LET THE STUDENTS EXPLORE ALGEBRA WITH CAS, TI89

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ABSTRACT

These symbolic calculating tools, CAS, Computer Algebra System, will change the way to teach mathematics more than the start of using graphing calculators did.
With these tools we will get more time over to discuss the concepts of mathematics, more time to let the students explore algebra themselves and more time to increase the understanding of mathematics. The question is not if, but when and how, we should use CAS in our math classes. In my presentation I will show some examples how to work with TI89 and simultaneously reinforce the concepts of mathematics. All students in my class use TI89 and the age of the students are 17-19 years old.

Let the students explore algebra with CAS, TI89.

All my students use TI89 as a tool in my math classes. It is some kind of a project because the school provide it with some financial funding. Further more there is another class that also works with TI89. In all we have 21 classes at a secondary upper school that takes mathematics.

Every test we do is divided in two separate parts. One part is without any helping technical tool and the other parts is with help of graphing calculator or symbolic graphing calculator, CAS. With this system we still demand a minimum bas of knowledge in mathematics with paper and pen.

I made a little investigation among my students and asked them what positive and negative reactions they have concern TI89.

The positive reactions are following:

1. The tool is often used as a control function. "When I work with a task, I often use "Solve" or other tools at TI89 to check if my thoughts are right.", said several students
2. If you get an answer from the TI89 that you really don't recognize, you get a very strong motivation to search and find out how this answer could appear.
3. Just working with CAS, TI89, is very nice and quick
4. You can easier see the general picture of the thoughts.

The negative reactions was:

1. You could sometimes be a little lazy with CAS, TI89
2. Some of the answers don't match the answer back in the book and sometimes you don't understand the answer.
3. TI89 seems little expensive comparing with ordinary graphic calculators.

Although my students now have worked with this tool one and a half year there are still many things to explore in mathematics and how to use this CAS tool.

We have the same tests for students at the same grade in our school. This tests are divided in two parts. One part where no technological tools are allowed, just paper and pen, and one part where CAS tools a are used. Here are some examples from the latest test:

A. If a function f(x) has the derivation f ´ (x) = 3x^2-7x+2. Use this derivation to make a sketch of the function f(x). Your sketch should be mathematical motivated. All calculating where you make conclusions from should be written down.

B. A cars price was from the beginning 265000 SKR. After 7 year has the value decreased to 150000 SKR. Suppose the value has changed exponential during the time.

a) How big is the exponential value decreasing each year?

b) After how long time is the value of the car 95000 SKR?

Motivate all your mathematical steps in the solution.

C. In a village with 974 people the number increase with 3,7% each year. At the same time in another village, with 1090 people, the growth is 2,4 % per year. How many year will it take until the both villages have the same number of inhabitants? Give the answer exactly and approximately. For maximum points you have to write down and motivate all your steps in the solution.

D. Decide exactly the equation(s) of the tangent(s) to the function y = 3x-x^3/4 and which are parallel with the line y = 1-9x.

We also have some national tests, because we have a compulsory curriculum in our schools.

My pupils have succeed very well in this tests.

The new way to teach math with CAS , Computer Algebra System, is now growing all over the world.
We have to face a new approach to make young students understand math better.

One way is perhaps to give them the answer of example from real life and let them make the right questions or the right polynomials. It will be some kind of **"jeopardy"** in math. It could be like this: The answer is that the two zeros are 1 or 6 and the maximum value is 4. *See picture 1*.

What will the question be?

The question will be: What zeros and maximum value will f(x) = - 1/16 (x-1)(x-6)

We can also randomly make a quadratic function equal zero and solve it in one order. You get two solutions (zeros). Look at the *picture 2*. Here from you ask your students if they can suggest one or two equations with these zeros. This is a way to make the students understand that every quadratic function can be at the form A(x-x1)(x-x2)

The students can with CAS simplify huge expression and get a very easy answer. But to understand this answer you also can use the CAS tool to explore and find out the steps to make a better understanding for this algebra. *See picture 3 and 4.*

Create and solve differential equations. *See pictures 5,6,7 and 8*

In a little village with 780 peoples a rumor is spreading with a velocity which is proportional to the number of peoples which do not know the rumor and the peoples that know. The constant of proportional is 0,05 %

Make a differential equation over the problem and then solve it exactly and numerical with a slope field. How many people know the rumor after 18 days?

