STUDYING THE EVOLUTION OF STUDENTS´ THINKING ABOUT VARIATION THROUGH USE OF THE TRANSFORMATIVE AND CONJECTURE-DRIVEN RESEARCH DESIGN

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ABSTRACT

The paper describes how the transformative and conjecture-driven research design, a research model that utilizes both theory and common core classroom conditions, was employed in a study examining introductory statistics students' understanding of the concept of variation. It describes how the approach was linked to classroom practice and was employed in terms of research design, data collection, and data analysis. The many possibilities that the design offered for systematically researching students' conceptual change are contrasted to the limitations of the prevailing methodology employed by researchers examining conceptions of data and chance.

Introduction

The prevailing methodology employed by researchers examining conceptions of data and chance is to take snapshots of how students might make sense of stochastic phenomena at a specific point in time. Rarely does one do any follow up of students' initial thinking to watch for future transitions (Shaughnessy, 1997). Researchers such as Pratt (1998) are casting doubt on the validity of this research tradition, which ignores the influence of the setting on the shaping of intuitions, and stress the need for investigation of students' conceptions and beliefs in natural school contexts, for a prolonged period of time.

A recent trend witnessed in educational research is an increase in the study of exemplary instructional practices based on the argument that new classroom practices need to evolve from these "best practices" (Confrey and Lachance, 1999). However, this type of research might not be ideal for wide-scale implementation. A pressing need exists for designs which allow a more speculative classroom research by relaxing some the constraints of typical classrooms while keeping others in force. The paper describes the experiences from adopting such a design, called the *transformative* and *conjecture-driven research design* (Confrey and Lachance, 1999), in a study examining introductory statistics students' understanding of variation. It provides an overview of how the conjecture guiding the study was developed and was linked to classroom practice and outlines how the transformative and conjecture-driven approach was employed in the study in terms of research design, data collection, and data analysis. The rich insights into the evolution of students' thinking that have originated from this research are then briefly discussed.

Developing The Conjecture

Definition of Conjecture

The conjecture is a very important aspect of the kind of research described in this paper. It has two dimensions, a content dimension and a pedagogical dimension. It is also situated within a theoretical framework, which helps to structure the activities and methodologies used in the teaching experiment and link together the content and pedagogical dimension of the conjecture. A conjecture is "*not* an assertion waiting to be proved or disproved", but "an inference based on inconclusive or incomplete evidence"(Confrey and Lachance, 1999, p. 235). In research following the positivistic paradigm, hypotheses or theses are set beforehand and the study's sole purpose is to confirm or disprove their truth. In contrast, a conjecture-driven research design perceives theory development as an inductive process. The purpose of the conjecture is to serve as a guide and not to constrict the collection of data. During the course of data collection and analysis, as experience with the setting increases, the conjecture is subjected to several alterations and modifications.

Variation as the Central Tenet of Statistics Instruction Conjecture

The conjecture driving this study was that if statistics curricula were to put more emphasis on helping students improve their intuitions about variation and its relevance to statistics, we would be able to witness improved comprehension of statistical concepts. The motivation for the study gave the results of a previous study of students that had just completed an introductory statistics course. The results of that study (Meletiou, Lee, and Myers, 1999), agreed with the main findings of research in the area of stochastics education. We had found that the students we interviewed, regardless of whether they came from a lecture-based classroom or from a course that made heavy use of technology and interesting activities, tended to have poor intuitions about the stochastic and to think deterministically. This led me to conclude that student difficulties might stem from inadequate emphasis paid by instruction to the role of variation in statistical reasoning, as well as to students' intuitions. I decided to conduct a teaching experiment that adopted a nontraditional approach to statistics instruction with variation as its central tenet. Pfannkuch's (1997) epistemological triangle, as shown Figure 1, seemed well suited for meeting my research aspirations.

