CRITICAL FACTORS AND PROGNOSTIC VALIDITY IN MATHEMATICS ASSESSMENT

Christer BERGSTEN
Department of Mathematics, Linköpings Universitet
SE 58183 Linköping, Sweden
e-mail: chber@mai.liu.se

ABSTRACT

High school mathematics is traditionally more procedural than conceptual in character, as well as formally less rigorous, than is mathematics at the university level, and hence puts less demand on logical reasoning and conceptual understanding. To find an instrument to make a reasonably good prognosis for success in undergraduate mathematical studies, it is therefore necessary to look closely at the demands of the future mathematical activities rather than only more narrowly at what has actually been accomplished at the high school level in terms of content and methods. In this paper the development of a short test for prognosticating academic performance in mathematics is discussed, and the results from a group doing the test when entering university is related to the results on their first mathematics courses.

Based on research literature and an analysis of the demand of the courses, the design of the test was built upon ten factors that were found to be critical for passing the mathematics courses in the educational programme being considered: conceptual depth, control, creativity, effort, flexibility, logic, method, organization, process, and speed. The critical factors cut across the content-process distinction and are expressions of a holistic view of mathematical performance. To prognosticate academic performance it is necessary to identify important nodes of integration in the web of mathematical ideas, concepts, skills, forms, affects, and so on. The **critical factors** constitute vertices where the different dimensions of mathematical thinking meet.

In the paper the construction of the test is discussed, and the results show a strongly significant correlation to performance on the target undergraduate mathematics course. A notion of **prognostic validity** of the test is outlined and discussed. The paper shows how test construction, analysis and interpretation of the outcome, depends heavily on what the result is going to be used for, and how a mathematics assessment design by necessity leads into discussions about the nature of mathematics and the understanding/performance of mathematics. What seems to be typical in mathematical problem solving is that many of the critical factors are involved in one problem solving process and must be combined for success.

1. Introduction

School marks in mathematics alone may have limited value for prognosticating performance in mathematics at the university level. High school mathematics is traditionally more procedural than conceptual (cf. Hiebert, 1986) in character, as well as formally less rigorous, and hence puts less demand on logical reasoning and conceptual understanding. To find an instrument to make a reasonably good prognosis for future success in college mathematics, it is therefore necessary to look more closely at the demands of the future mathematical activities than only more narrowly at what has actually been accomplished at high school.

The development of an assessment instrument to prognosticate academic performance in mathematics is discussed, along with test results, compared to results from the first university course in mathematics for one group of students. An underlying assumption is that some of the general problems of assessment in mathematics become visible through the window of an example.

In mathematics assessment it is common to make the distinction between content and process variables, thus forming a matrix of combinations of different aspects of these two objectives[1]. In the NAEP mathematics assessment there are five content and four process variables. To the framework of the APU secondary assessment further dimensions affecting the assessment outcome have been added to the matrix, such as the mode of assessment, context, and attitudes. The content and process categorization is used also in The National Criteria for Mathematics, where as much as 17 process objectives are listed. (See Ernest, 1989, for descriptions and references) Content and process knowledge, or domain-specific and general-strategic knowledge, are closely related or dependent of each other (Alexander & Judy, 1988; Perkins & Salomon, 1989), making it difficult to separate them in a meaningful way in an assessment situation.

During the work with the assessment standards in the USA it has been stressed that any assessment in mathematics should deal with **important** mathematics: "Answers to the question *What is the important mathematics here?* Should be reflected in: • the plans for the assessment, • each assessment task and activity, • the interpretation of students' responses, and • the intended uses of assessment results" (NCTM, 1993, p. 29). It is part of the nature of prognostic testing that the mathematics achievement one is trying to predict deals with content unknown for the students at the time of the testing. Therefore it is necessary to look at what *aspects* of mathematical thinking are important for the future studies, and then find relevant known content.