An investigation of cubic functions can be a good test of CAS tool.

If you draw f(x)= (x-1)(x-3)(x-6)=x^3-10x^2+27x-18 you discover the three zeros. If you take the mean value of two of the zeros and draw a tangent at this value you will hit the third zero with the tangent. *See picture 9*

After this investigation I ask my students to show that this is true for all cubic functions with three real zeros. Here is the CAS tool TI89 really a good help. The proof of this is at the pictures 10 and 11.

Perhaps this is correct for every line who has three intersections of the cubic function? See picture 12 It seems correct but the proof of this will I turn over to the reader.

Here is another example how to use CAS tool.

If you draw a tangent to the square root function y= √x at any point, you will notice that the tangent will intersect with the x- axe . Investigate how far from origin this intersection will be if you select the tangent point at x = a. See picture 13

Give a proof that the intersection value is equal to –a . See picture 14

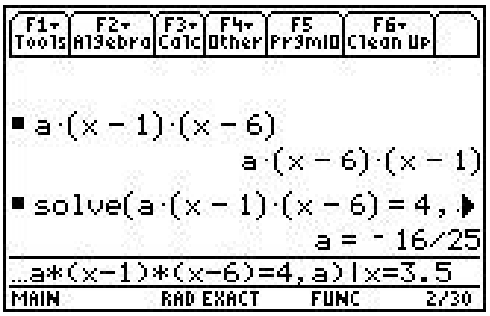
If you have a beam that will hit the square root function at a and reflect at the tangent, the reflection beam will intersect with x-axe at c. Independent what beam you start with the reflection beam always will hit the same value. Try to evaluate the value c. See picture 15.,16,17 and 18. Here you have to notice some conclusions from the above example.

Conclusion: Your students can faster reach the goal and have some time over to reflect about the problems. Even rather weak students will quickly understand the meaning with the

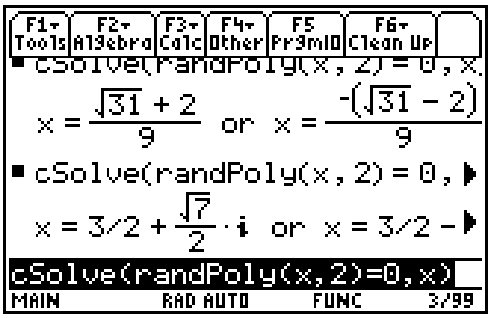
mathematical thoughts and will not be disoriented in all the algebraic labyrinth with no exit. They can rise their heads and see the entire meaning of the thoughts.

Bert Waits:
"Some mathematics becomes more important because technology requires it.
Some mathematics becomes less important because technology replaces it.
Some mathematics becomes possible because technology allows it."

Pictures to “Let the students explore algebra with CAS tool, TI89”



Picture 1



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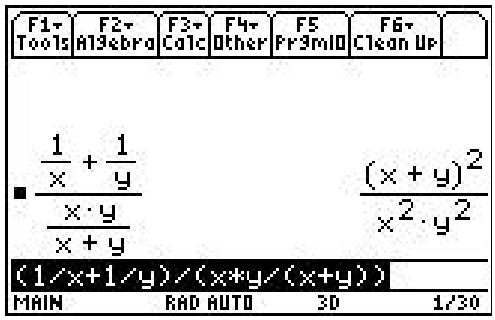

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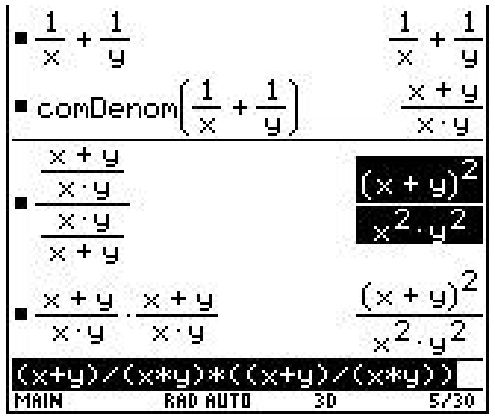

