MODELLING AND INTERPRETING EXPERIMENTAL DATA

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ABSTRACT

A lot of time is spent in traditional math courses for analysing graphs, computing minimal and maximal values, intersections and asymptotes of given functions. In some other examples the students have to find functions fitting to some given data, having special properties. Data are always given by the teachers and the situations treated seem rather artificial to the students.

Instead of this the students can do some experiments from physics and chemistry by themselves and try to model the data obtained by these experiments using mathematical concepts. However, it is rather time consuming to gather large lists of experimental data.

The collection of data during real experiments is supported by the CBL (Calculator Based Laboratory) and CBR (Calculator Based Ranger) from Texas Instruments. It is quite easy to transfer these data to graphic calculators for visualisation and further mathematical manipulations.

Various practical as well as mathematical skills of the students are trained by carrying out experiments, analysing the results and finally using functions for fitting data points obtained by the experiments.

We report about experiments being carried out in the years 1999 until 2001. In eight different classes consisting of students at the age of 16 to 18 experimenting with CBL, CBR and TI-92 was integrated within regular classes. About 50% of the students were girls. A special course for high ability students at the age of 14 was installed during the school year 2000/01 also carrying out experiments with CBL. In 2000 a group of students were testing the water quality in regular classes using CBL and ion selective probes from Vernier. The main goal of these projects was to train cross curriculum reasoning by the students.

The main basic skill in mathematics was to recognise the functional interdependency of experimental data and to find suitable fitting functions. For this reason the students needed some knowledge about different types of functions. They should know the typical shapes of the graphs and how the graphs change if the occurring parameters are varied.

Writing summaries of the experiments they understood the background of the respective experiments and some of the students wished to repeat the experiments to obtain better results. Interpreting results was difficult for the pupils especially in the course of testing water quality.

The students were really motivated. According to questionnaires and feedback forms they enjoyed practical work and felt free of the “pressure of learning”.

It was also new for the students to work in groups. They had to distribute work to different group members in accordance to their abilities. Finally, it was quite difficult to find a fair grading for the students according to their individual achievements.

1. Introduction

Cross curriculum reasoning is a principal objective of natural sciences in Austrian high schools [Aspetsberger 2001]. In science courses students have to understand laws from physics and chemistry written in mathematical terms and to apply them to real situations [Aspetsberger 1999]. On the other hand they should be able to model processes in physics and chemistry by mathematical functions, to see the interdependencies of experimental data and to interpret the results of calculations and the occurring parameters in fitting functions [Aspetsberger 2000 b].

In math courses students often do not see the necessity for introducing new mathematical concepts. Examples demonstrating the use of these new concepts are often quite artificial and the students have even problems to understand these examples. [Laughbaum 2000] suggests to use data collected during experiments carried out by the students themselves to promote mathematical understanding of (new) concepts.

It is very motivating for the students to carry out some experiments in science courses. However, they have to learn experimenting, how to obtain good results, how to document their work and to write reports and how to work in groups. It takes a lot of time to reach these goals, but they seem worth for doing this additional effort.

Graphic pocket calculators like TI-92 from Texas Instruments help to visualise mathematical concepts, to plot graphs and to execute tedious and complicated calculations like determining regression curves. Experimental data can be investigated and visualised quite comfortable by using the TI-92. The main problem is to obtain a large set of experimental data of high accuracy.

CBL from Texas Instruments is a Calculator Based Laboratory which allows to collect data during physical and chemical experiments. Data are stored directly to a calculator e.g. the TI-92 for graphical visualisation and further manipulation. CBR from Texas Instruments is a motion detector which allows to gather a large amount of data points from an object in motion. CBL, CBR and TI-92 support data collection and manipulation. However careful experimenting is absolutely important for obtaining good quantitative results, which are necessary for functional modelling of experimental data.

We report about experiments being carried out in the years 1999 to 2001 at the Bundesrealgymnasium Landwiedstrasse, which is an Austrian Grammar school in Linz. In eight different classes consisting of approximately 100 students at the age of 16 to 18 experimenting with the CBL and TI-92 was integrated within regular science classes. About 50% of the students were girls. It was an important goal of these courses to train cross curriculum reasoning by the students. Most of the experiences mentioned in this paper concern to these science classes projects.

In one of these projects a group of 20 students at the age of 16 was testing the water quality of freshwater during regular biology classes using CBL and ion selective probes from Vernier. Their quantitative results of several water samples were compared to the official data obtained from the local government. So the students had a good feedback according to the accuracy of their experimental work.

