Ακουστικά Κύματα

Η Ακουστική εξίσωση

Γενικές Λύσεις

Εισαγωγή στην Ακουστική Ωκεανογραφία

Εξίσωση για την ακουστική πίεση

$$\nabla^2 p_1(\vec{x},t) = \frac{1}{c(\vec{x})^2} \frac{\partial^2 p_1(\vec{x},t)}{\partial t^2}$$

Γραμμικοποιημένη εξίσωση

$$\rho = \rho_0(\vec{x},t) + \varepsilon \rho_1(\vec{x},t)$$

$$p = p_0(\vec{x},t) + \varepsilon\, p_1(\vec{x},t)$$

$$\vec{u} = \vec{u}_0(\vec{x},t) + \varepsilon \vec{u}_1(\vec{x},t)$$

$c(\vec{x})$: Ταχύτητα διάδοσης του κύματος (ήχου)

$$\nabla^2 p_1(\vec{x},t) = \frac{1}{c(\vec{x})^2}\frac{\partial^2 p_1(\vec{x},t)}{\partial t^2}$$

$$\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$ Καρτεσιανό σύστημα

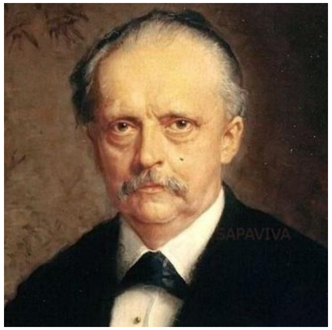
Χωρισμός μεταβλητών

$$p_1(\vec{x},t) = \overline{p}(\vec{x})T(t)$$

$$\overline{p}(\vec{x}) \equiv p(\vec{x})$$

$$T(t)\nabla^2 p(\vec{x}) = \frac{1}{c(\vec{x})^2}\, p(\vec{x})\frac{d^2T(t)}{dt^2}$$

$$\nabla^2 p_1(\vec{x},t) = \frac{1}{c(\vec{x})^2} \frac{\partial^2 p_1(\vec{x},t)}{\partial t^2}$$



1821-1894

Εξίσωση Helmholtz

$$T(t)\nabla^2 p(\vec{x}) = \frac{1}{c(\vec{x})^2} p(\vec{x}) \frac{d^2T(t)}{dt^2}$$

$$\frac{c(\vec{x})^2}{p(\vec{x})} \nabla^2 p(\vec{x}) = \frac{1}{T(t)} \frac{d^2T(t)}{dt^2}$$

$$\frac{c^2}{p}\nabla^2 p = \frac{1}{T}\frac{d^2T}{dt^2} = -\omega^2 \left(= l\right)$$

$$\nabla^2 p(\vec{x}) + \frac{\omega^2}{c(\vec{x})^2} p(\vec{x}) = 0$$

$$\frac{d^2T(t)}{dt^2} + \omega^2 T(t) = 0$$

$$\frac{d^2T(t)}{dt^2} + \omega^2 T(t) = 0$$

$$T(t) = Ae^{i\omega t} + Be^{-i\omega t} \qquad T(t) = A' \cos(\omega t) + B' \sin(\omega t)$$

Θεωρούμε χρονική εξάρτηση $T = e^{-i\omega t}$

$$p_1(\vec{x}, t) = p(\vec{x})T(t)$$

$$p_1(\vec{x}, t) = p(\vec{x})e^{-i\omega t}$$

Αρμονικά κύματα

Χωρισμός μεταβλητών

Καρτεσιανό σύστημα

$$\nabla^2 p + \frac{\omega^2}{c^2} p = 0 \qquad \nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$p(\vec{x}) = p(x, y, z) = p_x(x)\, p_y(y)\, p_z(z)$$

$$\frac{d^2 p_x}{dx^2} \cdot \frac{1}{p_x} + \frac{d^2 p_y}{dy^2} \cdot \frac{1}{p_y} + \frac{d^2 p_z}{dz^2} \cdot \frac{1}{p_z} + \left(\frac{\omega}{c}\right)^2 = 0$$

$$\frac{d^2 p_x}{dx^2} \cdot \frac{1}{p_x} = -k_x^2 \qquad \frac{d^2 p_y}{dy^2} \cdot \frac{1}{p_y} = -k_y^2 \qquad \frac{d^2 p_z}{dz^2} \cdot \frac{1}{p_z} = -k_z^2$$

$$\frac{d^2 p_x}{dx^2} \cdot \frac{1}{p_x} + \frac{d^2 p_y}{dy^2} \cdot \frac{1}{p_y} + \frac{d^2 p_z}{dz^2} \cdot \frac{1}{p_z} + \left(\frac{\omega}{c}\right)^2 = 0$$

$$k_x^2 + k_y^2 + k_z^2 = \left(\frac{\omega}{c}\right)^2$$

$$k_x^2 + k_y^2 + k_z^2 = k^2$$

Αριθμός κύματος $k = \frac{\omega}{c} = \frac{2\pi f}{c}$

Υποθέτουμε ότι k_x, k_y, k_z σταθερά

$$\frac{d^2 p_x}{dx^2} + k_x^2 p_x = 0 \qquad p_x(x) = A_1 e^{ik_x x} + A_2 e^{-ik_x x}$$

$$\frac{d^2 p_y}{dy^2} + k_y^2 p_y = 0 \qquad p_y(y) = B_1 e^{ik_y y} + B_2 e^{-ik_y y}$$

$$\frac{d^2 p_z}{dz^2} + k_z^2 p_z = 0 \qquad p_z(z) = C_1 e^{ik_z z} + C_2 e^{-ik_z z}$$

Υποθέτουμε ότι k_x, k_y, k_z σταθερά

$$\frac{d^2 p_x}{dx^2} + k_x^2 p_x = 0 \qquad p_x(x) = A_1^{'} \cos(k_x x) + A_2^{'} \sin(k_x x)$$