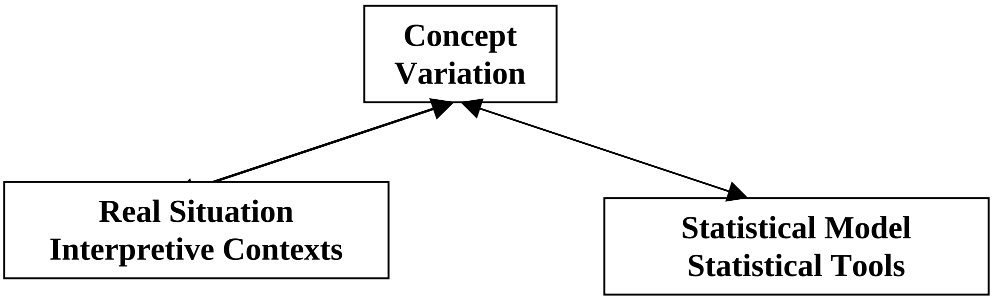

Figure 1: **Pfannkuch's epistemological triangle**

Pfannkuch's epistemological triangle views variation as the broader construct underlying statistical reasoning. In encouraging students to develop their understanding of the concept of variation it, at the same time, aims at promoting richer understanding of all the other main statistical ideas. The epistemological triangle indicates that for conceptualization of variation, a combination of subject and context knowledge is essential (Pfannkuch, 1997). The inter-linked arrows indicate the strong linkage that has to be created between the statistical tools and the context of the problem. The assumption underlying the epistemological triangle is that the concept of variation would be subject to development over a long period of time, through a variety of tools and contexts (Pfannkuch, 1997).

Pfannkuch's model, which bases instruction on contexts directly connected to students' experience, was a good alternative to typical approaches to statistics, which attempt to develop probabilistic reasoning through standard probability tasks. The model recognizes that adequate statistical reasoning requires more than understanding of the different ideas in isolation. It demands "*integration* between students' skills, knowledge and dispositions and ability to manage meaningful, realistic questions, problems, or situations" (Gal and Garfield, 1997, p. 7). Content is no longer a sequenced list of curricular topics taught in isolation, but "an interrelated repertoire of conceptual tools that can assist one in making sense of, and gaining insight and prediction over interesting phenomena" (Confrey, 1996).

Developing The Teaching Experiment

A transformative and conjecture-driven experiment is a planned intervention, taking place in a regular classroom over a significant period of time. What makes this research model unique and leads to a re-definition of the research-practice relationship is the dialectical relationship between the conjecture and the different components of instruction. Its research questions and methods of data collection are informed both by the conjecture and the components of instruction. Classroom research is speculative and while some of the constraints of typical classrooms are relaxed, others do remain in force (Confrey and Lachance, 1999).

Due to the need to continuously discuss and refine plans and interpretations, a transformative, conjecture-driven teaching experiment requires a team of researchers working together. In this study, I worked jointly with Dr. Lee, the course instructor towards designing the different aspects of the curriculum, towards refining and elaborating the conjecture and the components of instruction. Dr. Lee is a statistics education researcher with whom I had been collaborating for three years. He was therefore very familiar with the conjecture and acted as a research collaborator, providing invaluable insights that led to much better understanding and elaboration of the conjecture.

Context and Participants

The site for the study was an introductory statistics course in a mid-size Midwestern university in the United States. There were thirty-three students in the class (nineteen males and fourteen females). Most of these students were majoring in a business-related field of study. Only few had studied mathematics at the pre-calculus level or higher.

Curriculum and Classroom Setting

The design of the intervention was guided by the conjecture, while at the same time taking the time constraints and confines of the curriculum into account. Instruction included the set curriculum typically covered in an introductory statistics course, but was expanded in such a way as to include throughout the course activities that aimed at raising students' awareness of variation. The different topics were approached through the lens of the conjecture. The instructional approach employed in the course was based on the following two principles (adapted from Wild and Pfannkuch, 1999):

1. Complementarity of theory and experience: Statistical thinking necessitates a synthesis of statistical knowledge, context knowledge, and the information in the data in order to produce implications, insights and conjectures. If the statistics classroom is to be an authentic model of the statistical culture, it should model realistic statistical investigations rather than teaching methods and procedures in a sequential manner and in isolation. The teaching of the different statistical tools should be achieved through putting students in authentic contexts where they need those tools to make sense of the situation. Students should come to view to value statistical tools as a means to describe and quantify the variation inherent in almost any real-world process.

