ASSESSMENT OF SOLVING TYPES OF PROBLEMS IN ALGEBRA

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ABSTRACT

A classroom experience in algebra which considered and interaction-bound view of the didactic s of Mathematics regarding the categorization of types of mathematical problems was carried out. This experience aimed at relating the teaching of concepts and the resolution of algebraic problems previously validated and classified according to their nature into routine, non-routine problems; and according to their context into real, realistic, fantasy and purely mathematical problems. To that effect, a study of the learning of the unit about proportion variation for the first year high school based on the syllabus suggested by the Chilean Educational Reform was devised.

Learner-centered actions which gave way to group discussions and active interaction with the teacher were preferred in an attempt to reach the concept of proportion variations. The class work was carried out following a constructivist view of learning and was supported by class material specially prepared for such purpose.

Considering the students' actions in the classroom plus their interaction with the types of problems studied, it is possible to claim that the students could gain highly significant algebraic learning, demonstrating ability to recognize relations and transform the given data from a problem. By this means, they showed evidence of understanding the concept of proportion and is relations, knowledge and understanding of mathematical processes with accurate and fast calculations and an ability to reason in order to solve routine problems - preferably of purely mathematical, fantasy and realistic context - as encouraged by the Chilean Educational Reform.

Background information

All forms of learning imply the search for adequate knowledge or an effective skill. Knowledge about skills has increased in the past decades and this has contributed to the identification and development of cognitive skills such as problem solving.

In trying to find an appropriate heuristics to aid the solving of problems, Polya (1957) described a number of general strategies that could facilitate the procedure. Such concern increased later on as can be seen in articles about the same topic by Schoenfeld and Herman(1982), Mayer (1985), Sweller and Cooper (1985), Gick (1986), Minsky (1988), Schoenfeld and Herman, Mayer, Sweller and Cooper, Bifk, Minasky in Valenzuela, 1992).

Curricular tendencies for the teaching of Mathematics of the last decade have stressed 'the need to place capacities of higher level on a first position...'(Abrantes, 1996); that is, those capacities that are linked to the identification and resolution of problems, to critical thinking, and the use of metacognitive strategies.

Problems solving has been recognised as a main aspect of the learning process; hence, it has given rise to the formulation of operative strategies, to the classification of the different problems, and to the varied approaches given to the studies about solving types of problems. However, there is a need for further studies that suggest didactic strategies which allow an improvement in the teaching of mathematics, specifically of algebra, within the context of educational reforms (Díaz, Poblete, 2000).

At present, problem solving continues to be a topic of interest, especially from the perspective of the educational reforms that are being implemented in recent years. This consideration to problem solving tasks has been recognized by different educational reforms in Latin America. In Chile, it constitutes a fundamental element in the current teaching of mathematics at different levels, due to their relevance in everyday life application and usefulness (Díaz, Poblete, 1999). Such considerations of the conception of a problem have been expanded by means of a distinction between them that categorises them as routine and non-routine.

In our view, categorizing problems provides the conceptual basis for any didactic procedure in the school curriculum. To that effect, we have devised a classification that considers the nature and the context of the problem, and have devised the following categorization. Based on their nature, problems are categorized as Routine and Non-Routine problems; and based on their context they are categorized as Real, Realistic, Fantasy and Purely Mathematical problems (Díaz, Poblete, Fondecyt Project Number 1990558, 1999).

Routine Problems

Based on their context, we have classified problems as:

Real context problems: A context is real if it is effectively produced in reality and the student is involved in it.

Example: using a piece of thread, to measure the diameter and length of the circumference of three coins of different ze each. Find the ratio between the diameter and the length of each coin. What can you conclude from these ratios?

Realistic context problems: A context is realistic if it is likely to be really produced. It deals with a simulation of reality or of a part of reality.

Example: An industrial washing machine, working 8 continuous hours for 6 days has washed 1200 kilograms of clothes. How many kilograms of clothes will it wash in 20 days working for 10 hours daily?