Important factors for doing mathematics successfully have been analysed for example by Krutetski (1976), and an increasing number of studies also of advanced mathematical thinking have appeared (e.g. Tall, 1991; Holton, 2001). The choice of such factors must be based on literature studies and on experienced teacher judgement, including the marking of exams protocols (cf. Webb, 1992, p. 672). The term **critical factor** has been chosen here to indicate that with low 'levels' of these factors students will (most likely) meet problems to pass the mathematics courses considered. Also belief factors influence study results significantly (Niss, 1993; Webb, 1992), but will not be considered here.

---

[1] The meaning of the term 'process' is here vague, as it could refer to a specific mathematical skill, or to a general cognitive strategy.

2. Critical factors

In the present study 119 civil engineering students were enrolled in a four-and-a-half years programme with four different branches: Computer science (D), Industrial engineering (I), Mechanical enginee-ring (M), and Applied physics and electrical engineering (Y). Ten factors have been found to be critical here.

Conceptual depth – That mathematical concepts and procedures have been learned by root is often observed in students' attempts to solve well chosen problems. Conceptual depth shows for example when solutions are "simple" and accurate, right to the point without unnecessary complications over a number of tasks, but is often hard to trace in protocols.

Control – There are at least two aspects of control that are critical in this context. One refers to the "looking back" process of checking up a result that has been obtained and the feeling that it is reasonable. The other aspect is more delicate to describe but may be captured by the phrase 'I know what I'm doing', I'm controlling the mathematical entities I'm working with because I'm familiar with their properties (cf. Bergsten, 1993).

Creativity – In school mathematics fantasy, or originality in mathematical thought, is seldom emphasised, but when it shows is an indicator of problem solving ability. In the international mathematics education community there are now special conferences on creativity.

Effort – It can sometimes show in a protocol that the student has tried hard to work out the problem. For weaker students effort is one of the most critical factors. However, as this is an affective factor, it can't always be judged from a written response protocol alone.

Flexibility – The ability to change to a thinking mode suitable for the particular problem, for example to alter between a numeric, graphic, or symbolic form of representing mathematical ideas (sometimes called *versatile thinking*; see Tall 1991), is important for solving a wide range of mathematical problems. Included here is the ability to view mathematical symbols representing either a mathematical object or a mathematical operation to be performed (Gray & Tall, 1991; Sfard, 1991).

Logic – There are many faces of logic involved in a problem solving process. One is *rigour*, i.e. the extent to which a conclusion in the solution process is logically valid, and the necessary assumptions pointed out. Another face is *consistency*, i.e. the absence of contradictions. *Completeness, accuracy*, and *generality* in reasoning may also be regarded faces of logic.

Method – There are 'natural' and easy ways to solve a problem, and there are 'clumsy' ways. Choice of method with respect to its *efficacy* is a critical factor. Student often loose track in producing an increasing amount of 'algebraic mess'. The degree of *simplicity* may be viewed as a logical factor but may be considered a factor of its own as it goes beyond logic.

Organisation – This factor refers to the 'layout' of the written student response to a given problem. Is the logic visible, or are different points made randomly, as it looks, over the page?

Process – Is the student response predominantly *procedural* or *conceptual* in character? Messy algebraic manipulations are often indicators of a procedural approach, detached from conceptual understanding. Conclusions based on such an approach are often mathematically incorrect or meaningless. Figurative components, such as diagrams, reveal the presence of imagistic thinking, indicating that a conceptual approach has been used (cf. Goldin, 1987). Another such indicator is the text inclusion of reasoning in words, or short algebraic solutions. An integration of the procedural and conceptual process aspects is often stated a characteristic of understanding mathematics (Hiebert, 1986; Gray & Tall, 1991).

Speed – In university mathematics exams the speed factor can of course be significant for both the amount and the quality of the outcome.