During the last two years we introduced CBL, CBR and TI-92 to math and science teachers within several in-service teacher training courses promoting cross curriculum teaching. It was surprising to see that they had problems similar to the students when treating experimental data.

2. Experiments carried out

The experiments carried out by the students should lead to a better understanding of physical and chemical laws. On the other hand the students should learn to model and interpret data using mathematical methods. Cross curriculum reasoning was a major objective of the project. We have been inspired by [Holmquist, Randall, Volz 1998] and reports of Texas Instruments. In these articles and books we also found detailed descriptions of the experiments and handouts which could be used within lessons directly. The experiments were carried out by the students in groups of two or three.

We started with simple experiments measuring the temperature of endo- and exothermic processes to demonstrate how to use the CBL-system. In the next experiments the students had to determine melting heat and melting temperature of ice. Due to inaccuracies the students did not obtain 0°C exactly for the melting temperature. It was interesting that not all students wondered about that fact, some of them also documented very unrealistic values in their reports. Most of the students documented all the digits displayed on the CBL and did not care about the significant ones.

Investigating the laws of Boyle-Mariotte and of Gay-Lyssac the students used pressure sensors. In these experiments the students had to learn to model data by functions. They tried to find the parameters for the fitting functions by themselves as well as to determine regressions curves with the graphical pocket calculator.

By determining the concentration of an unknown solution using a colorimeter, plotting the curves of titration using a pH-probe or measuring the concentration of salty solutions using a conductivity probe the students had to solve some typical problems from chemistry.

In traditional physics courses it is very complicated or almost impossible to investigate the movement of a body in motion by measuring the distance of the body according to time. Using the CBR of Texas Instruments it was very convenient for the students to obtain big lists of (distance/time) - pairs describing the motion of a body. Fitting data points by functions lead to a mathematical description and analysis of several processes of motion.

Being familiar with the handling of the CBL and TI-92 the students analysed the quality of freshwater (see [Johnson, Holman, Holmquist 1999]) by using ion selective probes of Vernier in laboratory and outside. An intensive and very accurate calibrating of the probes was absolutely necessary for obtaining good results. This was completely new for the students. On the other hand having good calibration values it was really easy to measure the concentration of several ions in freshwater. Having only single point measurements there was no sense for a mathematical analysis. It was much more interesting to interpret the results and to compare them with official limits. Furthermore the students learned about the methods of how to analyse freshwater quality. Visiting the local institution for freshwater control the pupils learned that the same methods were used there.

During pre- and in-service teacher training courses we treated experiments concerning the cooling process of liquids and the unloading process of a capacitor in addition to the experiments mentioned above. It was surprising to see that they had problems similar to the students when treating experimental data, since they were used to operate with “exact data” solely.

3. Functional modelling and Interpreting

It was a major goal of the project to find fitting functions for the data obtained during the experiments and to give physical and chemical interpretations of the results. The students had to find functions fitting data by hand as well as to determine regression curves by the calculator. However, we preferred the method of varying the parameters of functions, because it required some mathematical reasoning of the students to choose suitable parameters for fitting functions. In case of determining fitting functions by the computers automatically the students had to interpret the parameters of the regression equations obtained from the calculator. We had to give the students a short introduction concerning the least square method and the meaning of the coefficient of correlation. In the following we discuss several types of functions being required for modelling data obtained in the experiments above.

3.1. Linear functions and direct relationship

Modelling data obtained during the experiments proving the laws of Beer and Gay-Lyssac we use linear functions.

According Lambert Beer´s Law light absorption is directly proportional to the concentration of the solution. In this experiment the students had to determine the concentration of an unknown green coloured solution. The students had first to make a sequence of different solutions from a stock solution of known concentration and to measure their light absorption using a colorimeter. Due to Beer´s Law the concentration/absorption data points lie on a straight line (see fig. 1). Now the students had to determine a regression line by hand or automatically by using the pocket calculator. Obtaining an almost homogeneous straight line indicates the accuracy of the sequence of different solutions produced by the students. Next the students had to measure the absorption of light of the unknown solution and to determine its concentration using the regression line by hand calculation or from the graph directly (see fig. 1). Note the steady change of mathematical reasoning, chemical interpretations and practical work in this example.

Homogeneous linear functions are typical for direct relationship. It was quite unusual for students (and even teachers) to combine the concept of relations with the concept of functions (see also the comments concerning rational functions). Of course, they were aware of the fact that a direct relationship can be illustrated by a straight line, however it was new that homogeneous linear functions indicate direct relationships.