Fantasy context problems: A context is considered a fantasy if it is the product of imagination not founded on reality.

Example: Two inhabitants from Krypton planet have been brought to Earth: Superman and Supergirl. In order for them not to be affected by Kryptonite, they need to drink an amount of liquid equivalent to one ninth of their weight. If Superman drank 21 litters of liquid in 3 days, how much liquid does he need to drink in a week?

Purely mathematical context problems: A context is purely mathematical if it refers exclusively to mathematical objects such as numbers, relations and arithmetic operations, geometry figures, etc.

Example: The sides of two squares have a ratio of 1:3. What is the ratio of their perimeters?

Non- Routine Problems:

Non- Routine Problems : these are those for which the student does not know an answer nor a previously established procedure or routine to find the answer.

Example: Think of two everyday life situations that are inversely proportional and determine the value of proportion constant in each case.

Note that non-routine problems can also be classified according to their context into real, realistic, fantasy and purely mathematical.

Development of the study

A didactic experience to articulate the mathematical concepts regarding a specific teaching unit and problem solving types for secondary school was devised.

The problems presented were algebra problems and the unit dealt with was that of proportion variations. The contents were structured as indicated by the Chilean Educational Reform for secondary schools. Primary education in Chile comprises 8 years and secondary school comprises 4 years. The qualitative research was mainly conducted as participating observations, and took place in May 2001, within a class of 40 students at a Science and Humanities Secondary School.

The whole process taking place within the class was described, analyzed and interpreted continuously by means of the interaction with the students. Three pairs of students were voluntarily chosen to be observed during the research process.

Active learner-centered actions rather than teacher-oriented actions were chosen as the methodology. Students worked with didactic materials designed by the researchers that included 55 tasks involving situations and problems based on the categorization devised.

Data were collected through observations in class. Individual interviews on some occasions and open-ended opinion questionnaires complemented such observations. These were passed at the

beginning and at the end of the experience and aimed at finding out the meaning students give to their individual actions and to get valid and accurate conclusions about the study.

Students started work with the materials given from the second session, could do the tasks either individually or in groups and ask questions either to the teacher or the observer. All sessions were recorded, so it was possible to get information about the conversations among the six students observed, the kinds of questions raised about the text and the questions asked to both the teacher and the observer.

The teaching material was used as a facilitating element of the didactic procedure; that is, as a means to improve and complement the teaching and learning procedure of the study unit. The material included problems, situations and questions related to direct and inverse proportionality, proportionality constant, its relation with a quotient or a constant product, composed proportionality, graphs, charts with values, and algebra expressions. All of it based on types of problems classified depending on their nature as routine and non-routine; and depending on their context as real context, fantasy and purely mathematical.

The activities consisting of didactic situations based on types of problems allowed the introduction of the concept of proportion variations. Most of them were accompanied by drawings and charts to visually explain the relations involved. Thus, the situations presented through types of problems and associated to charts allowed the students to identify themselves with different contexts and use their own learning styles.

Some real context problems were solved in order to relate the equivalence of the ratios with the constant of proportionality in each case and introduce the concept of proportion. Some examples of this are the following: "Measure the sides of your desk. Measure the sides of your teacher's desk. Establish the ratio between them and decide if they make a proportion"; "Draw three squares of different sizes. Draw the diagonal in each and determine the ratio between the side of the square and the diagonal".

Some other tasks based on realistic context problems were performed in class. Examples of these are: "A farmer from the south of Chile, in Osorno, needs 750 kilos of pasture to feed 50 cows for 10 days. How many days will he be able to feed 40 cows with 800 kilos of pasture?"; "Two cities that are at a distance of 18 km one from the other appear 6 cm. apart on a map. What is the real distance between two cities that appear 21 cm. apart on that same map? Both of these problems are likely to occur in real life; they correspond to a simulation of reality or part of reality.