A point of discussion is how the critical factors relate to the content-process distinction. Now, the distinction in itself is fuzzy (cf. above, and e.g. Lerman, 1989), and Perkins and Salomon (1989) advocate a synthesis. In any reasonable meaning of the terms, clearly content is involved in the conceptual depth factor, and process for example in the logic and method factors. In fact, method is the outcome of a content-process integration. Thus, the critical factors cut across the content-process distinction, and are expressions of a synthesis of the kind just mentioned, i.e. of a holistic view of mathematical performance. To prognosticate academic performance it is necessary to identify important nodes of integration in the web of mathematical ideas, concepts, procedures, skills, and so on. The critical factors constitute vertices where the different dimensions of mathematical thinking meet. That is why they are considered critical for prognosticating mathematical performance.

3. Test construction and results

With the previous discussion in mind, how should a written test be designed to predict the degree of successful academic performance in mathematics, and how should the responses be analysed and interpreted? It should not be possible to solve an item by direct reproduction of memorised techniques only, excluding pure routine tasks (cf. Christiansen & Walter, 1986) in favour of more complex problem solving. It is also obvious that all the critical factors above cannot be 'covered' in each one of the items. Therefore the design and the interpretation of the results must be based on an integrated local-global analysis. The rationales behind the selection of the items of the prognostic test[2] will be briefly discussed.

Mathematics consists, among other things, of ideas and the formal representations of ideas (cf. Mac Lane, 1986). One important idea is that of generalisation, often formalised by using algebraic symbolism (item 8). In mathematical problem solving the input is often an algebraic expression. The problem can consist of reasoning in algebraic terms (items 1 and 7), using only procedural knowledge or a combined (integral) procedural-conceptual approach by using for example numbers or diagrams. Mathematical reasoning can start with a diagram that needs to be analysed (item 6) and/or linked to algebraic symbolism (item 5), or a diagram may be constructed as a support for reasoning (item 4, possibly also items 3, 7, 8 and 10). Solving quadratic equations (item 3) is an example of a skill that can become highly automated, and is here used to reveal the presence of the control function. Control is critical also in items 1, 2 and 7. Hypothetical thinking is implicit in most mathematical problem solving, and has therefore been chosen as the core of a problem (item 9), keeping the 'technical' parts at a low level of difficulty. Pattern recognition is often fundamental for finding a solution to a mathematical problem (items 3 and 10).

To ensure effort the level of difficulty has been kept rather high, considering the students in focus and the time constraint (the speed factor). A test with tasks where only little effort is needed will not capture the status of the critical factors.

---

[2] The DP test, with 10 items; see Appendix A.

The achievement level may be viewed as a product or as a synthesis of the critical factors and content knowledge. A point of discussion is if the levels of the critical factors can be quantified and scored separately, or if they should be integrated in the achievement score.[3]

The person constructing the DP test (i.e. the present author) did not teach the calculus courses in question (but has done so previous years), nor did collaborate with the examiners, nor did they take part in the construction or evaluation of the test.

Out of a total of approximately 600 beginners at the civil engineering programme of the university, one group (i.e. class) from each of the branches D, I, M, and Y was randomly chosen, making a total of 119 students doing the DP test. The test[4] was administered before the beginning of the first regular mathematics course (calculus). Calculators or mathematical tables were not allowed. On each item the scoring was 0, 1, 2, or 3, where 3 was indicating a correct solution, with high levels on the relevant critical factors, 0 or 1 an insufficient solution, with low levels of the factors. Thus the range of the total score (sum) was from 0 to 30. Group means and standard deviations are shown in table 1, frequencies of different sums in table 2, and means of items in table 3.[5]

As can be seen from the tables some groups differ significantly in achievement, differences that are not explained by their school marks in mathematics. That the items 9 and 10 scored very low (table 3) cannot be explained by their difficulty alone but also by the time constraints. The correlation between item scores and total score (table 3) are relatively even (i.e. homogenous test), the lowest explained by the low variance of the item. A factor analysis (table 3) reveals only one dominating factor, possibly a general reasoning factor[6]. The second factor in size is related to items of an algebraic character. Item 5, with low variance, did not correlate with the other items.