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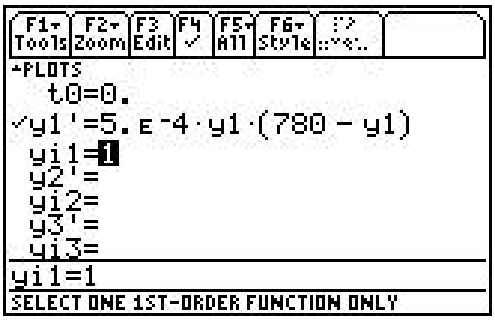
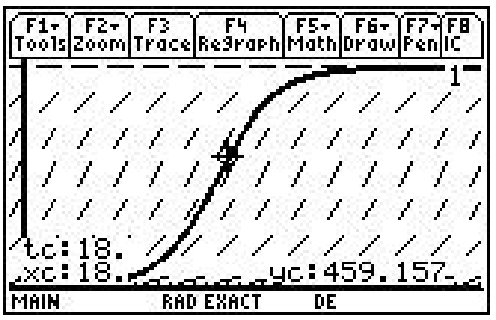


Picture 5



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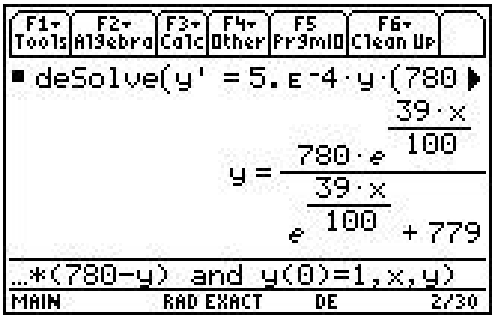

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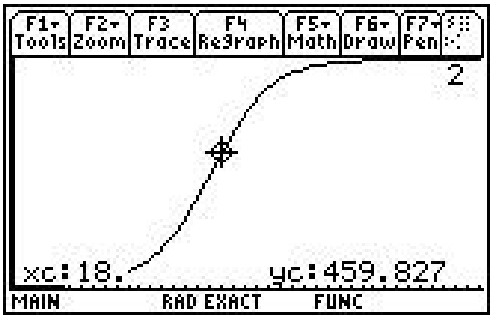

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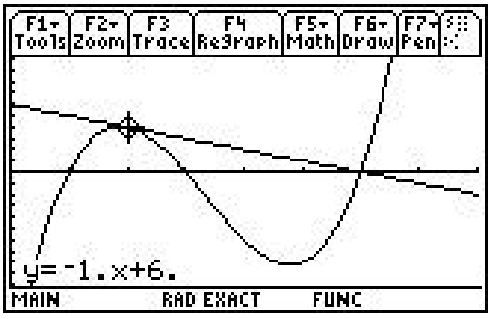
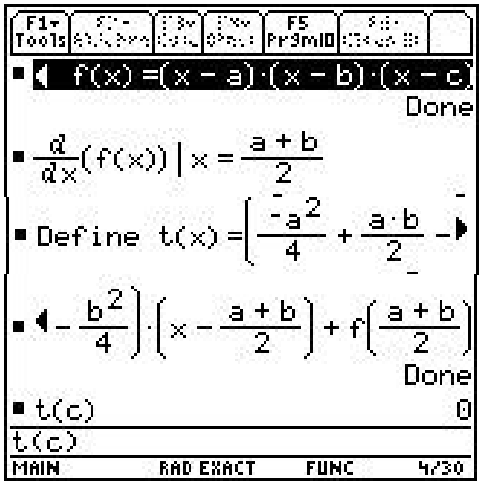
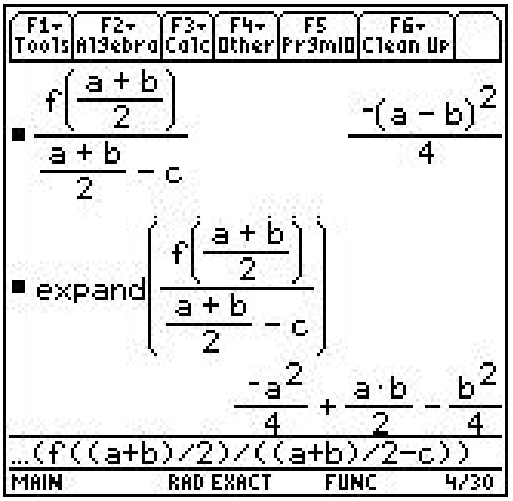