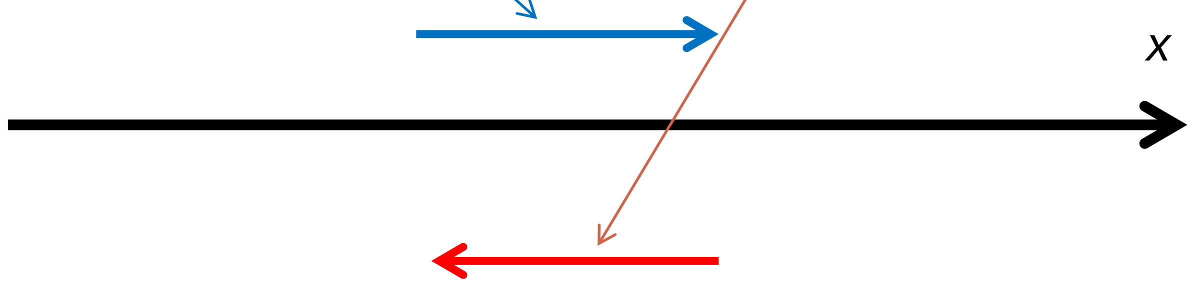
$$\frac{d^2 p_y}{dy^2} + k_y^2 p_y = 0 \qquad p_y(y) = B_1^{'} \cos(k_y y) + B_2^{'} \sin(k_y y)$$

$$\frac{d^2 p_z}{dz^2} + k_z^2 p_z = 0 \qquad p_z(z) = C_1^{'} \cos(k_z z) + C_2^{'} \sin(k_z z)$$

Σημασία κάθε όρου στην κυματική διάδοση

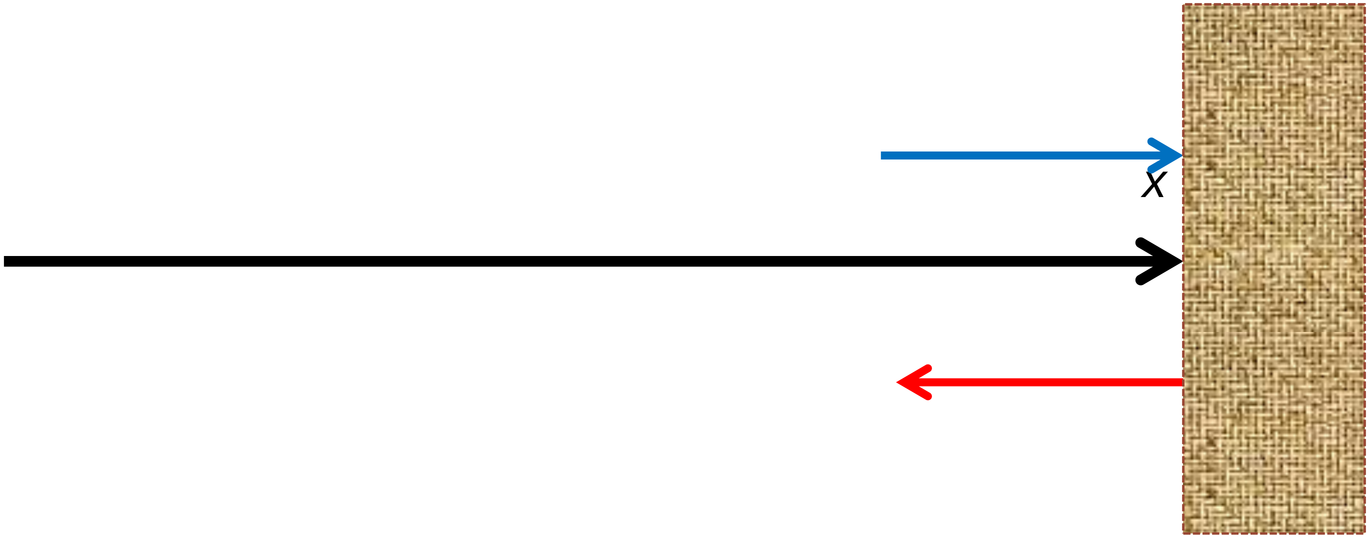
$$p_1(x,t) = p_x(x)e^{-i\omega t} = (A_1 e^{ik_x x} + A_2 e^{-ik_x x})e^{-i\omega t}$$

