SAVING MATH JOBS:
Keeping Math Courses within Math Departments

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ABSTRACT

There has been a trend for post-secondary math courses to move to other departments (statistics being taught in the business school, for example) and for math requirements to be reduced. Since math jobs are at stake, how can we stem or reverse this trend? In this paper, we talk about a successful curriculum innovation project involving calculus for business students, and some lessons learned about working with other departments and colleges. The project involved collaboration with the business school and members of all of its departments from the beginning. We first listened carefully to the needs of our client disciplines, both in terms of overall philosophy as well as specific topics. Then we looked to see what course concepts and texts already existed that might meet our needs, but soon realised that nothing really fit well and that we would have to craft a new solution ourselves. Our concept was to make a two-semester course with integrity that was problem-driven, and relate it to students' other courses, careers, and personal lives as closely as possible. We applied for and received grants from FIPSE, NSF, Villanova University, and Prentice Hall, which helped give us the time needed to develop new materials and the foresight and discipline to organise evaluations of the new course sequence. We worked extensively and sometimes agonisingly with an Advisory Committee from the business school as well as their Curriculum Committee and the math department, but made sure everyone was on board. We were careful to provide gradual and plentiful training and development for our math colleagues. The bottom line is that the new course has been a great success at all levels (student learning and attitudes, business school enthusiasm, and math faculty satisfaction). In this paper we will discuss details about our process and lessons learned.

Introduction: Storm Clouds on the Horizon?

Mathematics as a discipline is centuries old - quite old, as current post-secondary disciplines go. Probability/statistics, operations research, and computer science were developed initially mainly within mathematics, then later often evolved into separate disciplines within academia (and so, separate departments and programs within colleges and universities). Thus there is a fairly common pattern of subject areas being sired and developed within math, but then "moving out of the house" to strike out on their own, with the effect of reducing the size of the "household." This means there is a natural ebb and flow to the size of a math department over time, swelling to add new topics, then shrinking as they split off.

At the present moment in history, math departments seem to be a bit on the "ebb" side of the cycle. Many are still in the process of having computer science and/or statistics split off, are having statistics or discrete math courses taken over by other departments, or are seeing math requirements reduced, often to allow for requirements in new areas such as computer science, writing, or diversity. The loss or reduction of a single semester required math course in a program can mean the loss of several full-time faculty positions in a math department. Currently this requirement reduction seems to be happening in liberal arts and business programs most notably. But it is not happening at all institutions. Is there anything that a math department can do to prevent or minimise such losses?

In this paper, we will describe a project that we have been working on for the last decade to re-engineer the 2-semester first-year math service course sequence for our business school at Villanova University. The course incorporates most of the topics from courses usually called Finite Mathematics and Business Calculus in the U.S., including single variable calculus (both differential and integral), probability, matrices, partial derivatives and multivariable optimisation, including Lagrange multipliers and linear programming. We will describe our process in working with the business school, our math colleagues, and social science departments within our college of liberal arts and sciences to totally rethink this course sequence, implement the changes, and evaluate and monitor the results. We will also share a number of lessons we learned along the way, and give our advice for our math colleagues at other institutions around the world who wish to do all they can to keep from losing faculty positions in their departments. At Villanova, about 2/3 of the classes we teach in the math department are service courses, and nearly 1/4 of our classes are in the business calculus sequence (with comparable enrolment proportions). We will not focus here on issues of trying to increase the number of math majors, but on the provision of math service courses for other departments and colleges.

Recognising Symptoms: Houston, We Have a Problem!

About 10 years ago, some of us in the math department realised that we didn't really enjoy teaching our business calculus courses. One major reason was that the students really hated the courses. The students seemed to see the courses as pure torture, like a "hazing" ritual required to be inducted into the "fraternity/sorority" of business majors, to be tolerated and forgotten as soon as possible afterwards. On their evaluation forms they always wrote comments such as "When am I ever going to **use** this stuff?" Another problem was that the topics in the course felt very disjointed: it was a mishmash of unconnected fragments with no unity or flow to it.

