TARTINVILLE AND CABRI II

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ABSTRACT

We have examined the role that "Tartinville's method" can have in the qualitative analysis of parametric second degree equation, and in the teaching of geometry using Cabri II software.

Modern students do not know this method of analysis while they know Cabri II software.

First we examined the methods used to solve second degree problems since Euclid up to now, through the contributions of Diofanto, Pappo, Brahmagupta, Descartes, Newton.

Then we demonstrated how the Tartinville method could be geometrically interpreted via Cabri II, and how the different geometrical situations could be dawn up as a graph.

The most important aspect of the situation was reproduced through the drawing of Macros which made the solution of the problem easier.

At the end, we showed two relevant examples.

Keywords: parametric second order's equations, Tartinville's method, Cabri II^

1. Introduction

This article is the result of a debate evolved from a complicated subject like “Tartinville’s method” [3], and how it can become pleasant at school exploiting the main property of Cabri II, that is, the movement associated to a geometrical figure.

Tartinville (1847-1896) was a professor at Lycée Saint Louis in Paris.

He was well known because of his method of displacing the elements that permit to analyse and to solve a mixed system with one unknown, made up of a second degree equation f(x) = 0, whose coefficients depend on a real parameter k and one or two linear inequality of the type x   â, or x   á, or á   x   â.

2. Historical outline

The solutions of a second-degree problem by the intersection of straight lines and circle are a subject already present in Euclid’s *Elements* (3^{rd} century b.c.).

Exact or approximate numerical solutions of particular equations are in Heron of Alexandria (who lived a century before or a century after the vulgar era) and also in Diophantus (250AD) who never accepts either the negative solutions or the irrational ones.

The algebraic solution appears in Brahmagupta’s works (850); Descartes (1596-1650), who was the first to introduce the method of coordinates, quotes him in his own works.

In his work *Geometrie*, in three books, the rule of the signs, called Cartesius’, and the problem of Pappus is treated: draw through a point a straight line so that the part determined on it by two other straight lines is equal to a given segment.

At the beginning of the 18^{th} century, Newton’s *Arithmetica Universalis* (he was born in 1642 and died in 1727) was published, in which the most famous arithmetical problems are examined, the methods of separation of the roots and their approximation; for example Pappus’ problem is used for the drawing of the roots of a cubic or biquadrate equation by the intersections of a straight line with a conchoid.

The qualitative analysis of the problems, in the modern sense of the word, has been the result of the conquest of Algebra, from the second half of the 19^{th} century to the beginning of last century, and above all, from the discovery of Fourier and Sturm’s theorem, which allows to assign the number of real roots of an algebraic equation falling in a certain interval, solving implicitly, the problem of the qualitative analysis of the equation of degree 2,3, 4,… without having to look for the algebraic solution.

Secondary teaching and Mathematics, in particular, was flowering since the second half of the 19^{th} century. In the admission exams to several types of schools, problems requiring the qualitative analysis and/or the solution of second-degree problems or leading to them, were assigned.

From the publications of Academies and purely scientific memoirs, Mathematics started taking part in the debates of a more and more numerous audience. The so-called democratisation of elementary Mathematics started and the necessity of disciplining the methods of the qualitative analysis of the elementary problems had its origin.

3. Tartinville and Cabri II

The first scholar who dealt with such subject in a very direct and clear way was Tartinville, once “sadly” famous among the students of Liceo Scientifico.

The name of the French mathematician, Tartinville, is exclusively linked to the problem of the qualitative analysis of the second degree equations, and this method and he was widely studied at Liceo Scientifico, actually, that was the only method used to solve the problems assigned at the final exam at Liceo Scientifico up to 1969.

Thanks to the protest carried out by B. de Finetti, this method in particular and the qualitative analysis of the problems, in general, disappeared both from the syllabi and from the final exam at Liceo Scientifico.

Teachers who have taught this method usually believe that Tartinville's qualitative analysis is boring and cumbersome and it does not provoke any curiosity in the students but only a passive study of the subject; but we think that, not only the qualitative analysis of the problems, but also the method can be newly presented at school, using Cabri II software as useful tool for the teaching of geometry.

This idea should be placed in a wider cultural environment recognizing the importance of "external" events in the development of the mathematical thought.

So we decided to use the computer to make the teaching of geometry lively, to animate the geometrical object, a characteristic diffused in the geometry treatises of the 16^{th} and 17^{th} centuries.

Cabri II is an excellent support, in this sense.

4. General question

Once the figure representing a given problem has been drawn, we associate the unknown length x of a straight line segment with a point of abscissa x on the real line. After drawing the parabola which graphs the equation solving the problem, the pupil can verify that:

1- as k varies, the parabola of the sheaf varies with it and consequently the intersections with the segment vary;

2- as x varies, k varies with it and consequently it is possible to deduce the k values connected with one or two or no solution;

3- to each particular x value corresponds a geometrical interpretation of the problem of immediate representation;

4- in particular cases the figure degenerates and we can see why that happens.