$$= A_1 e^{i(k_x x - \omega t)} + A_2 e^{i(-k_x x - \omega t)}$$



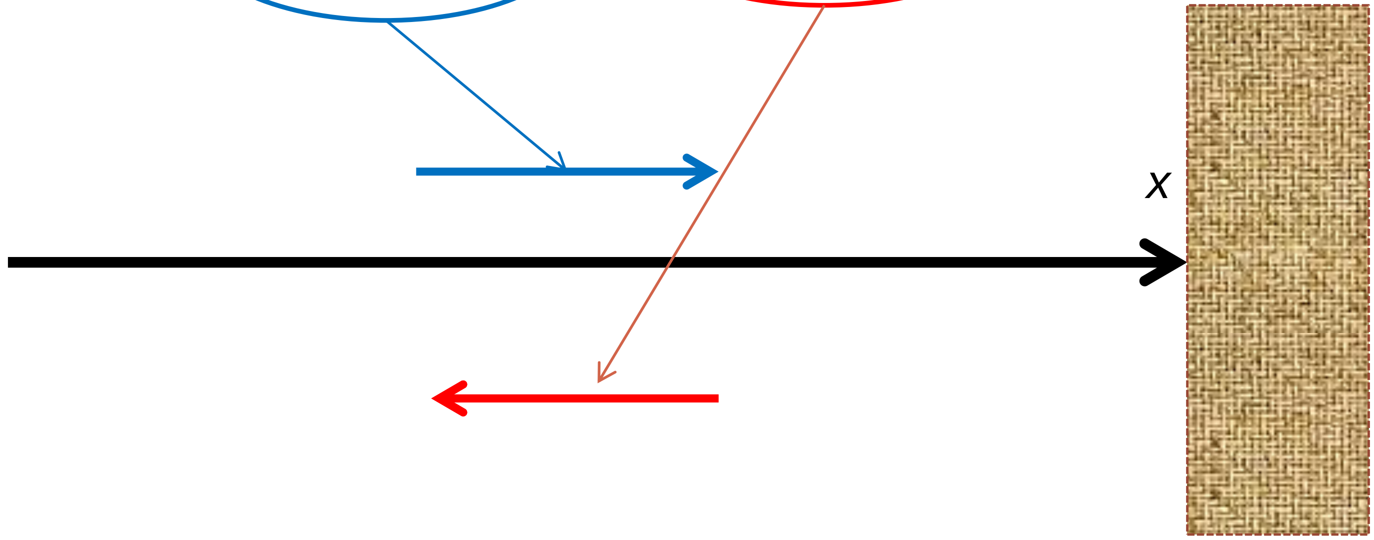
$$p_1(x,t) = p_x(x)e^{-i\omega t} = (A_1 e^{ik_x x} + A_2 e^{-ik_x x})e^{-i\omega t}$$

$$= A_1 e^{i(k_x x - \omega t)} + A_2 e^{i(-k_x x - \omega t)}$$



$$p_1(x,t) = p_x(x)e^{-i\omega t} = (A_1 e^{ik_x x} + A_2 e^{-ik_x x})e^{-i\omega t}$$

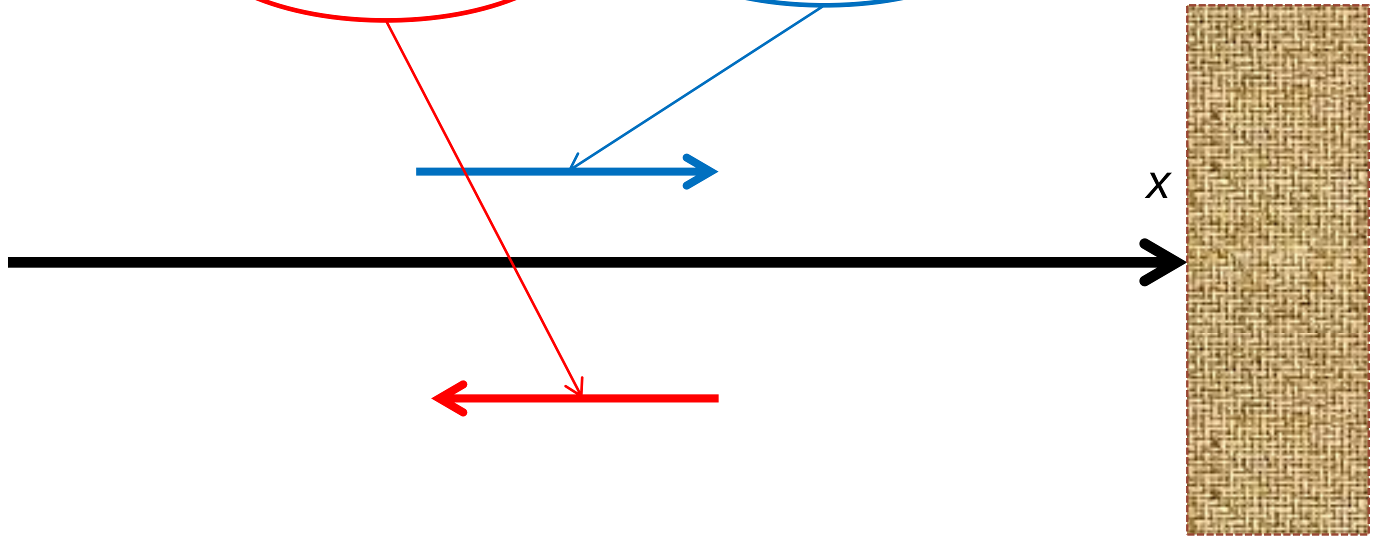
$$= A_1 e^{i(k_x x - \omega t)} + A_2 e^{i(-k_x x - \omega t)}$$



Εάν $T = e^{i\omega t}$

$$p_1(x,t) = p_x(x)e^{i\omega t} = (A_1 e^{ik_x x} + A_2 e^{-ik_x x})e^{i\omega t}$$