The prognostic value of the DP test can be measured by its correlation to the results of the mathematics courses that the same students took during their studies. For this paper results from the calculus course that followed immediately after the DP test will be discussed. This was a one semester course with one mid term exam and one final exam. The courses and the exams were identical for the groups D and Y. Groups I and M had separate (similar, less demanding) courses. On all exams there were seven items of problem solving with a maximum score of 3 on each item. In tables 4a (mid term exam) and 4b (final exam) group means, standard deviations, and correlations with the DP test are shown.[7] All correlations in table 4 are significant or strongly significant.

4. Comments on response protocols

Some general comments to each item of the DP test are given below.

Item 1 – A vast majority of the students seemed to bring a purely procedural approach from high school when it came to dealing with inequalities. After ‘simplifying’ the conclusions were often incorrect, irrelevant, or nonsense. There were often low levels on the logic, method, and control factors.

---

3 An integral approach was chosen here

4 Announced as a 90 minutes long diagnostic test

5 See Appendix B

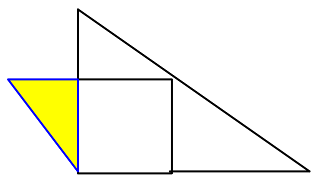
6 Often labelled *g* in the literature (e.g. Gustafsson, 1988)

7 See Appendix B

Item 2 – The most common mistake on this item was to not multiply the denominator outside and inside the parenthesis.

Item 3 – Only a third of the students observed that one of the roots of the equation they obtained was false (in most cases due to 'squaring' the equation). Here the control factor was indeed critical, process being purely procedural.

Item 4 – Most students based their solutions on the notion of similar triangles, and only a few applied the Pythagorean theorem. The direct solution obtained by transforming the triangle to a trapezoid with equal area (related to the creativity factor; see figure below) was not found in the response protocols.



Item 5 – Students used different identification methods, the most common checking up the value of y for one or two values of x. Only a few seemed to have argued on asymptotic behaviours.

Item 6 – Methods differed a lot in simplicity. Some students displayed a bunch of remembered area formulas, without knowing what to do with them.

Item 7 – Comments made on item 1 apply also here. The conceptual depth factor scored low on this item. 'Rules' from solving an equation were in some cases translated to an inequality. With low conceptual depth the control factor cannot work, and the procedural approach can lead almost anywhere. The inclusion of a parameter has put an extra load on the logic and the conceptual depth factors. As already noted, the absence of graphical solutions is here, most likely, an indicator of a low level of the conceptual depth factor.

Item 8 – This item puts emphasis on many aspects of the logic factor. Levels of explanation differed considerably (rigour), and difficulties of expressing the general formula in a simple form were frequent.

Item 9 – The problem was attempted by 46% of the students but solved (score 2) only by 8 %. Many students got lost in algebraic manipulations, an indicator of not understanding the logic, not really knowing what to do with all the algebra.

Item 10 – The problem was attempted by 51% of the students but solved (score 2) only by 3 %. In most of the attempts the equation $x^4 - x^2 = 0$ was solved, or a graph was drawn.

As an overall comment, what seems to be typical in mathematical problem solving is that many of the critical factors are involved in one problem solving process and must be combined for success.

5. Prognostic validity

The validity of a test depends on what the information (test result) is to be used for. For the kind of test discussed here it seems proper to talk about **prognostic validity**. This means that the analysis and interpretation of the results (i.e. the written test protocols) must be based on how well they may prognosticate academic performance in mathematics. The prognostic validity of the test may then be valued from the outcomes of this process, and is thus a function of such factors as design, selection of tasks, content specification, and rationale for protocol analysis (see e.g. Webb, 1992, p. 674).

In this paper, with prognostic validity in focus, a framework for the protocol analysis has been suggested consisting of ten factors that have been judged to be critical for future academic performance in mathematics. In a traditional achievement score, which within a mathematics department has a considerable reliability and validity[8], according to the standards of the faculty, critical factors may implicitly be evoked. There is seldom, however, a stated 'general manual' for the correcting procedure. The validity and reliability of the markings are normally based on teacher experience and judgement only.