Modelling data points obtained in the experiment (Gay-Lyssac) investigating the pressure p of a confined gas according to temperature T (measured in °C) the students used linear functions of type $p(T) = m \cdot T + b$. The vertical intercept b indicates the pressure at a temperature of 0°C. However there is also a physical interpretation of the intercept with the horizontal T-axis. It indicates the absolute zero of temperature (-273,15°C).

3.2. Quadratic functions

Modelling the motion of a bouncing ball or a ball rolling down a ramp quadratic functions were required. This was due to the fact that the motion of a bouncing ball as well as the motion of a ball rolling down a ramp were special cases of the free fall which could be described by the formula $\frac{a}{2}t^2 + v_0 t + s_0$, where s_0 denoted the starting point and v_0 the starting velocity of the ball.

For modelling the motion of a bouncing ball it was more convenient to use quadratic functions in perfect-square form $a \cdot (t-b)^2 + c$, where b and c denoted the coordinates of the vertex of the parabola (see fig. 2). Suited values of the parameter b and c could be found using the Trace mode in the graph window. Many students (and even some of the teachers within in-service teacher training courses) tried to match the graph with positive values for a at their first attempt. It took some time to find out that a suitable value was $a = -4.9$. The students had also to give a physical interpretation of the value of a ($a = -\frac{g}{2}$, where g is the gravitational constant of acceleration $g = 9.81\ ms^{-2}$).

Some of the teachers within in-service teacher training courses tried to find the parameters in the following way: They chose the parameters b and c as the coordinates of the vertex of the parabola as described above. For determining the third parameter a they selected a sample point from the graph and tried to solve an equation depending on the variable a solely. However they had chosen a sample point very near to the vertex, so they obtained an unsuitable function. This was due to the fact that the data measured were not totally exact and even minor round-off error might cause wrong results. It turned out that even teachers were not used to handle "real data".

3.3. Rational functions and inverse relationship

Rational functions were required for modelling graphs of pressure p against volume V obtained by the experiment of Boyle´s Law. The students had to find suitable values for the parameter a within the functions $p(V) = \frac{a}{V}$ (see fig. 3). Functions of this type are typically for inverse relationships.

The inverse relationship between pressure p and volume V in the experiment above could also be proven by testing the relation $p \cdot V = const$. The students had to compute the product $p \cdot V$ in a data/matrix window of the TI-92 pocket calculator and to verify whether the product was constant. Although the students had done the calculation of the product they did not see the simple interdependency $a = const$ of the parameter a in the rational function and $const$ in the relation above by themselves. However, it was easy for the students to verify the transformation $p \cdot V = a \Leftrightarrow p = \frac{a}{V}$ algebraically. It was a problem for the students to combine the concept of functions with the concept of relations and algebraic manipulations.

3.4. Exponential functions

For modelling the decrease/increase of temperature during a cooling/heating process or the decay of voltage when unloading a capacitor exponential functions of type $a \cdot b^x + c$ were required.

Decreasing exponential functions of type $a \cdot b^x$ having the x-axis as an asymptote were suitable functions for modelling the decay of the voltage of a capacitor during the process of unloading. A typical graph of voltage against time of an unloading process can be seen in fig. 4. We had to choose suitable values for the parameters. For the parameter a we substituted the initial voltage 4.8. However it was not so easy to find a suitable value for the parameter b. Instead of this we chose an equivalent formulation of the type $a \cdot e^{-\lambda x}$ for decreasing exponential functions. For determining a value for λ we used the relation $\lambda = \frac{\ln 2}{\tau}$, with τ being half life which was the quantity of time for a decaying process to be reduced to half. We determined $\tau = 0.8$ from the graph (see fig. 4), i.e. after 0.8 s the voltage of the capacitor had decreased to 2.4 V which was half of the initial voltage. Since the unloading process started after 0.8 s we had to shift the graph of the exponential function $4.8 \cdot e^{-\frac{\ln 2}{0.8}x}$ in direction to the right by subtracting 0.8 from the x-values. Thus we obtained the following function $4.8 \cdot e^{-\frac{\ln 2}{0.8}(x-0.8)}$ fitting our unloading process.

