7.6

1) $T(x,y,z) = 3x^2 + 3z^2$
$x^2 + z^2 = 2$, $0 \le y \le 2$, $k = 1$

$F = -\nabla T = -(6xi + 6zk) = (-6x, 0, -6z)$

S_1: $r_1(x,z) = (x, 0, z)$

$r_{1x} = (1,0,0)$, $r_{1z} = (0,0,1)$

$r_{1x} \times r_{1z} = (0,1,0)$

$\|r_{1x} \times r_{1z}\| = 1$

$$\iint_{S_1} (0,1,0) \cdot (-6x, 0, -6z)\, dx\, dz = 0$$

S_2: $r_2(x,z) = (x, 2, z)$

$$\iint_{S_2} (0,1,0)(-6x, 0, -6z)\, dx\, dz = 0$$

$r(\theta, z) = (\sqrt{2}\cos\theta, \sqrt{2}\sin\theta, z)$

$r_\theta = (-\sqrt{2}\sin\theta, \sqrt{2}\cos\theta, 0)$

$r_z = (0, 0, 1)$

(3) Κλείνω την S σας:

S_1 : το ημισφαίριο με παραμέτρηση

$$r_1(x,y) = (x, y, \sqrt{1-x^2-y^2})$$

S_2 : το επίπεδο μέρος της με

$$r_2(x,y) = (x, y, 0)$$

$$\left\| \frac{\partial r_1}{\partial x} \times \frac{\partial r_1}{\partial y} \right\| = \frac{1}{\sqrt{1-x^2-y^2}}$$

$\frac{\partial r_1}{\partial x} \times \frac{\partial r_1}{\partial y}$ έχει θετική z-συνιστώσα

οπότε κατευθύνεται προς το εξωτερικό της S.

$$\iint_{S_1} F\,dS = \iint_D (2x, 2y, 2z)\,(x,y,z)\,\frac{1}{z}\,dx\,dy =$$

$$= 2 \iint \frac{1}{\sqrt{1-x^2-y^2}}\,dx\,dy \overset{\text{πολικές}}{=}$$

$$= 2\int_0^1 \int_0^{2\pi} \frac{r}{\sqrt{1-r^2}}\,d\theta\,dr = 4\pi \int_0^1 \frac{r}{\sqrt{1-r^2}}\,dr =$$

$$= 4\pi \left[-\sqrt{1-r^2}\right]_0^1 = 4\pi$$

$$\frac{\partial r_2}{\partial x} \times \frac{\partial r_2}{\partial y} = (0,0,1)$$
$\downarrow$
>0

άρα κατεύθυνση προς το εσωτερικό της S

άρα θέτω $n=(0,0,-1)$

$$\iint_{S_2} E\,dS = \iint_D (2x,2y,2z)(0,0,-1)\,dx\,dy =$$

$$= -2\iint_D z\,dx\,dy =$$

$$= -2z\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dy\,dx = 0$$

($z=0$ στην S_2)

άρα $\boxed{4\pi}$

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$$r(x,y) = \left(x, y, -\frac{1}{3}\sqrt{1-x^2-y^2}\right)$$

$$\frac{\partial r}{\partial x} \times \frac{\partial r}{\partial y} = \left(-\frac{x}{3\sqrt{1-x^2-y^2}}, \frac{-y}{\sqrt{1-x^2-y^2}}, 1\right)$$

$$\left\|\frac{\partial r}{\partial x} \times \frac{\partial r}{\partial y}\right\| = \frac{\sqrt{9-8x^2-8y^2}}{3\sqrt{1-x^2-y^2}}$$

άρα $n = \left(-\frac{x}{\sqrt{9-8x^2-8y^2}}, \frac{-y}{\sqrt{9-8x^2-8y^2}}, \frac{3\sqrt{1-x^2-y^2}}{\sqrt{9-8x^2-8y^2}}\right)$

$$\iint_S (\nabla \times F) n \, dA = \frac{1}{9}\iint_D (2x^4 - 3x^2y^3 - 18)\, dx\, dy =$$

$$= 2\pi$$

$$D = \{(x,y) : x^2+y^2 \le 1\}$$

$$F = (y, -x, zx^3y^2)$$

$$2yzx^3, 1, -2)$$

(9) $\varphi(x,y,z) = (3xy^2, 3x^2y, z^3)$

$r(\theta,\varphi) = (\cos\theta\sin\varphi, \sin\theta\sin\varphi, \cos\varphi)$

$\frac{\partial r}{\partial \theta} \times \frac{\partial r}{\partial \varphi} = -\sin\varphi \cdot r(\theta,\varphi)$