A theory of how to find appropriate forms for analysing responses to an assessment situation is still lacking (Webb, 1992). One has to take into account not only the four components mentioned above[9] but also the interaction between them. This means that one must keep some kind of control of the whole assessment process, so that all its aspects are in alignment with the purpose of the assessment.

One preliminary quantitative measure of the prognostic validity of the DP test is given by the correlation between the total score on the DP test and the total score on the university mathematics exams. As can be seen from table 4, these ranged from .52 to .90 on the first calculus exam, and from .44 to .86 on the second. For the Y and D programmes, this prognostic validity of the DP test was quite substantial.

A fact that must be considered here is that the DP test and the first calculus exam both are written problem solving mathematics tests, given with a delay of only two months. Therefore a positive correlation between those tests was to be expected and explained maybe by the general cognitive ability factor g (cf. the factor analysis in table 3). However, the point made here is that the DP test, based on high school mathematics content only, was designed to have a strong correlation with the exams results, and it may well be the case that in basic university mathematics the g factor shows by its influence on the critical factors (cf Gustafsson, 1988). More data will be needed to further evaluate the prognostic validity of the DP test.

6. Discussion

It is becoming generally acknowledged that to provide a good picture of a student's mathematical ability an assessment 'package' is needed (Niss, 1993). It is, however, also necessary to ask the 'reverse' question: How much, and what kind of information can you get from only one written test? After all, written tests are often all you can get. What information there is hidden in a response protocol is a result of the interaction of the student with the test tasks and the assessment situation. The test will measure the kind of mathematical performance that the items and the situation will evoke.

The items of the DP test were constructed to show the students' levels on the critical factors. This influenced the analysis of the protocols in such a way that the scoring was made against the relevant critical factors. The judging of the levels of these factors from the protocols are related to what is expected from this group of students, which means that they cannot be objective or absolute but are socially referenced. During the work it was impossible to keep these factors apart from mathematical content or achievement. An attempt was made to mark one achievement score (locally on each item) and one global score on the critical factors, to give a summarised or adjusted score of mathematical

8 Validity in relation to course objectives

9 As there quoted from NCTM, 1993, p. 29

performance. However, the achievement score was, indeed, always based on the level of some critical factor(s). Therefore, only one integral score was chosen.

One shortcoming in all testing is that students may have the knowledge but don't use it (cf. Schoenfeld, 1987). To what extent the critical factors are related to the ability of evoking relevant knowledge in a problem solving situation is an open question. This is related to the cognitive status of the critical factors, which is beyond the present scope.

This paper illustrates how test construction, analysis and interpretation of the outcome, depend heavily on what the result is going to be used for. It also shows how a mathematics assessment design by necessity leads into discussions about the nature of mathematics and of doing and understanding mathematics. The prognostic test DP was designed to make the critical factors visible. The results indicated a substantial prognostic validity of the test, and further developments and experiences will show the degree of substance in this conceptualisation.

REFERENCES

-Alexander, P., Judy, J., 1988, "The interaction of domain-specific and strategic knowledge in academic performance". *Review of Educational Research*, **58**, 375-400.

-Bergsten, C., 1993, "On analysis computer labs", in *Technology in mathematics teaching: A bridge between teaching and learning, Conference proceedings*, B. Jaworski (ed.), University of Birmingham, pp. 133-140.

-Christiansen, B., Walther, G., 1986, "Task and activity", in B. Christiansen, A. Howson, M. Otte (eds.), *Perspectives on mathematics education*, Dordrecht: Reidel.

-Ernest, P., 1989, "Developments in assessing mathematics", in P. Ernest (ed.), *Mathematics teaching: The state of the art*, New York: The Falmer Press.

-Goldin, G., 1987, "Cognitive representational systems for mathematical problem solving", in C. Janvier (ed.), *Problems of representation in the teaching and learning of mathematics*, Hillsdale, NJ: Lawrence Erlbaum Associates.