Temperature vs. time graphs of cooling processes typically have horizontal lines $y = c$ as an asymptote, where c denotes room temperature. Similar to the modelling process above we had to find functions of the type $a \cdot e^{-\lambda x} + c$ shifting the graphs vertically by adding the constant c.(see [SCHMIDT 1995]) However, it was not so easy to determine regression curves for cooling processes by the calculator, since the TI-92 was able to compute regression equations of type

$y = a \cdot b^x$ solely. To manage this problem we had to shift data points first by subtracting the room temperature *c*, to compute a regression curve for the transferred data points next and finally to re-shift the regression curve by adding the constant *c* again. (see fig. 5)

3.5. Trigonometric functions

Trigonometric functions were required for modelling periodic processes, e.g. the motion of a pendulum (see fig. 6) or of a spring. The students had to find suitable values for the parameters of functions of type $y(t) = a \cdot \sin(w \cdot (t - b)) + c$, where *a* was the amplitude of the motion. *b* and *c* allowed to shift the graph of the function to left or right and up and down respectively. The value of *c* was easily found by measuring the initial position of the pendulum/spring. The frequency ω was determined by measuring the time T of a period according to the relation $w = \frac{2p}{T}$.

4. Experiences and comments

The students were really motivated. According to questionnaires and feedback forms they enjoyed practical work and felt free of the "pressure of learning". Some of the students also mentioned the importance of learning how to use technical instruments.

Experimenting with the CBL, CBR and the TI-92 (or comparable graphic calculators) required and trained several basic skills in different areas, e.g. mathematical skills, verbal skills, practical skills and social skills (see [Aspetsberger 2000 a]).

The main goal of the regular science class project concerning mathematics education was to recognise the functional interdependency of experimental data and to find suitable fitting functions [Aspetsberger 2000 b]. For this reason the students needed some knowledge about different types of functions. They should know the typical shapes of the graphs and how the graphs change if the occurring parameters are varied. Although the students had already learned the underlying mathematical knowledge in regular math classes, it was new for them to apply this theoretical knowledge in real situations. This was also relevant for even quite simple mathematical concepts like direct and inverse relationships. The students had to decide which relationship is applicable. E.g. there is an inverse relationship between the pressure p and the volume V of a confined gas if the product p*V is constant or there is a strong argument for a direct relationships if the data points lie on a straight line running through the origin.

A further mathematical goal was to confront the students with real data. From regular math lessons the students were always used to obtain exact results from their calculations. It was quite surprising for them to obtain from the experiments e.g. 0.576°C or 1.25°C for the freezing temperature of water instead of the expected 0.000°C. They had to learn that experimental data could be inexact and to see the need of statistical methods.

It was surprising how difficult it was for the students to read and carry out instructions stepwise without additional explanations of the teacher. However, it was much more unfamiliar for the students to document their work writing reports and interpreting the results obtained. This was really an important verbal skill, which the students had to learn. Writing summaries of the experiments they understood the background of the respective experiments and some of the students wished to repeat the experiments to obtain better results.

Concerning the use of the TI-92 during experimenting a secure handling of the various commands and features for the different representations of data would be very helpful however it was not absolutely necessary. In this case - at the beginning of the courses - we had to give more detailed instructions. At the end of the courses we gave only short descriptions of the experiments. Extensive instructions were sometimes confusing for the students.

One of the major problems was the lack of time. In regular lessons – lasting only 50 minutes - we had to explain the goals and the background of the experiments and afterwards the students had to do the experiments. However there was often no more time left to discuss the problems occurred during the experiments. The discussion and interpretation of the results obtained by the students had to be delegated to the next lesson, which sometimes was one week later.

Interpreting results was difficult for the pupils especially in the course of testing water quality. It was very hard to estimate the accuracy of the results obtained by the CBL-system. The students were not familiar with the necessity of calibrating the probes and they wrote in their protocols all (senseless) digits of the results indicated on the display.

It was also new for the students to work in groups. They had to distribute work to different group members in accordance to their abilities. The further problem for the group members was to accept a unique grade for the whole group. It was quite difficult to find a fair grading for the students according to their individual achievements.

