WRITTEN META- COGNITION AND PROCEDURAL KNOWLEDGE

William BAKER
Hostos Community College, CUNY, New York City
500 Grand Concourse, Bronx, NY 10451,USA
e-mail: wbaker@hostos.cuny.edu

Bronisuave CZARNOCHA
Hostos Community College, CUNY, New York City
500 Grand Concourse, Bronx, NY 10451,USA
e-mail: bczarnocha@hostos.cuny.edu

ABSTRACT

The ability to express one's mathematical thoughts in writing and computational proficiency can be viewed as reflecting different aspects of an individuals' understanding of mathematics. Computational proficiency is the primary means used by educators to assess student's understanding of mathematics and thus in the mathematics classroom cognitive development is measured for the most part through students' ability to apply their procedural knowledge in a problem solving environment. In contrast, in written mathematics one's thoughts are not so much involved with the application of procedural knowledge as with reflection upon the concepts and procedures themselves.

In this article we analyze the relationship between an individual's ability to apply their procedural knowledge and the ability for meta-cognitive reflection and conceptual thought during written mathematics, with writing exercises designed in accordance with the framework for conceptual development of Sfard. (Sfard,1992,1994) and graded according to the scoring rubric set forth in Countryman. (Countryman,1992) This teaching research was done at a community college with students enrolled in the remedial courses of elementary algebra and basic mathematics who frequently displayed difficulties with computational exercises.

Key words : conceptual, procedural, cognitive development, mathematics writing.

Anderson's model of learning.

Within cognitive psychology, thought or cognition is typically viewed as containing both procedural knowledge and conceptual knowledge. For Anderson, procedural knowledge is, "knowledge of how to do things," it is frequently unconscious, and is housed in task-orientated structures called production systems. In contrast, conceptual knowledge, what Anderson calls, "declarative knowledge," is "knowledge about facts and things," it conscious and fully housed in the hierarchical structures called schema, organized by degrees of generality. (Anderson,1995).

In Anderson' s model, learning begins with actions on existing conceptual knowledge and with increasing knowledge the individual begins to internalize the procedures involved, incorporating them into productions systems, leaving aside the conceptual knowledge upon which the procedures arose. A process called "proceduralization." Thus, the acquisition of procedural knowledge is dependent upon existing conceptual knowledge and the knowledge gained by the repeated use of procedures or actions. (Byrnes and Wasik,1991)

Piaget and Vygotsky

Sierpinska in (Sierpinska,1998) contrasts the positions that Vygostsky and Piaget have on the ability of writing to influence thought and development. She states that, "Piaget would not claim that the activity of communication can change the course of development. On the contrary, he would claim that development is a precondition for a person to express him or herself clearly in writing." In contrasts, speaking of Vygotsky she states: "Vygotsky was claiming that writing can have an actual impact upon development."

Piaget Model

One explanation for Piaget's position is based on the relationship between procedural knowledge and conceptual knowledge inherent in his model of learning and development. Piaget would essentially agree with Anderson that learning begins with actions on existing conceptual knowledge and for both Anderson and Piaget an individual's ability to internalize procedural knowledge is an essential component in learning. However, for Piaget the relationship between procedural and conceptual knowledge is more complex, because in Piaget's view, after the individual gains proficiency with and internalizes procedural knowledge he or she begins to reflect upon this process and as a result gains new conceptual knowledge. (Byrnes and Wasik, 1991) In particular, for Piaget conceptual knowledge and procedural knowledge are both integral parts of a single cognitive schema, they are not separate. Thus, with models of learning based upon Piaget, concepts are assimilated into cognitive schema. Furthermore, this assimilation occurs in the advanced stages of cognitive development , which are characterized by abstract, metacogntive reflection and conceptual thought and are dependent upon completion of the first stage, the internalization of procedural knowledge. (Sfard, 1992)

This characteristic of models of development based upon the work of Piaget has lead the authors of (Haapasalo and Kadijevich, 2000) to hypothesize that such models subscribe to the "genetic view," which states that, acquisition of conceptual knowledge is dependent upon efficiency with or internalization of procedural knowledge. Therefore, the genetic view supplies a good hypothesis for Piaget's position that, written mathematical thought is dependent upon cognitive development.

Vygotsky model

For Vygotsky writing and algebraic thought are similar in nature because they both require conscious reflection upon previously unconscious or intuitive thought. More specifically, for Vygotsky, both writing and algebraic thought, involve conscious reflection upon what he would call the "spontaneous concepts" of speech and arithmetic. (Vygotksy,1986)

Thus, for Vygotsky written thought and algebraic thought are linked together or related by conscious reflection furthermore, unlike Piaget's model, which requires procedural efficiency before metacognition and conceptual thought, for Vygotsky algebraic thought begins with conscious reflection upon existing unconscious or "spontaneous" conceptual knowledge. Specifically, for Vygotsky written mathematical thought is not dependent upon procedural knowledge or cognitive development rather, it is an active agent in promoting such growth.