$$= A_1 e^{i(k_x x + \omega t)} + A_2 e^{i(-k_x x + \omega t)}$$



Διάδοση σε τρεις διαστάσεις

$$p_1(\vec{x},t) = p(\vec{x})e^{-i\omega t}$$

$$p(x,y,z) = p_x(x)p_y(y)p_z(z)$$

$$p_x(x) = A_1 e^{ik_x x} + A_2 e^{-ik_x x}$$

$$p_y(x) = B_1 e^{ik_y y} + B_2 e^{-ik_y y}$$

$$p_z(x) = C_1 e^{ik_z z} + C_2 e^{-ik_z z}$$

$$p_1(x,y,z,t) = A_1 B_1 C_1 e^{i(k_x x + k_y y + k_z z - \omega t)}$$

$$p_1(x, y, z, t) = A_1 B_1 C_1 e^{i(k_x x + k_y y + k_z z - \omega t)}$$

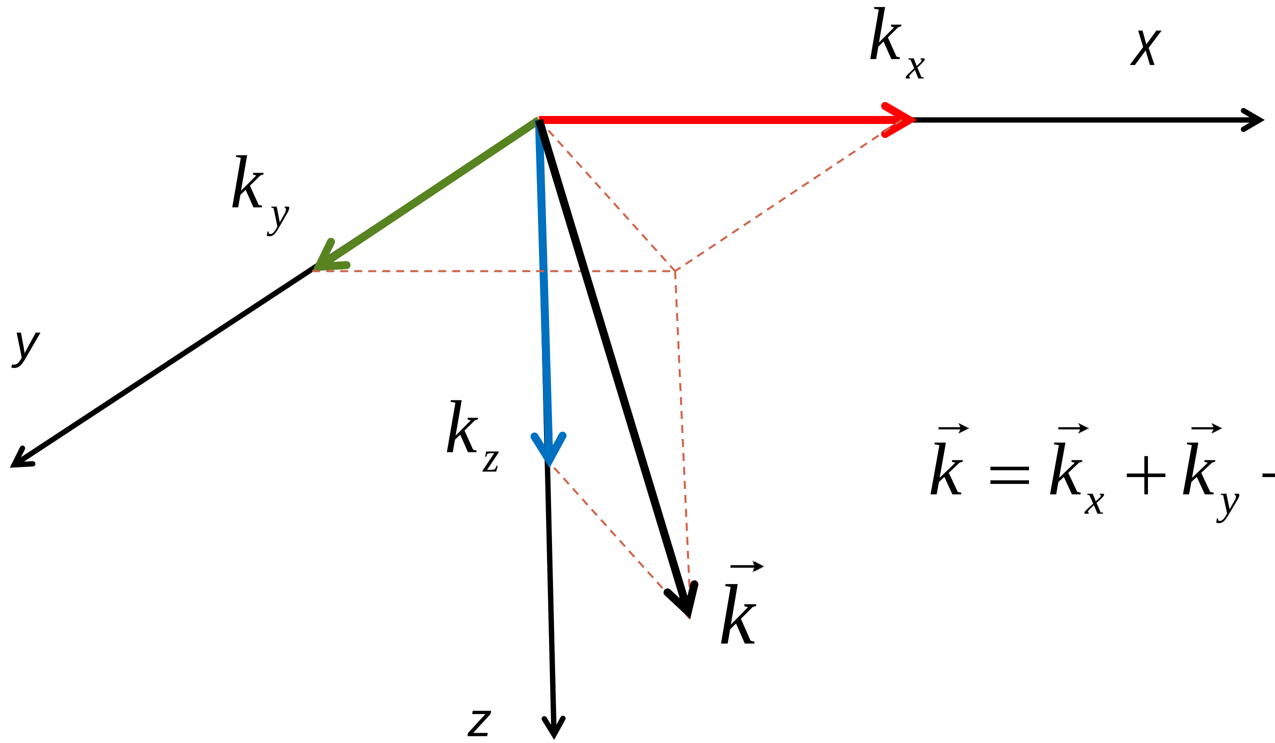
Όδευση προς την κατεύθυνση των θετικών των αντίστοιχων αξόνων

$$\vec{k} = \vec{k}_x + \vec{k}_y + \vec{k}_z = (k_x, k_y, k_z)$$

$$\vec{x} = (x, y, \mathrm{z})$$

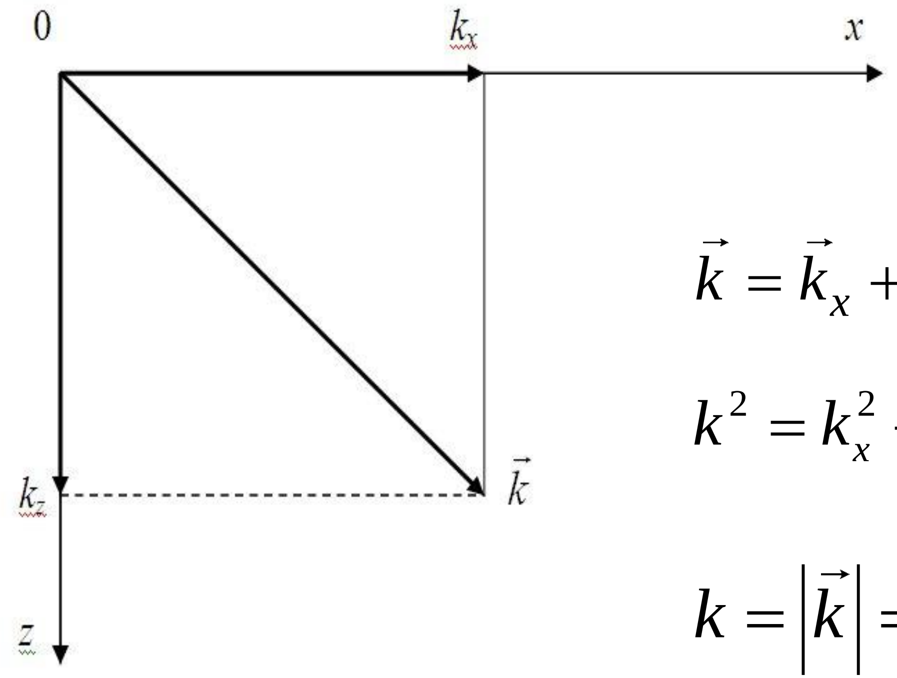
$$p_1(x, y, z, t) = D e^{i(\vec{k} \cdot \vec{x} - \omega t)}$$