Picture 9



Picture 10



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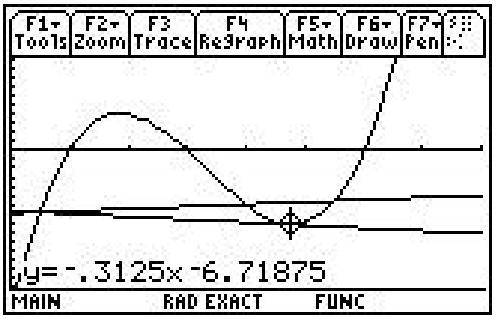
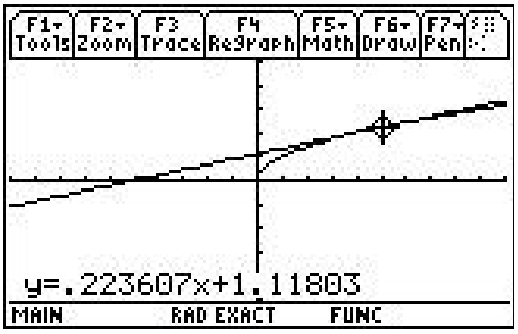
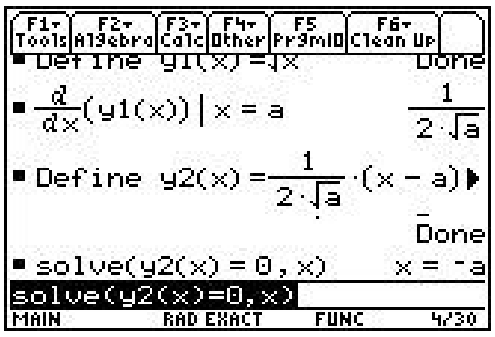


Picture 12



Picture 13



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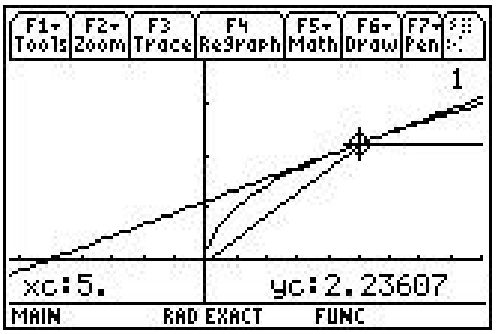

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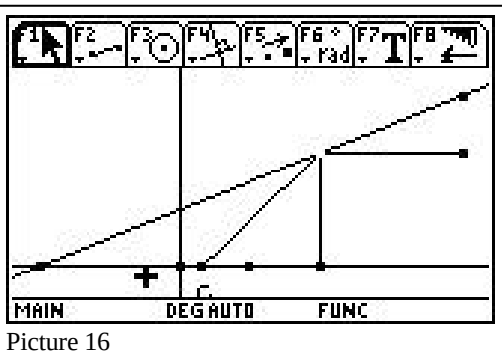

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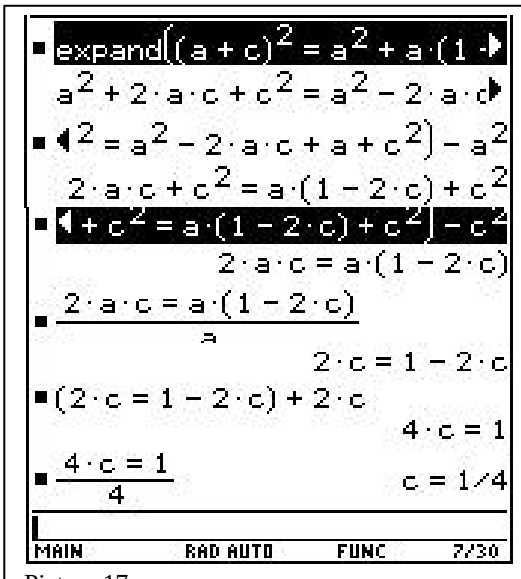

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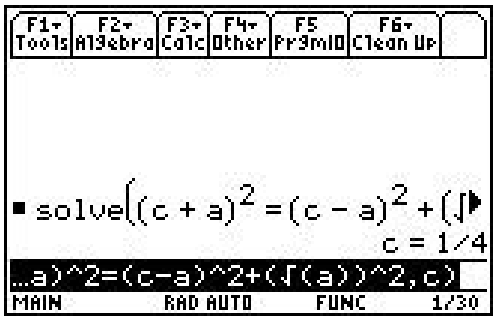

Picture 18