table 1.

2.4 Stage IV or post-hybrid...Evolution never stops

As indicated in Section 2.2, the primary student computing environment consists of general access labs not designed with pedagogical considerations. These labs are not appropriate for lecture, and are not readily adaptable for cooperative learning. Yet both common sense and research [5, 10] indicate that the teaching environment is critical.

[4] No assessments of student attitudes in the 4-0 model was ever conducted, so it is was not possible to compare attitudes of 3-2 with 4-0.

So during the development of the 3-2 model, faculty who were instrumental in the 2-2 and 3-2 models were applying for a CCLI[5] grant from National Science Foundation. An objective of the funded project [14] was to remodel a traditional classroom with round tables (instead of traditional desks) and laptop computers.

A central part of this project is to test the effectiveness of a classroom environment that allows for a seamless blending of lecture and lab. The project seeks to find an environment where

1. cooperation among students is natural,
2. students can easily go back and forth between lecture and exploration with computers, to emphasize the connection between lecture and lab, and
3. instructors can devote as much or as little class time as they please to computer work on any given day.

The second point above is informed by earlier assessments by the investigators. In particular, it is well documented [12] that women are an under-represented group in the computing sciences. Moreover, confidence has also been determined to be a significant predictor of mathematical achievement for women [6]. In one study, [13] it was found that math confidence was positively associated to computer confidence. In [7] there was some evidence that women might not be responding to the laboratory experiences in the same way as men, partly because of the computing environment. Therefore, it was felt by the investigators that an environment where students can more readily feel the connection between the lecture and lab might be particularly helpful for women students. The comparison of survey data [see Figures 1 and 2] collected in 3-2 classes with post-hybrid illustrates that this may be the case.

Currently some calculus instructors use the blended environment described above, while others use the 3-2 template with varying degrees of cooperation and coordination among the instructors. We hope the students will gain

1. a fluency with technology,
2. enhanced communication skills,
3. increased understanding,
4. problem solving abilities, and
5. an improved attitude toward mathematics

while maintaining necessary skills.

In summary the authors feel that this evolution of the calculus courses has had its ups and downs. But we feel that the process has introduced the faculty within the department to ways in which they can work together to develop and deliver an innovative curriculum. Transforming traditional classrooms and general access labs into combined environments is expensive and slow, but it seems that this is the most promising way to provide a common experience for students while at the same time addressing concerns related to different learning and teaching styles.