Ο αριθμός κύματος ως διανυσματικό μέγεθος



$$\vec{k} = \vec{k}_x + \vec{k}_y + \vec{k}_z$$

Ο αριθμός κύματος ως διανυσματικό μέγεθος

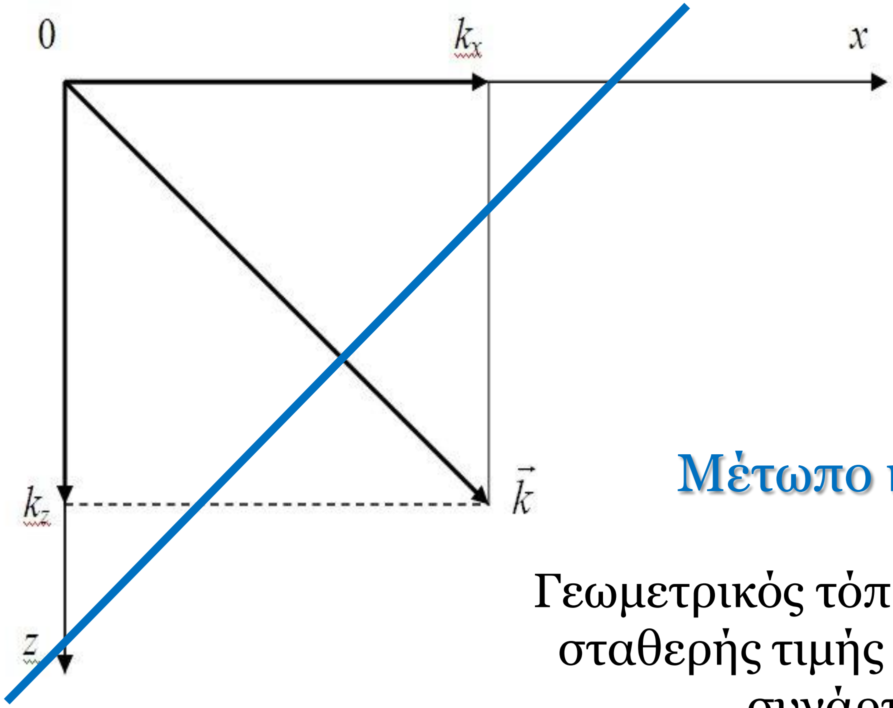


$$\vec{k} = \vec{k}_x + \vec{k}_y$$

$$k^2 = k_x^2 + k_y^2$$

$$k = \left|\vec{k}\right| = \frac{\omega}{c}$$

Ο αριθμός κύματος ως διανυσματικό μέγεθος

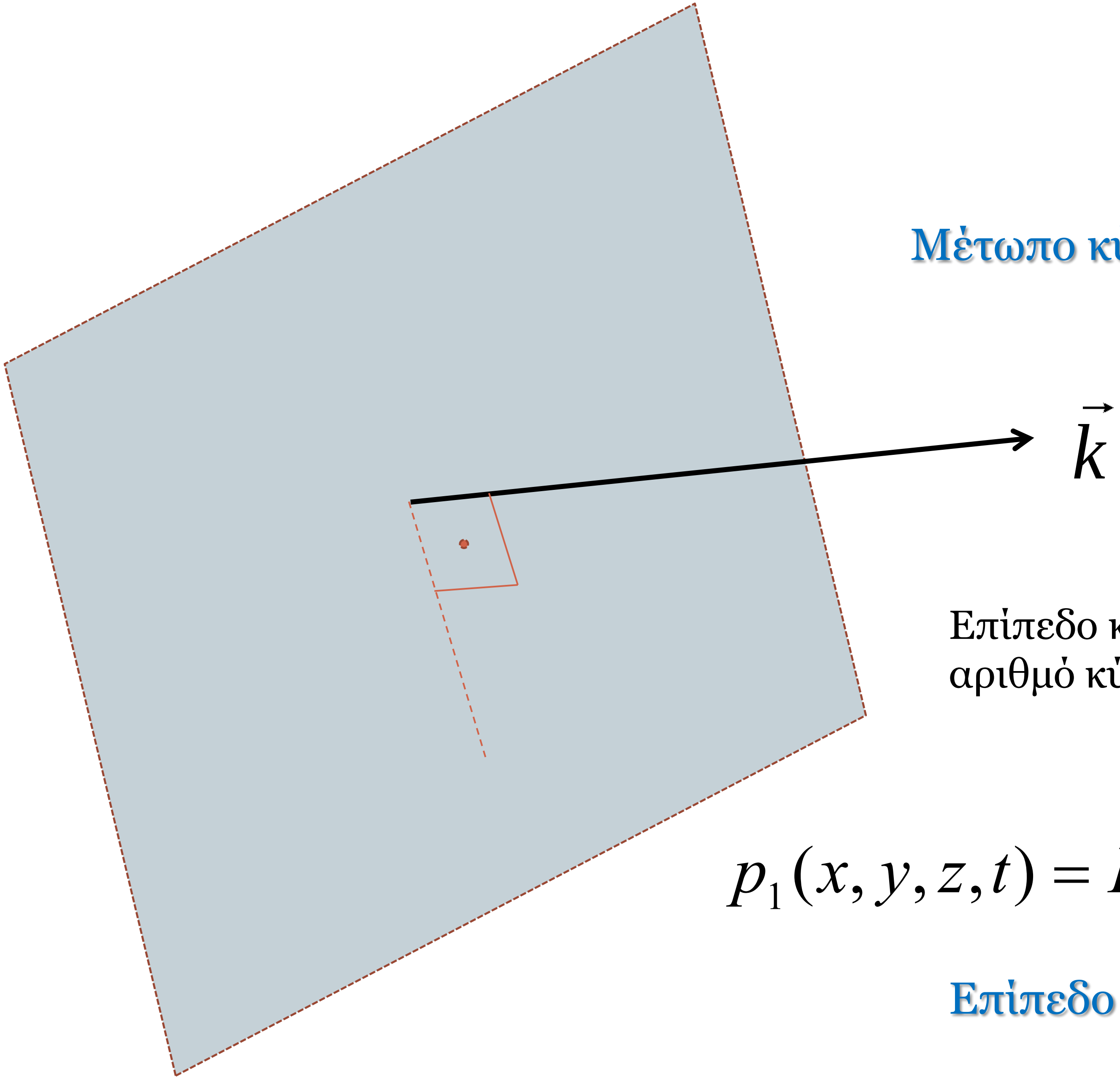