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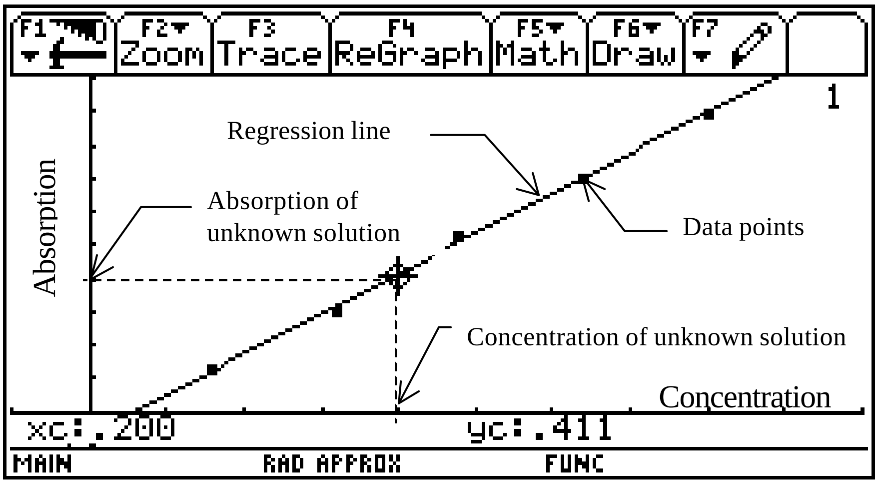

Fig 1: Determining the concentration of an unknown solution using Beer´s Law

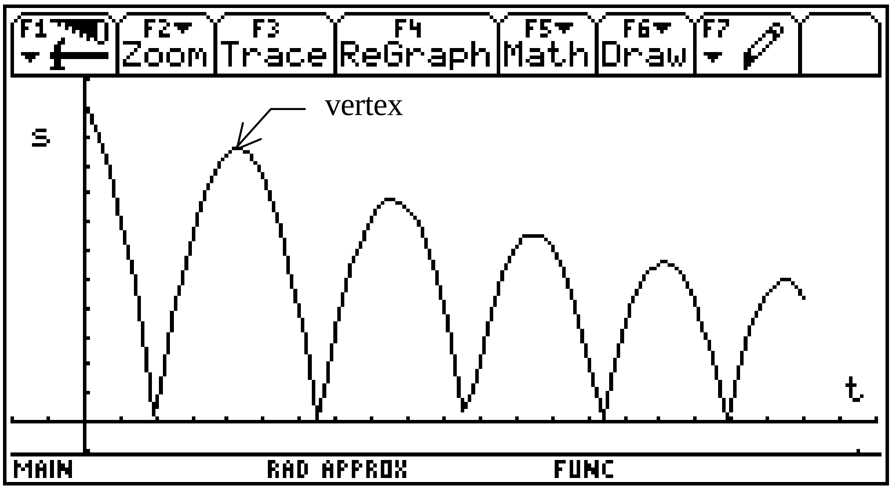

Fig 2: Motion of a bouncing ball. Quadratic functions in perfect square form $a \cdot (t-b)^2 + c$ are suitable for fitting the parabolas. The coordinates (b;c) of the vertices of the parabolas can be determined from the graph using the Trace mode.

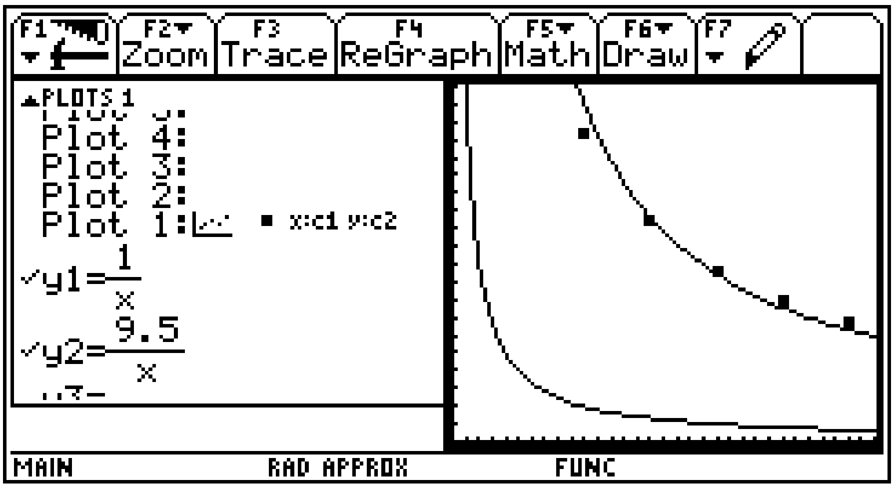

Fig 3: Using rational functions for modelling Boyle´s Law.