Similarly, fantastic context problems were also included. Some of them were: "A specimen from Saturn has been brought to Earth. It covers 21 meters in three jumps. What distance would it cover in 5 jumps?"; "If three cats eat three mice in three minutes each, how many cats are necessary to eat nine mice in nine minutes?." Both problems are just part of imagination.

The purely mathematical problems used are similar to those that normally appear in traditional coursebooks regarding proportion variations. Those have to do exclusively with mathematical objects: numbers, mathematical relations, geometry figures, etc. Students did not have major problems solving these problems.

Students appeared highly motivated with the methodology employed and were actively engaged in their groups trying to figure out the solving procedures.

Similarly, students solved non-routine problems such as: "Write down the mathematical formulation of the following variations: a) the N number of long distance calls between two cities is

inversely proportional to the distance between both, b) The D distance, expressed in meters, covered by a vehicle in 15 minutes in inversely proportional to the average speed V, expressed as m/min." "Invent a problem that combines both types of proportionality." Faced with these problems, students discussed them for a few minutes and then tried solving them by means of numeric rather than by algebraic expressions.

In order to assess the students' previous knowledge about the topic, a pre- and meta-test were applied. This was useful to compare the level of achievement obtained by the students during the didactic experience and also as an assessment instrument for the unit.

Six protocols were developed during the research: they involved class observations, individual interviews and an opinion questionnaire.

Results

The application of the pre- and meta- test in this didactic experience based on routine and non-routine context problem solving has enlightened our understanding of the processes involved by means of a comparative study of individual performance.

Student 1 got 15.6% in the pre-test and throughout the development of the research increased his performance to reach 87.5% achievement. He developed a significant ability in solving routine problems of either realistic, fantasy or real context. He developed ability to identify relations and convert data from a given problem into other form, thus showing evidence of understanding the concept of proportionality and its relations.

Student 2 got 6.25% in the pre-test and by the end of the research had increased his performance to 75% achievement. He showed evidence of knowledge and understanding of the concept of proportionality, of mathematical procedures with fast and accurate calculations and ability to reason and solve routine problems, mainly those of purely mathematical context, fantasy and realistic context.

Student 3 got 6.25% in the pre-test and improved greatly throughout the development of the research to get 90.6% achievement in the final test. He showed greater skills in reading and interpreting routine problems of purely mathematical, realistic and fantasy context. He also developed the routine problem of real context with a certain degree of success, though minor errors led him to a wrong answer.

Student 4 was only able to solve one problem in the pre-test, getting 3.12% of achievement, improved his performance so much that he finally got 96.8% of achievement by the end of the experience, thus getting the highest achievement percentage of all students observed. The student developed the skills necessary to do mathematical reasoning, and showed evidence of knowledge and understanding of generalizations about asking and answering routine problems of all contexts. Similarly, he was the only one able to solve the non-routine problem presented in the test, thus indicating that he was able to apply prior knowledge to solve an uncommon type of problem, by making use of more complex mental processes since this problem required a higher kind of analysis.

Student 5 went from a 9.3% achievement in the pre-test to a 68.7% in the final test. He showed evidence of a discrete skill development regarding problems solving. He demonstrated greater

ability doing routine problems of realistic and fantasy context and an adequate interpretation of the data given for such problems.

Finally, student 6 only did 3 problems in the pre-test, getting a 9.3% achievement

and increased it to 87.5% in the meta-test. His achievement improved greatly, specifically in his reading, interpreting, analyzing the data from a problem, and finally solving practically all routine problems successfully, except the one of real context. Certain data show that his answer was sensibly oriented yet a wrong interpretation misguided his answer.

Categorization of the research

Based on the data obtained, the class observation, literally transcribed at the end of each working session, plus the pre- and post- opinion questionnaire, all the information was classified in order to find the convergence. The idea was to obtain a corpus of data that allowed a more systematic analysis of each situation, that led to the formation of categories from the similarities in order to maintain internal homogeneity, or the differences related to external heterogeneity, trying to establish clear and coherent criteria for the classification and ordering of the information obtained. Similarities found for the three pairs of students observed are detailed below.