Μέτωπο κύματος

Γεωμετρικός τόπος των σημείων σταθερής τιμής της κυματικής συνάρτησης

$$p_1(x,y,z,t) = De^{i(\vec{k}\cdot\vec{x}-\omega t)}$$

$$\vec{k}\cdot\vec{x} = k_x x + k_y y = C$$

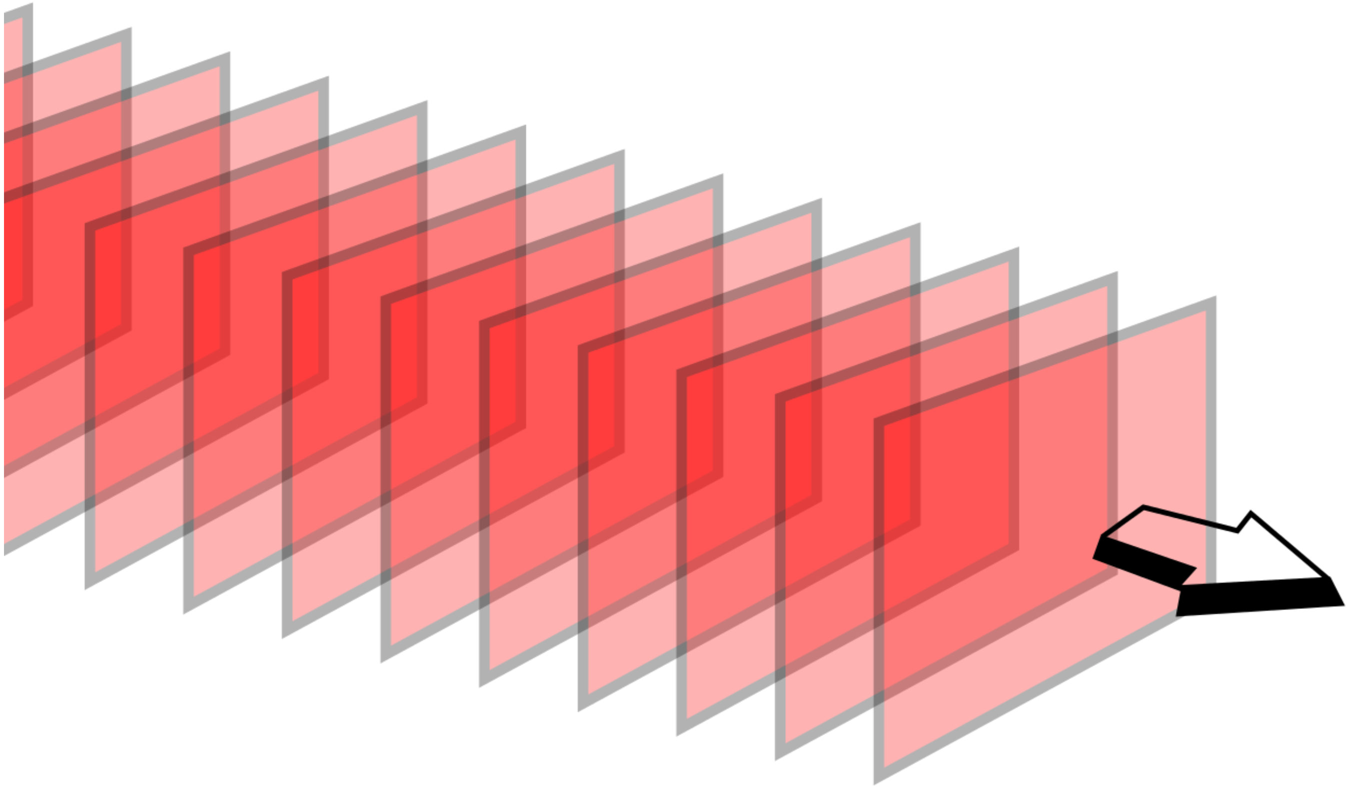


Μέτωπο κύματος

Επίπεδο κάθετο στον αριθμό κύματος

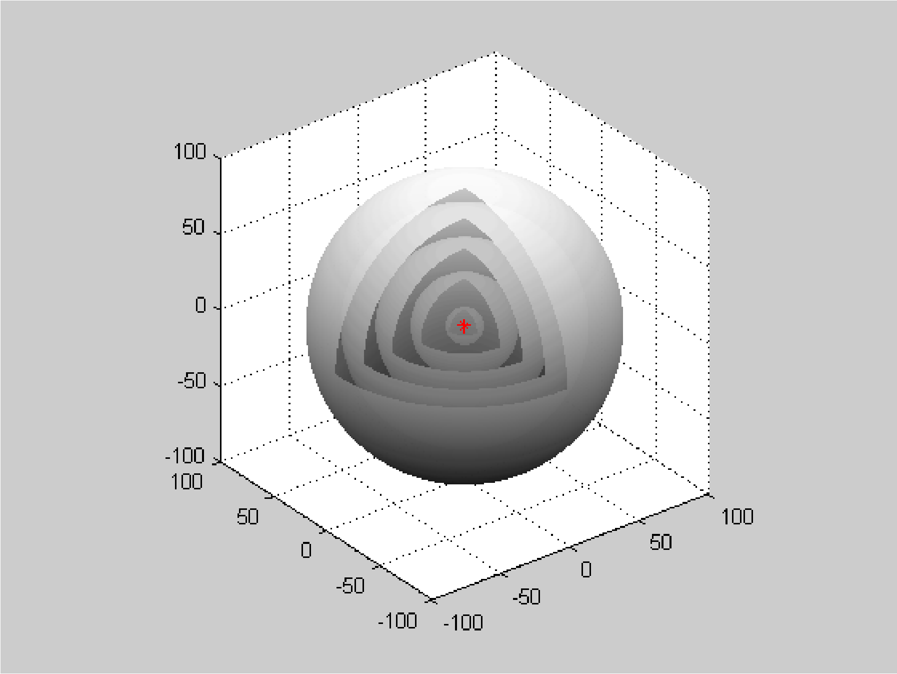
$$p_1(x,y,z,t) = De^{i(\vec{k}\cdot\vec{x}-\omega t)}$$