↳ έχει κανονική προς τα εσωτερικό της S

$\left\| \frac{\partial r}{\partial \theta} \times \frac{\partial r}{\partial \varphi} \right\| = \sin\varphi$

άρα: $\eta = r(\theta,\varphi)$

$$\iint_S \varphi \cdot \eta \, dS = \int_0^{\pi}\int_0^{2\pi} (6\cos^2\theta\sin^2\theta\sin^5\varphi + \cos^4\varphi\sin\varphi)\, d\theta\, d\varphi = $$

$$= \frac{12}{5}\pi$$

(10) S_1: κάτω μέρος

$r_1(x,y) = (x,y,0)$

$\frac{\partial r_1}{\partial x} \times \frac{\partial r_1}{\partial y} = (0,0,1)$

$\left\| \frac{\partial r_1}{\partial x} \times \frac{\partial r_1}{\partial y} \right\| = 1$

Θέλω η προς τα εξωτερικό της S άρα

$\eta = (0,0,-1)$ άρα $\iint_{S_1} F \cdot \eta \, dA = 0.$

$= -\iint_D z(x^2+y^2)^2 \, dxdy$

$z = 0$ στην S_1

S_2: $r_2(x,y) = (x, y, 1)$ (πάνω πλευρά)

$$\frac{\partial r_2}{\partial x} \times \frac{\partial r_2}{\partial y} = (0, 0, 1)$$

$$\left\| \frac{\partial r_2}{\partial x} \times \frac{\partial r_2}{\partial y} \right\| = 1.$$

$$n = (0, 0, 1)$$

$$\iint_{S_2} F n = \iint_D z(x^2+y^2)^2 \, dx dy =$$

$$= \int_0^1 \int_0^{2\pi} r^5 \, d\theta dr = 2\pi \left[\frac{r^6}{6}\right]_0^1 = \frac{\pi}{3}$$

S_3: κυλινδρική επιφάνεια

$$r_3(\theta, z) = (\cos\theta, \sin\theta, z)$$

$$\frac{\partial r_3}{\partial \theta} \times \frac{\partial r_3}{\partial z} = (\cos\theta, \sin\theta, 0)$$

$$\left\| \frac{\partial r_3}{\partial \theta} \times \frac{\partial r_3}{\partial z} \right\| = 1$$

$$n = (\cos\theta, \sin\theta, 0)$$

Άρα $\iint_{S_3} F n \, dA = \int_0^1 \int_0^{2\pi} (\cos\theta + \sin\theta) d\theta dz = 0$.

(17) $r(\vartheta, \varphi) = (\alpha\cos\vartheta\sin\varphi, b\sin\vartheta\sin\varphi, c\cdot\cos\varphi)$

$r\vartheta = (-\alpha\sin\vartheta\sin\varphi, b\cos\vartheta\sin\varphi, 0)$

$r\varphi = (\alpha\cos\vartheta\cos\varphi, b\sin\vartheta\cos\varphi, -c\sin\varphi)$

$r\vartheta \times r\varphi = (-bc\sin^2\varphi\cos\vartheta, -\alpha c\sin^2\varphi\sin\vartheta, -\alpha b\sin\varphi\cos\varphi)$

$$\iint_D -(\alpha^3\cos^3\vartheta\sin^3\varphi)(-bc\sin^2\varphi\cos\vartheta)\,d\vartheta\,d\varphi =$$

$$= \int_0^{\pi/2}\int_0^{2\pi} \alpha^3 bc\cos^4\vartheta\sin^5\varphi\,d\vartheta\,d\varphi =$$

$$= \int_0^{\pi/2} \frac{3\pi}{4}\alpha^3 bc\sin^5\varphi\,d\varphi =$$

$$= \frac{2}{3}\pi\alpha^3 bc$$

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$$r(x,y) = \left(x, y, \sqrt{1-x^2-y^2}\right)$$

$$r_x = \left(1, 0, -\frac{x}{\sqrt{1-x^2-y^2}}\right)$$

$$r_y = \left(0, 1, \frac{-y}{\sqrt{1-x^2-y^2}}\right)$$

$$r_x \times r_y = \left(\frac{x}{\sqrt{1-x^2-y^2}}, \frac{y}{\sqrt{1-x^2-y^2}}, 1\right)$$

$$\int_S F\,dS = \iint \frac{x^2+y^2}{\sqrt{1-x^2-y^2}}\,dx\,dy =$$

[illegible]

$$= \int_0^1 \int_0^{\pi} \frac{r^3}{\sqrt{1-r^2}}\,d\theta\,dr =$$

$$= \pi \int_0^1 \frac{r^3}{\sqrt{1-r^2}}\,dr = \frac{2\pi}{3}$$