At this stage, the student realizes that the link between mathematics and reality is very strong and he is urged to "materialize" an animated drawing, which reproduces the mental scheme of the mathematical tool.

However, being involved by the power of the images in movement on the screen, we have to avoid underestimating the necessity of proofs.

We conclude with a few reflections:

1- before examining a problem, it is necessary to make sure that the students own the necessary requisites;

2- under this circumstance, group work should be encouraged as it allows more constructive comparisons and debates, while the teacher should keep a discrete role;

3- during the correction of the mistakes, the teacher's presence should be more active and the completed works should be commented.

5. Operations whit Cabri II

In order to proceed to the drawing by means of successive **Macros**, it is necessary to define a few fundamental operations; i.e.:

Given two segments x and y, draw the segment of length x^2, the segment x+y, the segment xy.

MACRO: X^2

Draw a straight line r, on which pick the points 0,1,X

Draw the straight lines orthogonal to r through the point 0

Draw the circumference with centre 0 and radius 1

Draw the intersections between the straight-line s and the previous **circle**

Draw a straight line t through 0 different from s (*e.g.*.: the bisector of the angle r0s)

Draw the straight line k through 1 perpendicular to r
Draw B = t ∩ k
Draw the segment BX
Draw the straight line p through X perpendicular to r
Draw A = p ∩ t
Draw the straight line m through A parallel to BX
Draw r ∩ m = C = X^2
Initial objects: (r,0,1,X)
Final objects X^2
Name: SQUARE OF X.

MACRO X+Y

Given the straight line r, the point 0, the point 1, the point X, the point Y, the segment sum x+y is obtained drawing the symmetric of 0 with respect to (X+Y)/2.

Draw the straight line r, and on it, the points 0,1,X,Y
Draw M = midpoint of X and Y
Draw the symmetric of 0 with respect to M
Initial objects: (r,0,1,X,Y)
Final objects: X+Y
Name: SUM OF X+Y.

MACRO XY

Draw r,0,1,X,Y
Draw the straight line s through 0 orthogonal to r
Draw the circumference (0,1)
Draw a straight line b through 0 different from r (e.g.: the bisector of s0r)
Draw the straight line p through 1 perpendicular to r
Draw A = b ∩ p
Draw the segment AX
Draw the straight line m perpendicular to r through Y
Draw C = m ∩ b
Draw the straight line k through C parallel to AX
Draw r ∩ k = XY
Initial objects: (r,0,1,X,Y)
Final objects: XY
Name: PRODUCT XY.

By means of these Macros, varying the order of the initial objects we can obtain the following results:
the segment x-y (r,Y,1,0,X) on X+Y
the segment x/y (r,0,Y,1,X,) on XY
the segment x^3 (0,X,X^2) on X^2
the segment 1/x (0,X,1) on X^2.

6. Applications

We show a couple of examples; in the first example the student is asked to analyse qualitatively a parametric second degree equation applying successive Macros and to draw the parabola whose graph represents the solving equation in such a way he can see that, as the parameter k varies, the intersections of the parabola with the x axis vary with it and he can proceed to calculate the x value associated with the value of k. The values are pointed out by means of colours in the figure.

It is a useful exercise in the application of geometry to do algebra**.**

We suppose we have the parametric second-degree equation

$$x^2 + x(4-k) + k-5 = 0$$

with the conditions $-1 \leq x \leq 2$.

Given a system of Cartesian orthogonal axis, as Cabri II does not recognize the two axis separately, we draw the straight line r coinciding with the x axis and choose the points 0,1,x,k, on it (preferably 0 and 1 coinciding with points of the grid).

Next, we draw the segments x^2, x(4-k), k-5 and, at the end, the segment sum of x^2+x(k-4)+k-5 with M as one of its extreme points.

Such an end point will belong to the x-axis. We draw its symmetric with respect to the bisector of the first and third quadrant in order to move it on the y-axis. We draw the straight line through x perpendicular to the x-axis, the straight line through M perpendicular to the y-axis; their intersection point P generates the appropriate locus when x varies.

We can see how the parabola varies as k varies.

At this stage, if we want to work on the parabola as a conic section we should draw it picking 5 points belonging to the locus. Selecting the segment with end points –1 and 2 on the x-axis, clicking on k, the parabola moves and its intersections with the x-axis vary, accordingly.