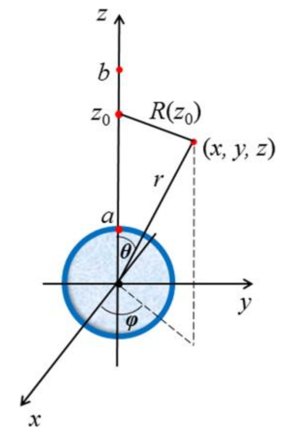
Επίπεδο Κύμα

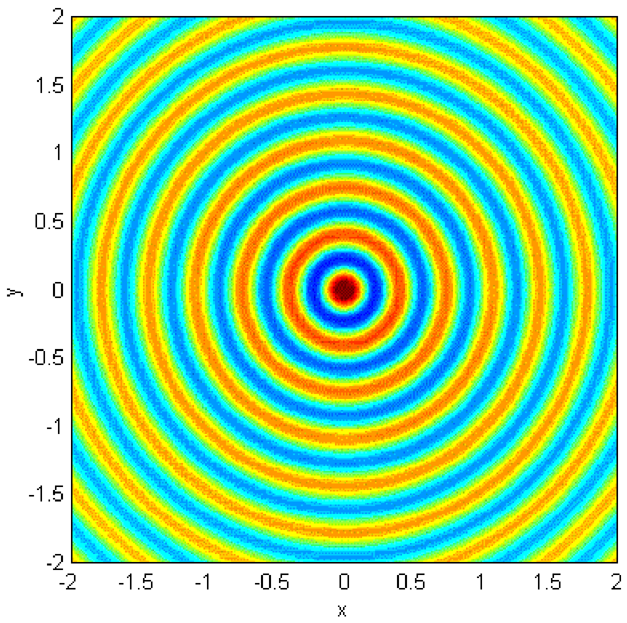


Επίπεδο Κύμα

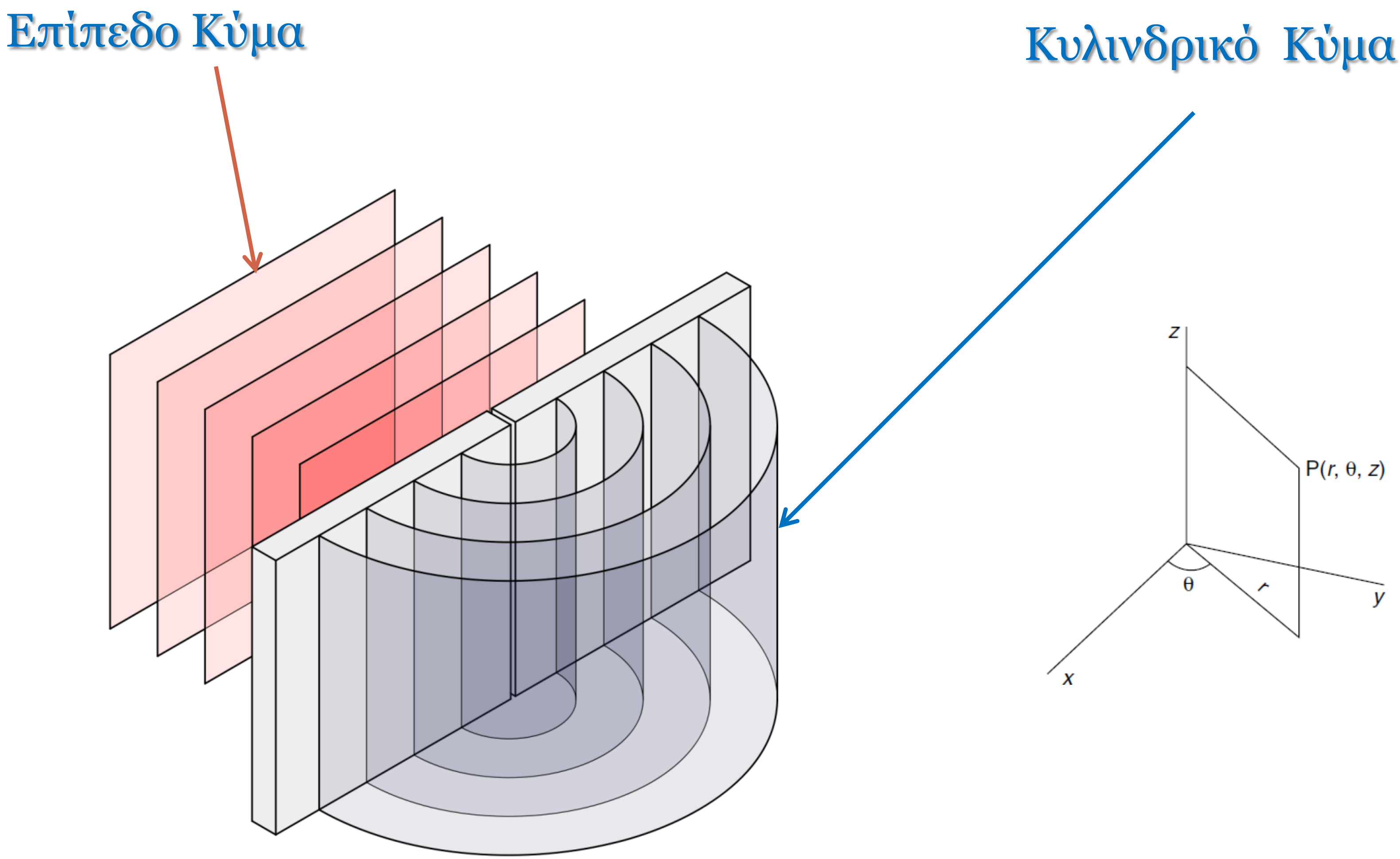
Σφαιρικό Κύμα







$$\frac{\partial^2 p(r,t)}{\partial r^2} + \frac{2}{r}\frac{\partial p}{\partial r} = \frac{1}{c^2}\frac{\partial^2 p(r,t)}{\partial t^2}$$