For further information, including other perspectives on these events, see

http://www/Academic/Curric/smithaj/curricpg.html

[5] Course, Curriculum and Laboratory Improvement

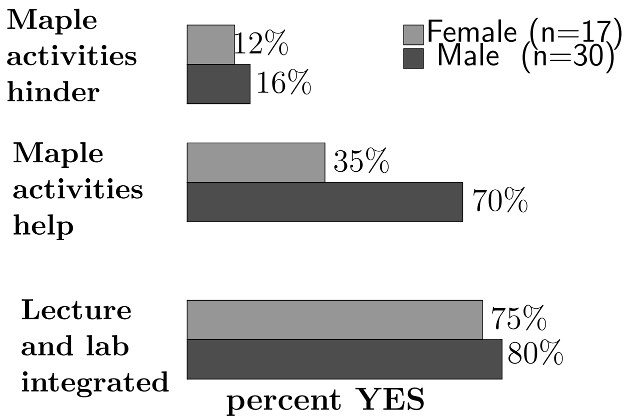

Figure 1: **3+2** Comparison of Female versus Male Responses

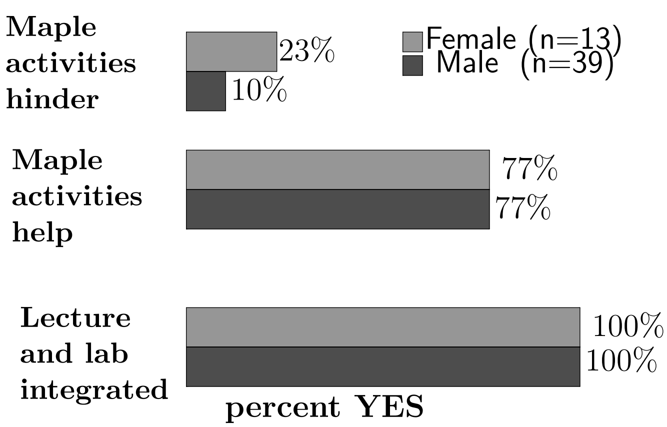

Figure 2: **Post-hybrid (Blended)** Comparison of Female versus Male Responses

Topic	% Correct (4-0)	% Correct (3-2)	Change
Fundamental Theorem of Calculus	21	100	+79
Integration of products of $\sin x$ and $\cos x$	68	82	+14
Improper Integral	32	82	+14
Arc Length	75	94	+19
Volume of Solid of Revolution	75	100	+25
Integrating a Rational Function	50	82	+32
Absolute Convergence vs. Convergence	54	53	−1
Limit of a Sequence	18	71	+53
Test for Absolute Convergence	71	71	+0
Geometric Series	50	41	−9
Maclaurin Series	50	71	+21

Table 1: Comparison of performance on final exam topics for a conventional lecture class and a hybrid calculus course [same instructor].

References

[1] A. Balas, B. Bansenauer, J. Clay, M. Goulet, and A. Smith, *Assessing calculus reform at UWEC*, Proceedings of the International Conference on the Teaching of Mathematics, Samos, Greece (D. Hughes-Hallett and I. Vakalis, eds.), John Wiley & Sons, Inc, 1998, pp. 35–37.

[2] A. Balas and J. Clay, *Student perceptions of success in a reformed calculus class*, Preprint, 1998.

[3] Ed Dubinsky, *Calculus, concepts and computers DUE-9053432*, NSF-DUE Course and Curriculum Development Award, 1990.

[4] Ed Dubinsky, *Meaning and formalism in mathematics*, International Journal of Computers for Mathematical Learning **5** (2000), no. 3, 211–240.

[5] Ed Dubinsky, *A better way of learning requires major changes in the teaching environment*, Proceedings of the Regional Campus Mathematics Conference, in press.

[6] E. Fennema and G. Leder (eds.), *Mathematics and gender*, New York: Teachers College Press, 1990.

[7] M. Goulet and J. Clay, *A comparison of student attitudes towards technology driven calculus activities versus technology flexible calculus activities*, Preprint, 1999.

[8] Steven G. Krantz, *How to teach mathematics.*, second ed., American Mathematical Society, 1998.

[9] C. Ney, J. Ross, and L. Stempel (eds.), *Flickering clusters: Women, science, and collaborative transformations.*, Madison: UW Press, 2001.

[10] M. Prosser and K. Trigwell, *Relations between perceptions of the teaching environment and approaches to teaching*, British Journal of Educational Psychology **67** (1997), 25–35.

[11] Alan Schoenfield, *Student assessment in calculus: A report of the NSF working group on assessment in calculus*, MAA Notes, Mathematical Association of America, 1997.

[12] L. Selby, K. Ryba, and A. Young, *Women in computing: What does the data show?*, New Zealand Journal of Applied Computing and Information Technology **2** (1998), no. 1, 74–81.

[13] Lily Shashani, *Gender differences in computer attitudes and use among college students*, J. Educational Computing Research **16** (1997), no. 1, 37–51.

[14] A. Smith, M. Goulet, and M. Penkava, *A combined mathematics and laboratory classroom environment DUE-0088254*, NSF-DUE Course, Curriculum and Laboratory Improvement Award, 2001.

[15] A. Tucker (ed.), *Models that work: Case studies in effective undergraduate mathematics programs*, Mathematical Association of America, 1995.

[16] J.C. Wright, S.B. Millar, S.A. Kosuik, D.L. Penberthy, P.H. Williams, and B.E. Wampold, *A novel strategy for assessing the effects of curriculum reform on student competence*, J. Chem. Educ. **75** (1998), 986–992.