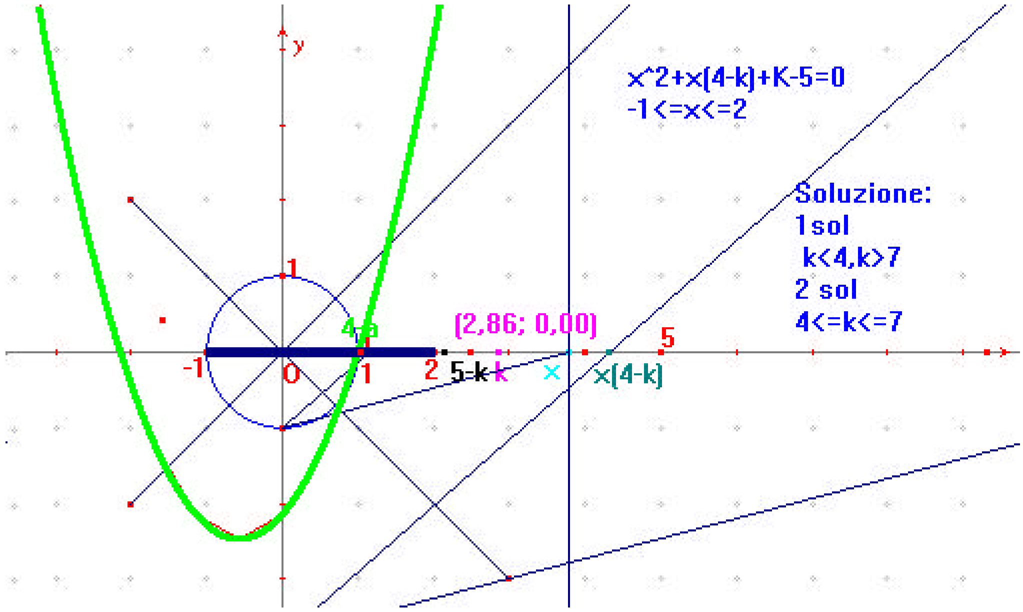

Fig. 1

The second example stems from the following problem:

Given the equilateral triangle ABC with length 1 sides determine on the side BC a point P so that AP^2+PB^2=k*AB^2. Qualitative analysis.

The student has to remember the drawing of the equilateral triangle with the compasses and the rule or else to open the Macro already defined in Cabri.

Taken X on the x-axis, we consider the point associated with it by means of the quadratic equation.

At this stage, we associate the point X on the x axis with the point P on the side BC so that PC=x. Animating the figure we can see how the initial figure is transformed as x varies.

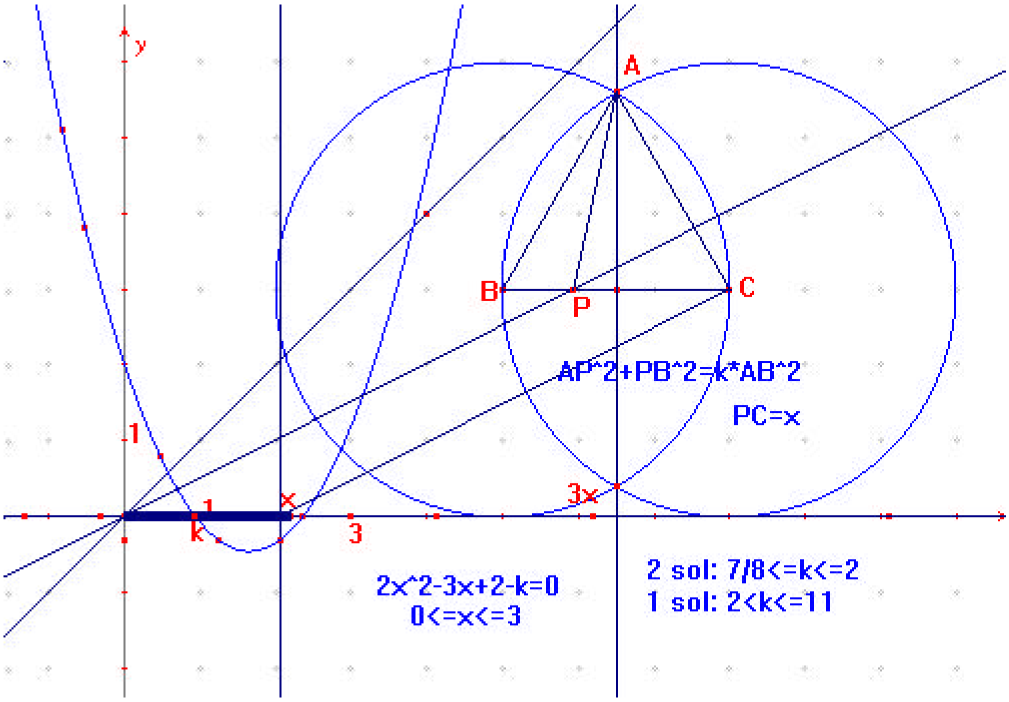

Fig. 2

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