-Gray, E., Tall, D., 1991, "Duality, ambiguity and flexibility in successful mathematical thinking", in *Proceedings Fifteenth PME Conference*, F. Furinghetti (ed.), Assisi, Italy, Vol II, pp. 72-79.

-Gustafsson, J-E., 1988, "Hierarchical models of individual differences in cognitive abilities", in R. Sternberg (ed.), *Advances in the psychology of human intelligence (vol 4)*, Hillsdale, NJ: Lawrence Erlbaum Associates.

-Hiebert, J. (ed.), 1986, *Conceptual and procedural knowledge: The case of mathematics*. Hillsdale, NJ: Lawrence Erlbaum Associates.

-Holton, D. (ed.), 2001, *The teaching and learning of mathematics at university level. An ICMI Study*, Dordrecht: Kluwer.

-Krutetski, V., 1976, *The psychology of mathematical abilities in school children*, Chicago: University of Chicago Press.

-Lerman, S., 1989, "Investigations: Where to now?", in P. Ernest (ed.), *Mathematics teaching: The state of the art*, New York: The Falmer Press.

-Mac Lane, S., 1986, *Mathematics: Form and function*, New York: Springer Verlag.

-National Council of Teachers of Mathematics, 1993, *Assessment standards for school mathematics*, Working Draft, October 1993, Reston, VA: NCTM.

-Niss, M. (ed.), 1993, *Investigations into assessment in mathematics education. An ICMI Study*, Dordrecht: Kluwer.

-Perkins, D., Salomon, G., 1989, "Are cognitive skills context-bound?", *Educational Researcher*, **18**, 16-25.

-Schoenfeld, A., 1987, "What's all the fuss about meta-cognition?", in A. Schoenfeld (ed.), Cognitive science and mathematics education, Hillsdale, NJ: Lawrence Erlbaum Associates.

-Sfard, A., 1991, "On the dual nature of mathematical conceptions: Reflections on processes and objects as different sides of the same coin", *Educational Studies in Mathematics*, **22**, 1-36.

-Tall, D. (ed.), 1991, *Advanced mathematical thinking*, Dordrecht: Kluwer.

-Webb, N., 1992, "Assessment of students' knowledge of mathematics: Steps toward a theory", in D. Grouws (ed.), *Handbook of research on mathematics teaching and learning*, New York: Macmillan.

APPENDIX A – The DP test

1. For what positive numbers a, b and c does the inequality $\frac{a}{b} < \frac{a+c}{b+c}$ hold?

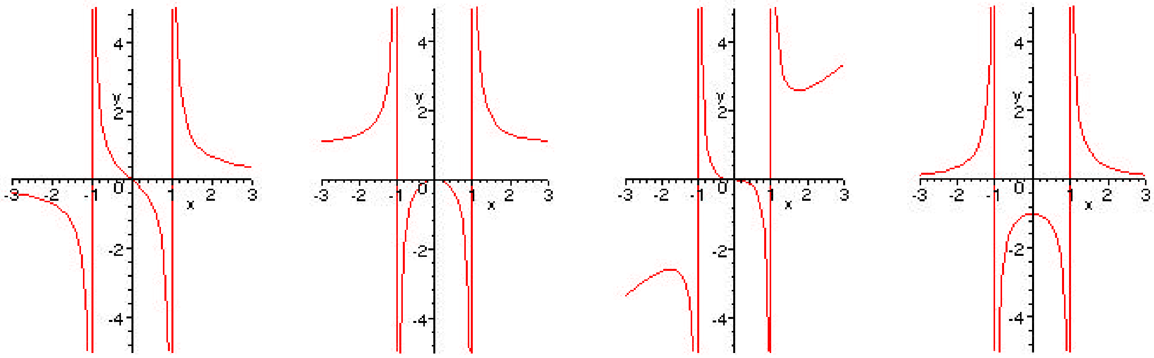
2. For the numbers $a_1, a_2, a_3, \ldots$ we have $a_1 = 0$, $a_2 = 1$, and $a_{n+2} = \frac{1}{4}(3a_{n+1} + a_n)$
   For all natural numbers n   1. Evaluate a_5 .