2. Balance between stochastic and deterministic reasoning: Instruction should view as an important precursor of statistical reasoning students' intuitive tendency to come up with causal explanations for any situation they have contextual knowledge about. It should present statistical thinking as a balance between stochastic and deterministic reasoning and should stress that statistical strategies, based on probabilistic modeling, are the best way to counteract our natural tendency to view patterns even when none exists, to distinguish between real causes and ephemeral patterns that are part of our imagination.

Instruction in a conjecture-driven teaching experiment changes over the course of the intervention in response to students' needs and inputs. In this study, curricular activities were designed to be flexible and open-ended. The instructor adapted them in response to feedback received from students. He would always situate instruction within contexts familiar to the learners. He would use analogies from students' everyday experience and would simplify mathematical relations in order to help build links to students' intuitions. He emphasized the complexity of real-life situations rather than making simplistic assumptions that would conflict with students' common sense. When, for example, discussing independent events, and after students had given typical examples of independent events such as coin tossing and die rolling, the instructor asked the class whether the success of a *"free throw"* of a basketball player is independent from the success of his previous *"free throw"*. Students argued that it depends on how the player responds to pressure, on how well he did on the previous throw etc. The instructor agreed remarking: *"In real life it's hard to say with a straight yes or no"*. He did not reject students' causal explanations although "hot hand" is an example often used by statistics educators and researchers to point out that people's tendency to detect patterns (hot hands) is often unwarranted. Tversky and Gilovich (1989) showed, using empirical data, that a binomial model well explains runs (streaks) in basketball player failures. According to this model, the chance of success in a shot is independent from the previous shot. One need not look for specific causes like nervousness since there is no other "pattern" than chance pattern explaining the data. However, the instructor understood what Biehler (1994) has pointed out – that even when the binomial model well explains the variation in a dataset, one should not exclude the possibility of alternative models, which give better prediction and which suggest causal dependence of individual throws. Similarly, when talking about slot machines in a casino, he noted: *"Although in theory when you put a coin and you pull it down and then you put another coin and you pull it down, although those two events should be independent, mechanically they might not be."*

The idea of making conjectures ran throughout the course. Students would state what they believed might or might not be true, and then looked critically at the data to evaluate their statements. Evaluation of conjectures would typically begin informally by using one or more graphical displays. The instructor would encourage students to describe the main features of the distribution displayed by the graph(s), always emphasizing the need to take into account not only the center, but also the spread. Students would look at the displays and try to give explanations for the patterns observed and for the departures from those patterns. Sometimes these explanations would be proposals for a possible model to summarize the dataset. The evaluation of conjectures would then become more quantitative. An analysis using appropriate numerical summaries would be made to support or refute the conjectures originally made by students. At the start of the course, the analysis was made using simple numerical summaries. Eventually, more tools were added to the students' repertoire and the mathematization of the data gradually became more formal.

Assessment

In order to enhance the understanding of the research setting and be able to provide answers to the research questions, a transformative and conjecture-driven experiment needs to use multiple forms of data generation. In examining students' learning progress and outcomes, a variety of both qualitative and quantitative data gathering techniques were employed. By assessing students' understanding prior to instruction, and then monitoring changes in their thinking throughout the course, the study attempted to develop a detailed description of the processes students go through in order to become able to intelligently deal with variability and uncertainty. The data gathering techniques employed included: (1) direct and participant observations, (2) interviews with the students and the instructor, (3) video-taping of group activities, (4) pre- and post-activity assessments, (5) fieldnotes, (6) samples of student work and (7) other relevant documents. Drawing data from several different sources permitted cross-checking of data and interpretations. The assessment strategies used to support and evaluate students' conceptual development helped students further clarify their reasoning strategies. The continuous monitoring of the effect of instruction on student learning was constantly supplying valuable information on their levels of concept attainment. This informed instruction, which was adjusted to promote deeper understandings, while also guiding the evolution of the conjecture.