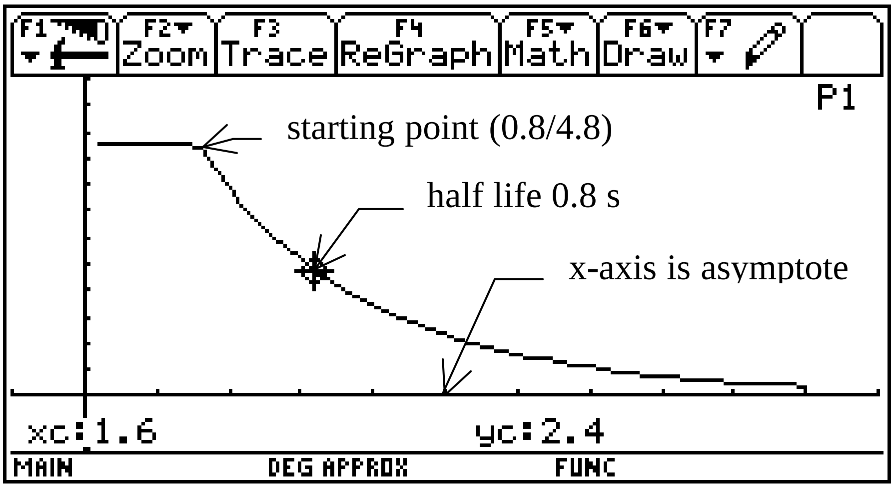

Fig 4: Using exponential functions for modelling the unloading process of a capacitor

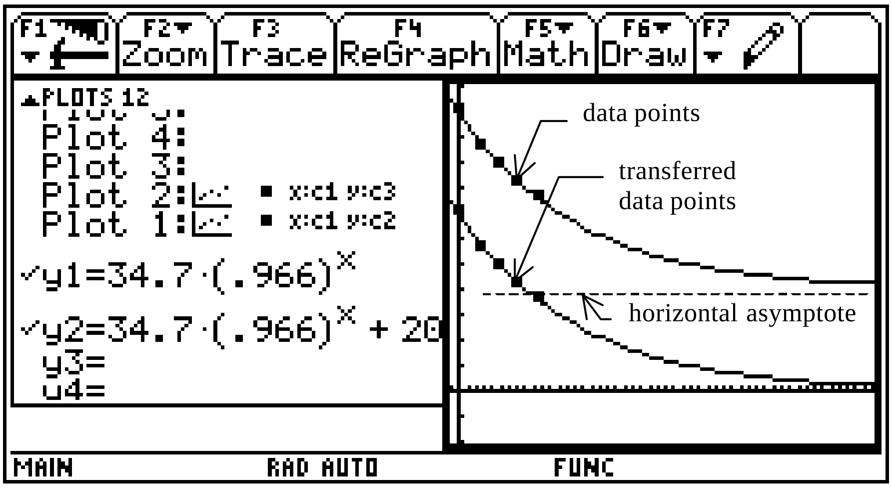

Fig 5: Using exponential functions of type $a \cdot e^{-lx} + c$ for modelling cooling processes having a horizontal asymptote $y = c$.

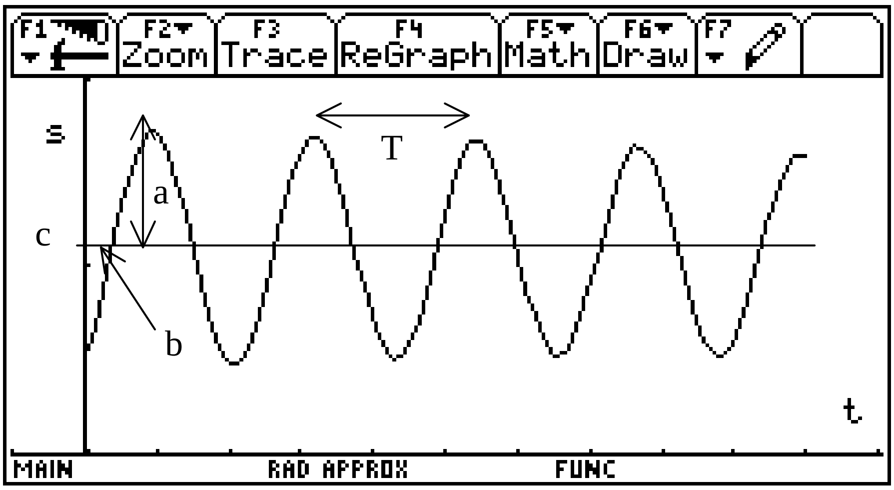

Fig 6: Modelling periodic motions of a pendulum using trigonometric functions of type $a \cdot \sin(w \cdot (t - b)) + c$.