We decided to check in with our business school to see how they felt about the course sequence. (Interestingly, and fortuitously, they later claimed that **they** had initiated the contact, so

both main actors felt ownership of the process. This is ideal, if you can swing it!) They expressed some concern that their students did not know the math that the business school really wanted and needed them to know. In some cases, the areas of deficiency were already on our math syllabus, but the students would often claim they had never seen the material before. This is not uncommon in such service courses, but we wanted to try to minimise the phenomenon. In our discussions with the business school, we realised that one factor in this disconnect could be a difference in notation and terminology in the two fields (math and business), so we tried to find where this occurred.

Our Solution

At this point, it seemed clear to all concerned that something needed to be done to modify the course, so an Ad Hoc Committee of math and business faculty was created to study the problem. This group decided that everyone's needs would be best served by making the course **problem-driven** rather than abstract and theoretical. We decided a good starting point would be to ask faculty in all of the business departments for **examples** of mathematical problems they used in their courses. This turned out to be very difficult. We got many lists of topics for different disciplines, but very few colleagues were able to give us concrete specific examples. Eventually we did get representative problems from each department.

Next we did a search of existing texts and courses to see if anything existed to do what we wanted to do. The closest we could find was the *Calculus Concepts* text out of Clemson University, in its early stages of development. This text focused on using real-world data and fitting curves to the data, and came much closer to what we wanted than anything else that existed at the time. We decided to adopt it in several experimental sections of our course sequence. Unfortunately, this text did not cover matrices and linear programming, which our business colleagues still wanted us to cover, so we realised that we would have to develop supplementary textual material on these topics ourselves to fit the style of the other topics.

As we used the Clemson text this first time, we realised that it was a great improvement over the traditional texts, but that it didn't go into as much detail about the process of math modelling as we wanted. It opened the door, but didn't walk all the way in, so to speak. At around the same time in our discussions with the business school, they expressed a preference for covering all of the single-variable calculus material in one semester. That way, students with AP credit could place out of that part of the course, but get the rest of the content in the other semester. Up to this point we had covered through derivatives in the first semester, then did integrals, partial derivatives, matrices, and linear programming in the second semester. As we discussed specific topics that the business school wanted and did not want, we realised that in fact we could cover all of the needed single-variable calculus in the first semester. This was possible because there were a number of topics we had been teaching that they did not care about, including implicit differentiation, related rates, the Mean Value Theorem, and most techniques of integration.

We then realised that we *could* put all of the single-variable topics in the first semester and the multivariable topics in the second semester. We weren't sure where to put the topics of compound interest and net present value, but saw that they could be thought of as involving functions of several variables (interest rate, time, etc.) and put them into the second semester. Now we started to see that we could go beyond the idea of math modelling, and could think of the course sequence as a course in **problem solving**: single-variable in the first semester and multivariable in the second semester. More specifically, we would be **teaching the entire process of solving real-**

world problems using math modelling, calculus, and technology. We could use the Clemson approach of using **graphing calculators** in the first semester to fit single-variable functions to data. After going to the first Harvard Consortium Conference on the Teaching of Calculus in 1992, we also realised that **spreadsheets** would be very helpful for the second semester, both for matrix calculations and to fit multivariable functions to data, paralleling what we did with the graphing calculators. This focus on problem solving and least squares regression would help give the course the **integrity** that we were looking for.

As we spelled out the entire process of solving real world problems, we realised that the very first step in the process is identifying and defining your problem in the first place. We knew that we wanted students to learn about and *experience* the *entire* process of problem solving. We had already done some experimenting with the use of **student-generated projects** (projects related to the course, but where the *student* chooses a topic based on their own life and interests) in this and other courses. We realised that a semester-long student-generated project was a perfect way to help students *learn* the entire process of problems solving and to see the **relevance** of the math as well. We like to use the analogy that the traditional approach to this course was like teaching students to fly at 5000 feet, but we wanted to teach them how to take off and land as well. Making those **connections** between the real world (the ground) and the world of math (up in the clouds) was exactly what the student-generated projects could do.