Research Question

In contrast to previous research directed towards establishing the effect that written mathematics has on promoting conceptual development or mathematical maturity we analyze the relationship between a student's ability to apply their procedural knowledge and his or her ability to reflect upon such knowledge during written mathematics. (Bell and Bell, 1985), (Lesnak, 1989) (Ganguli, 1989) In order to analyze this relationship, we measure procedural knowledge by the students' course average and we measure meta-cognition and conceptual knowledge through students' scores on writing exercises through out the semester. These two measurements of knowledge are then used as independent variables in a multivariate statistical model with cognitive development, measured by the students' GPA as the dependent variable.

Our goal was to analyze the relationship between written mathematical thought and procedural knowledge in terms of the contrasting viewpoints offered by Piaget and Vygotsky. On the one hand Piaget's position interpreted as the "genetic view" that, meta-cognitive reflection and conceptual thought during the act of writing are dependent upon procedural knowledge. On the other hand Vygostky position that conceptual thought and meta-cognitive reflection during written mathematics are beneficial in promoting development. Furthermore, the benefit an individual derives from such reflection is independent of his or her ability to apply their procedural knowledge.

Theoretical Framework

In order to employ writing as a tool to both measure and promote conceptual development we employ the three-step model due to Sfard (1991,1992,1994), which is based upon the work of Piaget. Then, we follow Shepard (Shepard, 1993) who matches levels of conceptual development with the appropriate writing categories due to Britton (Britton et. al.,1975). In the model of Sfard, concepts are assimilated into the schema in the last stage of a three-step abstraction process.

"A constant three step pattern can be identified in the successive transitions from operational to structural conceptions: first there must be a process performed on the already familiar objects, then the idea of turning this process into a more compact, self contained whole should emerge, and finally an ability to view this new entity as a permanent object in its own right must be acquired. These three steps will be called interiorization, condensation and reification." (Sfard, 1992, pp.64-65).

Interiorization

According to Sfard a procedure is interiorized when it, "can be carried out through mental representations, and in order to be considered analyzed and compared it needs no longer to be actually performed." We match the interiorization step, with the late initial learning phase and generalized narrative writing category in Shepard's work. In this phase Shepard recommends writings that produce, "personal examples of concepts" or that explain, "definitions of procedures in one's own words." In this phase we continually asked our students to translate algebraic expressions and expressions back and forth between language and symbolic language.

Condensation

According to Sfard, "at this stage a person becomes more and more capable of thinking about a given process as a whole without feeling an urge to go into details." In describing condensation, Sfard makes an analogy to computer algorithms when she writes that condensation allows the individual to look at a procedure as autonomous, "from now on the learner would refer to the process in terms of input-output relationships rather than by indicating any operations." On the effect that condensation has on an individual's ability for abstraction she writes, "Thanks to condensation, combining the process with other processes, making comparisons, and generalizations become much easier."

We match the condensation step of Sfard with the intermediate learning phase and the low level analogic and analogic writing categories in Shepard. In this phase Shepard suggests, "explaining how to solve a problem" and further, "explaining how concepts are relates," or explaining why, "concepts and procedures either do or do not apply." Thus, at this phase students' were required to turn their meta-cognitive reflection away from the definitions or rules of the procedures and towards the conditions that govern their use, as well as the difference and similarities between procedures or conceptual objects.

Reification

In the words of Sfard (Sfard, 1992), "the condensation phase lasts as long as a new entity is tightly connected to a certain process." Of reification she notes, "the new entity is soon detached from the process which produced it and begins to draw its meaning from the fact of its being a member of a certain category." In Sfard's model of conceptual development, like all models based upon Piaget's work conceptual development takes place in the framework of a cognitive schema. Thus, the last step of reification is identified with structuring and organization of one's cognitive schema, a step necessary for conceptual development. As explained by Sfard, for an individual who has not organized their schema, "information can only be stored in an unstructured sequential cognitive schemata." In contrast for an individual with a structural understanding, their cognitive schemata has a "compact whole" thus through a process of ordering or restructuring it becomes a "hierarchical schema." Furthermore without such an ordering, "there is hardly the place for what is usually called meaningful" (Sfard,1992).

We match the reification step of Sfard with the early terminal phase and the analogic-tautologic writing category used in Shepard, who recommends writing categories that involve, "speculating about several different ways to solve a novel problem." More specifically in our work we required students to focus their meta-cognitive reflection not on the procedures and not on the rules that govern their use but instead on the strategies involved when applying procedural knowledge in problem solving. Our objective was to encourage students' organization and structuring of their cognitive schema.

