DEMONSTRATIONS, EXPERIMENTS, AND SUPPLEMENTARY NOTES TO MOTIVATE STUDENTS IN DIFFERENTIAL EQUATIONS COURSES.

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ABSTRACT

We describe a series of demonstrations and experiments that are used in our differential equations course. These experiments are designed to be low-tech, low-cost alternatives to illustrate and motivate the modeling/prediction aspect of the course. A series of lecture notes are also being prepared to relieve students from the task of taking notes, and allowing them instead to concentrate on understanding what is being discussed. We have found that there is great enthusiasm for the experiments and demonstrations, even though most of them have been performed before in the Physics Lab. Apparently, many aspects of the experiments were not understood during that first encounter in the laboratory. Also, an effort is made to present the mathematics and the experimental confirmation simultaneously to enhance the effect. On the other hand, we have not noticed any increased performance from the students due to the introduction of these demonstrations. In any case, the instructor certainly has fun with the demonstrations.

1. Introduction

A series of experiments and demonstrations used by the author in the course *Introductory Differential Equations* offered at the University of Puerto Rico at Mayagüez will be described. These experiments do not require a sophisticated setup, computers, or measurement devices. They are designed to be low-tech, low cost alternatives for instructors who wish to illustrate the predictive and modeling aspects of differential equations.

Our institution is the Engineering campus of the University of Puerto Rico. A consequence of this is that most of our students in the course are Engineers. We also service Physics, Chemistry, and of course, Mathematics students. Because of the demographics it is particularly important for us to address the aspect of applications in the course. The demonstrations serve as a driving force, taking the course in this direction. We try to make the course very relevant to the students.

2. The experiments

The demonstrations/experiments are listed chronologically, in the order in which they are presented in the course.

First order differential equations.

For the topic of first order differential equations, no demonstrations are currently presented. However, possible demonstrations that could be introduced are: the leaking tank to illustrate Torricelli's law, or a cooling object to illustrate Newton's law of cooling. The reason we have not implemented these demos is that we have not found convenient, no- mess, low-cost ways of setting them up in the classroom, on the fly.

Second order, linear, constant-coefficient equations I: the harmonic oscillator.

For this topic we have two demonstrations: the mass/spring, and the pendulum.

Mass/spring

For the mass/spring the governing equation is derived assuming no friction, Hooke's law and no external forcing. The general solution is obtained and it is shown that regardless of the initial conditions, the solution is periodic with constant circular frequency. Immediately several experiments are performed: a mass/spring system is put in motion under different initial conditions and the frequency of oscillation is experimentally determined each time: it is found to be constant. The term natural frequency is then introduced and motivated. The differences between the assumptions made and the real setup are noted: no friction, Hooke's law. The surprise that even after making those "unrealistic assumptions" the model still yielded important qualitative information, verified by the experiment, is emphasized. Also, static and dynamic (using the natural frequency) methods for the determination of the spring constant are introduced and implemented.

Experimental setup: We bought a spring from our local Pep-Boys (auto shop) which cost \$5.00, and a large bolt from our local Home-Depot which cost \$2.00. A wooden classroom ruler is also convenient for measuring lengths and also serves as an anchor for the mass/spring (\$2.50) .Total cost: \$9.50.

Pendulum

Again, the governing equation is derived assuming no friction. A non-linear equation is obtained, and the idea of linearization is introduced. Assuming small oscillations, the equation is replaced by a linear, constant coefficient equation. It is noted that the equation is identical to the one obtained with the mass/spring, except for the interpretation of the coefficients. Several experiments are performed confirming the harmonicity of the oscillations, and the relationship between length of the pendulum and resulting natural frequency is confirmed by doubling or halving the length of the pendulum. Again, the assumptions are analyzed and the robustness of the analysis in emphasized by stressing that even under such assumptions, the model and analysis yielded conclusions confirmed by the experiments. Some commentary can also be made on dimensional analysis: from the parameters of the setup (length of pendulum, mass, gravity) one could conclude the dependency of the natural frequency on length and gravity.

Experimental setup: We bought a large nut from our local Home-Depot which cost $0.50, and nylon fishing line for the pendulum length in Walmart ($4.00). A wooden classroom ruler is also convenient for measuring lengths and also serves as an anchor for the pendulum ($2.50) .Total cost: $7.00.

Second order, linear, constant-coefficient equations II: forced oscillations and resonance.

Here, the emphasis will be on sinusoidal forcing of varying frequency, and the phenomenon of resonance. The mathematical analysis for undamped and damped mass/spring systems is performed predicting that a single frequency has the largest effect on the response. This resonant frequency is interpreted as dangerous for structures, in the sense that a large response can destroy the system. In the case of the RLC circuit, resonance is interpreted in terms of amplification and filtering.

Mass/spring

The same setup as before, except that we begin with trivial initial conditions and gently tap the ruler (from which hangs the mass/spring) from above and below in a periodic fashion. This experiment is imperfect in the sense that the force is not actually applied to the mass directly, but it is the best we could come up with in keeping with the low-frills philosophy! However, the results are quite spectacular. If the frequency of the stimulus is too slow or too fast, the response is negligible, but if it approximates the natural frequency (which can be observed by free oscillations), the response is considerable. One can remark that if stimulus and response are out of resonant phase, cancellations result in small response. This is visible in the experiment.