In the process of rethinking the course sequence from this perspective, we realised that there were also ways that we could make it flow more naturally and logically, and not feel so disjointed. We realised that the two semesters could be somewhat parallel in structure. They could start with defining functions, then focus on the process of formulating models, both from verbal descriptions and from data, then show how to take derivatives and optimise functions (with and without constraints), and talk about post-optimality analysis (including verification, validation, sensitivity analysis, and estimating margins of error). For the multivariable semester, we realised that we could cover matrices just before optimisation, just in time for solving the systems of linear equations that you obtain when setting partial derivatives of quadratic functions equal to zero. We could also cover Lagrange multipliers after partial derivatives, *after* which we could discuss shadow prices and linear programming, to give students a deeper understanding of shadow prices. For both semesters, it worked out conveniently that lower priority topics (integration and linear programming) came at the end of the semester, which meant that students would know all they needed for their projects about two-thirds of the way through the course. This meant that they could hand in drafts of their project reports, get extensive feedback and suggestions, and then hand in a revised report at the end of the semester, making it possible for them to produce a work of extremely high quality.

At around this time, we held discussions with the Math Curriculum Subcommittee of the business school's Curriculum Committee, to work out the details of topic coverage and course structure and philosophy. This was the hardest part of the entire process. There were several areas that turned out to be quite tricky and delicate to negotiate. One was what to do about integration. Economics and some Finance faculty wanted it covered, but not in great depth. We decided it should be covered, motivated largely by continuous probability (which the students would be using implicitly in statistics later) and Consumer and Producer Surplus. As mathematicians, we felt very strongly about covering the Fundamental Theorem of Calculus if we were going to teach integration, for its inherent beauty and importance in the history of ideas. They were convinced by our intellectual argument, happy that we were willing to drop techniques of integration and other topics they considered arcane.

The most difficult discussion involved the overall organisation of the course sequence. One member of the business curriculum math subcommittee felt very strongly that the course should be organised by having all of the **linear** topics in the first semester, and then all of the **non-linear** topics in the second semester. This was how he had learned it, and it is a very common structure, often broken up into Finite Math and then Business Calculus. We argued that this would totally destroy our idea of making the course about problem solving and using student-generated projects to reinforce the material and make it come alive for the students. The business school did like the idea of the student-generated projects very much. We told them that in our experience, there were almost no good project topics that were linear (a breakfast diet mix problem being about the only one), which would mean we couldn't really do the projects until the second semester. This would represent a huge loss in potential student motivation. After extensive discussion, we finally got approval for our structure by making some concessions in other areas that were not as critical to us.

Another difficult discussion revolved around technology, and this is quite common when working with faculty from other disciplines. Everyone agreed that spreadsheets were perfect for the second semester. But our business colleagues did not like the idea of using graphing calculators. Some of them required financial calculators for their students, and they felt this extra calculator was unnecessary. We talked about using spreadsheets in the first semester, but having taught with the Clemson text using graphing calculators, we knew that they were pedagogically far superior, since Villanova at the time did not have computer classrooms and the students did not have laptops. In fact, a very high percentage of the students came to college with a graphing calculator, so it was not a great burden. After demonstrating the power of the graphing calculator, and giving a free sample to each of the subcommittee members to see for themselves, they reluctantly agreed.

A final discussion with the subcommittee involved the topics for the student-generated projects. We said that examples of projects included finding the optimal amount of exercise in a day to maximise your energy level or the optimal amount of time warming up before a performance or athletic activity to maximise performance. They expressed concern that the topics were not of a business nature. We said that one of the main reasons for this was that the primary reason for the student-generated projects is **motivation**: to **connect** the math to something each student cares about in their own life. Secondly, we said that, while the topics do not appear strictly "business" in nature, most of them do involve optimal allocation of resources (such as time), which *is* very fundamentally an *economic* problem, quite analogous to many business problems. We did also say that we always encourage students to do real business examples (such as pricing T-shirts for a fund-raiser or cottage industry crafts, etc.). The combination of these arguments was strong enough for them to approve the idea, if somewhat reluctantly.

Implementation

Now that we had our concept clear, we knew that we had to do at lest some of the creation of materials ourselves. We talked with the Clemson authors to see if they were interested in working together, and they expressed openness to the possibility. We proceeded to write grant applications from the U.S. Department of Education's Fund for the Improvement of Post-Secondary Education (FIPSE) and the National Science Foundation (NSF), and in fact received both grants. These were especially helpful in giving us course relief to work on the course materials and in forcing us to develop a plan for evaluating our work. They also require us to form an Advisory Committee of business faculty, which was extremely helpful throughout the process.