Results

For slightly over 180 students (n=183) the correlation between course average and GPA was R = 0.398, which was significant at the 0.01 level (high degree of significance). The corresponding R -square value was 0.158 and thus approximately 15.8% of the GPA was determined by course average. The correlation between writing scores and GPA was 0.402, which was also significant at the 0.01 level. When we used both course average and writing scores as independent variables together to explain GPA the R-value was 0.455 and the corresponding R-square value was 0.207. Thus, approximately 20.7% of the GPA was explained using course average and writing scores. This represents an increase of 37% over the 15.8% explained using only course average. It is not to be expected that course average and writing scores in one class would explain most of a student's GPA through out their college career. However, the 37% increase of explained GPA when writing scores were added to course average is an indication of the important role written mathematical thought has in learning and cognitive development. The F-value of this multivariate model was 23.952, which had a 0.000 significance rating, thus the use of writing scores and course average resulted in a very significant model in which neither course average nor writing scores dominated the other. In particular, writing scores were not dependent upon the ability to apply one's procedural knowledge.

Analysis

We have argued that written mathematical thought by its reflective nature is predominately composed of conceptual thought and meta-cognitive reflection upon procedural knowledge, both of which characterize the more advanced stages of development in Sfard's model. In contrast, we have argued that computational proficiency is predominately composed of the ability to apply procedural knowledge, which epitomizes the initial stage of development. Our result that, written meta-cognitive reflection and conceptual thought are independent of an individual's ability to apply his or her procedural knowledge provides evidence against Piaget's position interpreted as the "genetic view," i.e., the more advanced stages of cognitive development are dependent upon completion of the first "interiorization" stage.

This result indicates that reflection upon procedural knowledge is not always a by-product of the repeated actions that characterize the "interiorization" stage. Instead, meta-cognitive reflection can proceed during the act of writing about mathematics as well as through the process of repeated actions. Moreover, this result provides evidence in support of Vygotsky's position that development can proceed through reflection, while writing, upon existing conceptual knowledge independently of the "interiorization" process, i.e., reflection due to repeated actions.

In obtaining this result we stress that we do not interpret this as evidence that, mathematical educators should ask students' to reflect upon a procedure before being asked to perform the procedure. Instead we interpret our result to be an indication that many students have the ability to reflect on a procedure during written mathematics before they are efficient in applying the procedure.

We conclude with reminding the reader that the setting of this research was a community college with a high percentage of students who had difficulties computational problems, i.e., application of procedural knowledge. This study was designed to test whether such students could use language reasoning skills during written mathematics to assist in developing their ability for meta-cognitive reflection upon procedural knowledge that would then carry over to procedural knowledge. Thus, as educators we were pleased with the results.

REFERENCES

Anderson, J.R. (1995) *Cognitive Psychology and its Implications*, 4th edition, W. H. Freeman and Company.

Arzarello, F. (1998) 'The role of language in prealgebraic and algebraic thinking', in M. Bartolini-Brussi, A. Sierpinska, and H. Steinberg (Eds*.), Language and Communication in the Mathemati cs Classroom*, NCTM, Reston, VA, pp.249-261.

Bell, E.S. and Bell, R.N. (1985) 'Writing and mathematical problem solving: arguments in favor of synthesis', *School Science and Mathematics*, 85(3), March, 210-221.

Britton, J., Burgess, T., Martin, N., McLeod, A., Rosen, H. (1975). The Development of Writing Abilities, pp.11-18, London: Macmilliam.

Byrnes, J. and Wasik, B. (1991) 'The role of conceptual knowledge in mathematical procedural learning', Developmental Psychology, 27(5), 777-786.

Countryman, J. (1992) Writing to Learn Mathematics: strategies that work, K-12, Heinemann Educational Books, Inc.

Haapasalo, L. and Kadijevich, D. (2000) 'Two Types of Mathematical Knowledge and Their Relation', Journal fÜr Mathematikdidatik, 21(2), 139-157.

Ganguli, A.B. (1989) 'Integrating writing in developmental mathematics,' *College Teaching*, **37**(4) pp.140-142.

Lesnak, R.J. (1989) 'Writing to Learn: An Experiment in Remedial Algebra in Writing to Learn Mathematics and Science'. In P. Connolly and T. Vilardi (Eds.), *Writing to Learn Mathematics and Science*, New York: Teachers College Press.

Sfard, A. (1991) 'On the dual nature of mathematical conceptions: reflection on processes and objects as different sides of the same coin', *Educational Studies in Mathematics*, **22**:1-36.

Sfard, A., (1992) 'Operational origins of mathematical objects and the quandary of reification-the case of function'. In Harel, G. and Dubinsky, E. (Eds). *The Concept of Function: Aspects of Epistemology and Pedagogy*, MAA Notes **25**, pp.59-84, Washington:MAA.

Sfard, A. (1994) 'The gains and the pitfalls of reification-the case of algebra', *Educational Studies in Mathematics*, **26**,191-228.

Sierpinska, A. (1998) Three Epistemologies, Three Views of Classroom Communication: Constructivism, Sociocultural Approaches, Interactionism, in Bartolini-Brussi, M., Sierpinska, A. and Steinberg, H. (Eds.), Language and Communication in the Mathematics Classroom, NCTM, Reston VA..

Shepard,R.S. (1993) 'Writing for conceptual development in mathematics', *Journal of Mathematical Behavior*, **12**, 287-293.

Vygotsky (1986) Thought and Language, MA:MIT press.