3. Find all real solutions to the equation $x = 1 + \sqrt{x}$.

4. Inscribe a square in a right-angled triangle so that two of its sides fall along the smaller sides of the triangle, and one vertex on the hypotenuse. Show that the inverted value of the side of the square equals the sum of the inverted values of the smaller sides of the right-angled triangle.

5. Match the function (a-d) to the corresponding graph (1-4):

   a) $\frac{1}{x^2-1}$ b) $\frac{x}{x^2-1}$ c) $\frac{x^2}{x^2-1}$ d) $\frac{x^3}{x^2-1}$



(1) (2) (3) (4)

6. A circle is inscribed in an equilateral triangle, which is inscribed in a circle that is inscribed in a square inscribed in a circle (with figure in test). What portion, in percentage, is the smallest circle area of the biggest circle area?

7. For what real numbers x is $x^2 < ax$? (a is a real constant)

8. A triangle has no diagonal. A square has two diagonals. A regular pentagon has five diagonals. How many diagonals are there in a regular

   a) hexagon ? b) n-polygon ? (n is a natural number   3)

9. Let a and b be any positive numbers. For the numbers $A = \frac{1}{2}(a+b)$, $G = \sqrt{ab}$ and H,
   where $\frac{1}{H} = \frac{1}{2}\left(\frac{1}{a} + \frac{1}{b}\right)$, we have (1) A G and (2) G H
   Show that (1) implies (2).

10. Find (without the use of derivatives) the minimum value of

    a) $x^4 - x^2$ b) $4^x - 2^x$

APPENDIX B – Tables

Group	m	s	min/max	n
D	10.9	6.4	2/30	29
I	8.6	4.4	3/22	31
M	8.8	2.5	3/12	31
Y	13.4	6.1	1/27	28
Total	10.3	5.3	1/30	119

Table 1. Means (m), standard deviations (s), minimum/maximum score, and group sizes on the diagnostic test DP

Sum	1-3	4-6	7-9	10-12	13-15	16-18	19-21	22-24	25-27	28-30
F	8	19	33	27	10	13	5	2	1	1

Table 2. Frequencies (f) of students with sums in given intervals (sum) on the diagnostic test DP

Item	Factor 1	Factor 2	Factor 3	r	m	%
1	.03	.65	.27	.58	1.0	87
2	.03	.03	.82	.42	1.9	92
3	-.04	.68	.18	.46	1.0	96
4	.43	.45	.39	.72	.9	77
5	.17	.15	.23	.34	2.6	96
6	.72	-.01	.26	.59	.9	64
7	.65	.27	.28	.65	.4	80
8	.69	.02	-.08	.45	1.1	85
9	.33	.72	-.19	.57	.3	46
10	.51	.49	-.23	.49	.1	51

Table 3. Factor loadings (varimax rotation) on DP items (eigenvalues 3.00, 1.09, and 1.08 respectively), and Pearson correlations item-sum (r). Means of items (m) and the proportion of students (%) that attempted items (n=119).

Group	m	s	min/max	n	r
D	4.4	5.3	0/21	25	.90
I	4.4	4.5	0/15	30	.57
M	5.4	3.4	0/11	28	.52
Y	7.7	4.1	2/17	27	.62
D+Y	6.1	4.9	0/21	52	.79

Table 4a.
Means (m), standard deviations (s), min/max scores, group size (n) and Pearson correlations (r) between mid term exam of calculus and the test DP

Group	m	s	min/max	n	r
D	5.7	5.3	0/17	23	.86
I	6.7	4.9	0/16	30	.44
M	7.0	4.8	0/15	27	.64
Y	9.1	3.9	2/19	27	.66
D+Y	7.5	4.9	0/19	50	.78

Table 4b.
Means (m), standard deviations (s), min/max scores, group size (n) and Pearson correlations (r) between final exam of calculus and the test DP