In pursuing our discussions with the Clemson authors, we realised that our concepts were too different to work together, and so we struck off on our own. Several publishers expressed interest in our project, and we signed on with Prentice Hall, who also gave us a grant to fund laptop computers and other hardware and software that were all very helpful in the development process.

The first year of our FIPSE grant, the two of us taught experimental sections of the course, while everyone else continued with the traditional text and approach. During that first year, we gave several workshops for the math faculty, explaining our concept for the course, teaching the technology on the graphing calculator and using spreadsheets, and going over the processes of real world problem solving, math modelling, and student-generated projects. The summer after that first year, we led the first of many annual summer workshops for faculty from Villanova and other colleges and universities, and videotaped them. Because of all this faculty development, we were able to offer the new course in *all* sections during the second year of our grant. In fact, we wanted to keep a few sections using the traditional approach for evaluation purposes, but the business school wouldn't *let* us! They liked the new course so much, they didn't want to *deprive* any students from being able to take it. We were sorry in some ways to not be able to complete our evaluation as planned, but on the other hand, the strong endorsement was a form of evaluation in itself.

Our approach was radically different from before. To follow the spirit of being problem-driven, we even changed the pedagogy. Instead of a deductive approach teaching abstract concepts, then numerical examples, then simplistic applications (if there was time), we use an inductive approach giving realistic problems to motive material, then working with specific numerical examples using students' intuition, then generalising the patterns to present concepts. We want to give the students an intuitive conceptual understanding of the material. They should be able to solve simplistic problems by hand to understand the processes, but then be able to use technology to solve real problems.

Because of these radical changes, we had initially expected resistance from some of our faculty. It never materialised. Everyone who came to our workshops was extremely enthusiastic, and no one else in the department ever objected. Initially, some wanted to make sure the mathematical level of rigor was adequate. Many of them later said that they believe the students of this new course understood multivariable calculus better than the students in our engineering calculus, and have brought over some of our concepts to those courses. The first year that all sections were using the new approach, we had weekly voluntary discussions about the course. These were a wonderful experience, many of us discussing teaching together for the first time in a significant and regular way. Everyone commented on how much more lively the students were in class, and how much more satisfying it was to teach.

Results and Conclusions

Based on our statistical evaluation results, where students were randomly placed into control (traditional) and experimental sections, students learned significantly more of what the business school wanted. Based on 19-question pre- and post-tests they helped us construct, the experimental group scored about 5 points higher on average out of 19 points, with $p = 0.01$. Furthermore, the students rated our course sequence as significantly more relevant to their other courses, their careers, and their personal lives (the experimental group rating each category about

0.5 points higher or more on a 5-point Likert scale, all with $p < 0.04$). Instead of asking "When will I ever *use* this?" students now often say "I never knew that math could be *useful* before!" Our faculty enjoy teaching the course much more than before, although the student-generated projects can take more time to grade. Our business school has held up our project as a model for other curriculum reform efforts, and has been extremely supportive and enthusiastic.

What are the lessons for trying to maintain requirements and service courses in math departments? One is to **be on the lookout** for feelings of dissatisfaction with a course. If a course is getting stale, something is needed to revitalise it. **Get together with the appropriate client disciplines**, and assess what is working and what is not. **Determine the needs** of the client discipline (these can and do change over time, especially as technology changes), and **reconcile these with maintaining mathematical integrity *together***. **Look for existing texts** and materials to meet the needs and goals of the course, and **if necessary create your own**. If you need to create your own materials, **look for grant support** to be able to do it well. You cannot *over*do **faculty development** to train people for curriculum changes. Finally, **evaluate and monitor your results** and **maintain communication** with all concerned.

REFERENCES

-LaTorre, D. et al., 1997, *Calculus Concepts*, New York: Houghton Mifflin.

-Pollack-Johnson, B., Borchardt, A.F., 1998, *Mathematical Connections: A Modeling Approach to Business Calculus*, Upper Saddle River, NY: Prentice Hall.

-Borchardt, A.F., Pollack-Johnson, B., 1998, *Mathematical Connections: A Modeling Approach to Business Calculus and Finite Mathematics*, Upper Saddle River, NY: Prentice Hall.