RLC circuit

We bought two kits from Marlin P. Jones & Assoc. Inc. (there are many distributors of electronic supplies that would do): a function generator and a small amplifier. Both kits and a small speaker cost less that $12.00. One has to put them together, which involves a bit of tinkering, but in less than two hours it is done. The idea is to use these kits to force an RLC circuit and to “hear” resonance as amplification and lack of it as filtering. Unfortunately, we have yet to construct the inductor of the RLC circuit to be stimulated, so this experiment has not been implemented yet.

Tuning forks, whistling tube, singing bars

These three experiments have been chosen to illustrate the fact that resonance is a fairly universal phenomenon: all objects are subject to vibrations, and all prefer to vibrate at certain frequencies. In fact, the natural frequencies give us information about the vibrating object. The demonstration kits were bought from Pasco (distributor of educational materials), and are the most expensive items (more than $300.00). However, the whistling tube is sold at toy stores for $3.00 and one can replace the catalog singing bars with an aluminum bar used as support in labs or with a crystal wine glass. The crystal wine glass demonstration is very spectacular and one does not even need resin to perform it: simply wet you finger and rub your finger slowly around the rim of the glass will holding with the other hand at the base. The glass will respond to the stimulus, and the response is usually quite audible. One can even tune the glass by adding water: More water means lower response frequency. In fact, with the frequency generator and an amplifier one could even tune the frequency generator to the natural frequency of the wine glass and perform the opera singer breaking of the glass!

The tuning forks are used to illustrate resonance in the following way: they are tuned with the adjustable mass until they resonate at the same frequency. They are placed close together, and one is struck with the mallet. After a few seconds it is muted. The second fork will still sound: it has been stimulated by the first fork at a resonant frequency and thus responds. Also, the phenomenon of beats can be illustrated by moving the mass slightly so the forks are no longer in tune, and striking both with the mallet. The periodically varying amplitude can be heard clearly. It is important to derive both phenomena from the governing equations of the mass/spring before or immediately after performing the experiments to achieve the desired effect.

The whistling tube is a corrugated hose. When spun, a pressure gradient is produced and a flow is induced. When spun at particular speeds this produces a tone. The faster the particular spinning speed, the higher the tone. The frequencies of these tones are in rational proportion. In fact, they happen to be related to the Pithagorean theory of harmony. One can hear an octave, a perfect fifth, etc. These tones are the result of resonant vibration patterns of pressure waves inside the tube (standing pressure profiles). This principle is what makes all winded musical instruments work. The moral is that resonance makes music and musical instruments possible. One can also illustrate with a rope or string to simulate string instruments, and show the standing waves or modes and corresponding natural frequencies, and their relationship with tension and mass in the rope.

Finally, the singing aluminum bars are held at some node, and stroked with fingers full of resin. The result is a very loud and powerful response at the corresponding natural frequencies (depending where one holds the bar). If one does not hold the bar close to a node of the first natural modes, then one does not hear a response. One can draw a parallel between the bar and any rigid structure (like a bridge or building) and talk about the dangers of resonance and the importance of avoiding it in certain situations.

Whirling rope

We no longer talk about boundary value problems in our introductory course, but the whirling rope is a perfect application/experiment for this topic. One can demonstrate what the mathematics predicts: the rope has a preference for whirling at certain natural frequencies. These frequencies depend on tension and mass (density). The whirling modes are also predicted by the theory. The experimental setup is as simple as a long piece of rope and the help of a couple of volunteers to spin the rope.

3. Lecture notes

We are in the process of completing a series of lecture notes. The purpose of these notes is not to replace the textbook, but to relieve the student from having to copy every little thing that is written on the blackboard. This will allow the student to concentrate on understanding what is being discussed and once in a while add a marginal note to the printed notes. Also, in our specific case, most of our students' mother tongue is Spanish, and we have found that for some of them it is difficult to understand the English textbook. The notes also give the student support in this respect, since they are written in Spanish. Finally, the notes conform to the specific blend of topics covered and limit themselves to them, presenting a concise, concrete synthesis of the course material.

4. Conclusions

There are many low-cost, practical alternatives to help bring the differential equations course to life. For some students this makes the difference between another typical mathematics course and a very relevant and interesting learning experience. Even for instructors, performing these quick experiments breaks the routine and makes lectures very pleasant. We must admit though, that we have not seen any noticeable improvement in students' performance due to use of the demonstrations. However, we hope that the impact on students' perceptions of mathematics and science has been and will continue to be positive.

One should note that some of the experiments (mass/spring, pendulum, RLC circuit) should be preceded and followed by complete mathematical derivations and analysis. However, some very spectacular demonstrations (singing bars, whistling tube, wine glass, etc…) cannot be fully justified in an introductory course in ordinary differential equations.

REFERENCES

-Edwards C.H., Penney D.E., 2000, Elementary Differential Equations with Boundary Value Problems, Prentice Hall

-Borelli R.L., Coleman C.S., 1999, Differential Equations: A Modelling Perspective, John Wiley and Sons

-Boyce W.E., DiPima R.C., 2000, Elementary Differential Equations and Boundary Value Problems, John Wiley and Sons