Data Analysis

In a transformative and conjecture-driven experiment, there are two types of data analysis. The first type is the ongoing preliminary analysis, taking place throughout the course, guiding instruction and pointing towards necessary curricular revisions. This preliminary analysis, which begins simultaneously with the data generation process, is necessitated by the design's anticipation of emerging issues. Throughout the course, I would meet with the instructor on an almost daily basis. Each time we met, I would present him with some preliminary analysis of the data I had collected since our previous meeting. The implications of the feedback gained from students guided our decisions as to how instruction should proceed and what modifications of our plans were necessary. In addition to substantial revisions of the curricular interventions, this initial analytical work of cycling back and forth the existing data also led to revisions and elaborations of the conjecture, which however were of a smaller magnitude than curricular changes. Fledging hypotheses continuously got tested and evidence began to build. This analysis generated ideas for collecting new and often better quality data.

After the data collection stage was completed and all data had been generated and transcribed, the process of analysis continued in a more formal and explicit way. At this final stage I attempted, using a variety of both qualitative and quantitative analysis techniques, "to construct a coherent story of the development of the students' ideas and their connection to the conjecture" (Confrey and Lachance, 1999, p. 255).

Findings

The conjecture driving this study was that the reform movement would be more successful in achieving its objectives if it were to put more emphasis on helping students build sound intuitions about variation and its relevance to statistics (Ballman, 1997). Findings from the study suggest that the emphasis of instruction on the omnipresence of variation and the complementarity of theory and experience was indeed helpful in building bridges between students' intuitions and statistical reasoning. Students' understanding of graphical tools and numerical measures of center and spread was much more sophisticated than that of students in the previous study we had conducted. Instruction proved quite effective in achieving one of its main goals – helping students move away from "uni-dimensional" thinking and integrate center and variation into their analyses and predictions. Although not totally letting go of their deterministic mindset, students were much more willing to interpret situations using a combination of stochastic and deterministic reasoning. The course increased significantly their awareness of variation and its effects.

The investigation of students' conceptions and beliefs in a real school setting has also allowed me to gain wealth of information about the source of student difficulties and to enrich my initial conceptualization of the conjecture. I found, for example, the different meanings that students attached to variation as being one of the main sources of difficulties they had with comprehending sampling distributions. Several students viewed variation as sample representativeness and thus argued that the variation of a sample increases with increase in sample size. Similarly, others who viewed variation as range also argued that variation goes up with increase in sample size. These beliefs regarding variation of individual samples affected how students perceived the relation between sample size and variation of sampling distribution. Both of these groups of students shared the belief that the bigger the sample size, the higher the variation of a sampling distribution. Other critical junctures and obstacles to the conceptual evolution of the role of variation that emerged included the following: (1) Understanding of histograms and other graphs; (2) Familiarity with abstract notation and with statistics language; (3) Appreciation of the need to be critical of data and always examine the method by which it was collected; (4) Distinguishing between population distribution, distribution of a single sample, and sampling distribution; and (4) Understanding the reasons behind finding confidence intervals when producing an estimate of some parameter based on a sample. A detailed description of the rich insights gained from the study can be found in my doctoral thesis (Meletiou, 2000).

Conclusion

Hawkins (1997) stresses the need for more systematic research to guide developments in statistics education. The transformative and conjecture-driven design proved to be a promising alternative to the prevailing methodology employed by researchers examining conceptions of data and chance. It allowed thorough investigation of introductory statistics students' intuitive understanding of variation and use of the knowledge acquired to design, implement, evaluate, and refine meaningful interventions that helped students develop and expand upon their understandings. By examining how students' intuitions evolved during the course, I was able to identify structures that facilitated, as well as structures that inhibited, the articulation of intuitions about the stochastic. The wealth of information that emerged from the study is an indication of the potential of this research model for expanding our understanding of the components that promote development and growth of students' understanding.

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