Similarities

(1)   About the problem solving

Regarding the pre-test:

They find it difficult, and argue that they cannot remember the contents studied in previous courses.

Their amount of knowledge was not enough to allow them to do the test.

Difficulties encountered mainly have to do with geometry.

Regarding the development:

They are more used to solving exercises than problems.

By the end of the study, students are able to discover applicability of algebra to everyday life situations with examples of realistic situations.

(2)   About their knowledge

Initially, students do not recognize the applicability of algebra to their everyday life.

They appear to have increasing difficulty understanding metalanguage.

Students make no distinction between Mathematics and Algebra.

They are able to define correctly, and without major difficulty the concept of direct and inverse proportionality exemplifying each with a routine problem of realistic context.

They claim to have studied and learned contents referring to proportion variation and are able to expand their answers.

(3)   About their teaching

Students can make a comparison and agree in identifying differences between the teaching of algebra in primary and secondary school.

They find the material entertaining and highlight the motivational feature of drawings.

Differences

(1)   About the mathematical concepts

The difficulties encountered during the classroom sessions were concentrated on the solution of equations, the problems that involved geometric concepts, real context problems that required the measuring and solving of problems related with compound proportionality.

Students define and exemplify with difficulty what they understand by compound proportionality.

(2)   About the methodology

For some students the methodology employed was adequate because it was entertaining and different to traditional classes, while for some others it was so because they could work in groups and share their ideas.

Students' views about the tasks performed are different: they are good and easy for some students, and very complicated for some others.

(3)   About their achievement

In the application of the pre and meta-test students obtained achievement that ranged from 3.1% and 15.6% in the pre-test to 58.3% and 100% in the meta-test.

It is worth mentioning that only one student got an achievement score below 70% in the meta-test. This indicates that there was a significant increase in the development of certain skills and the use of knowledge related to problem solving in algebra.

Conclusions

This research into the algebra area considered a didactic conception of mathematics that related the teaching of general algebraic concepts and problem solving. The investigation was aided by teaching material based on types of problems categorized according to their nature and their context as a means to reach the concept of proportion variations in the students.

The students showed ability to recognize relations and transform data from a problem given in one way into another, by this means they got to the understanding of the concept of proportionality and its relations. They displayed knowledge and understanding of proportionality concept and its relations. They showed evidence of knowledge and understanding of proportionality situations, of mathematical processes with fast and accurate calculations and ability to reason about and solve routine problems, preferably of purely mathematical, fantasy and realistic context.

By the end of the study, a group of students managed to transfer their previous knowledge to a non-habitual problem, making use of more complex mental processes, since this one belongs to a category of higher analysis. The students showed difficulties with geometry and were more skillful with arithmetic than with algebra. The notions and approaches the students used in arithmetic previously and that still maintain can explain this difficulty. Working with algebra requires students' change of mind so that they move away from concrete numeric situations to more general situations, like the ones given by non-routine problems. Transition from what can be labeled as an informal representation and problem solving to a more formal one is complex and disorientating for most students who start studying algebra, since they continue to use the approach that worked for them in arithmetic.

In general, solving a problem of real, realistic or fantasy context requires the mathematization of the given situation; that is, it has to be translated into mathematical language. Since we are dealing with a problem, the mathematization process requires that the students search for the solution. If the

student is able to 'mathematize' the situation in an automatic way without much effort, then he is not in the presence of a context problem but rather of a simple mathematization exercise. In everyday life there are concrete situations that can be made into problems. These situations can be given a mathematical formulation and can become isomorphic to those presented in the school curriculum, encouraging the students' constructive mental activity in the processes of knowledge acquisition and an effective development of the ability to deal with problem solving.

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