$$\frac{\partial^2 p(r,z,t)}{\partial r^2}+\frac{1}{r}\frac{\partial p(r,z,t)}{\partial r}+\frac{\partial^2 p(r,z,t)}{\partial z^2}=\frac{1}{c^2}\frac{\partial^2 p(r,zt)}{\partial t^2}$$

Διάδοση σε μία διεύθυνση (x)

$$p_1(x,t) = A_1 e^{i(k_x x - \omega t)}$$

Πραγματική πίεση

$$\mathrm{Re}\left(p_1(x,t) \right) = \mathrm{Re}\left(A_1 e^{i(k_x x - \omega t)} \right)$$

$$\tilde{p}_1(x,t) = A_1 \cos(k_x x - \omega t)$$

Με διαφορά φάσης $T(t) = \mathrm{e}^{-i(\omega t - \Delta\varphi)}$

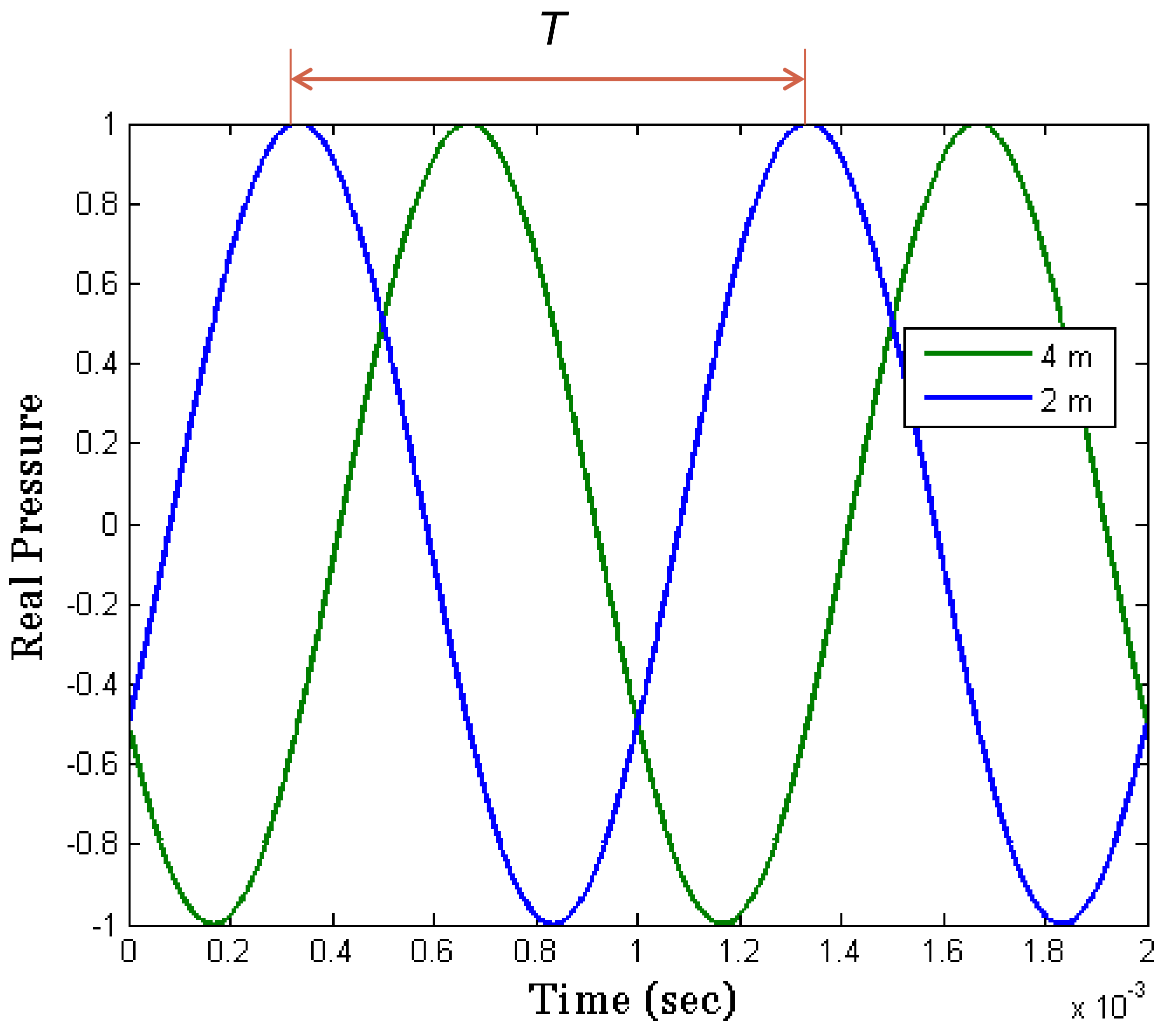
$$\tilde{p}_1(x,t) = A_1 \cos(k_x x - \omega t + \Delta\varphi)$$

Διάδοση σε μία διεύθυνση

$\omega = 2\pi f$ $\quad f$: συχνότητα σε Hz

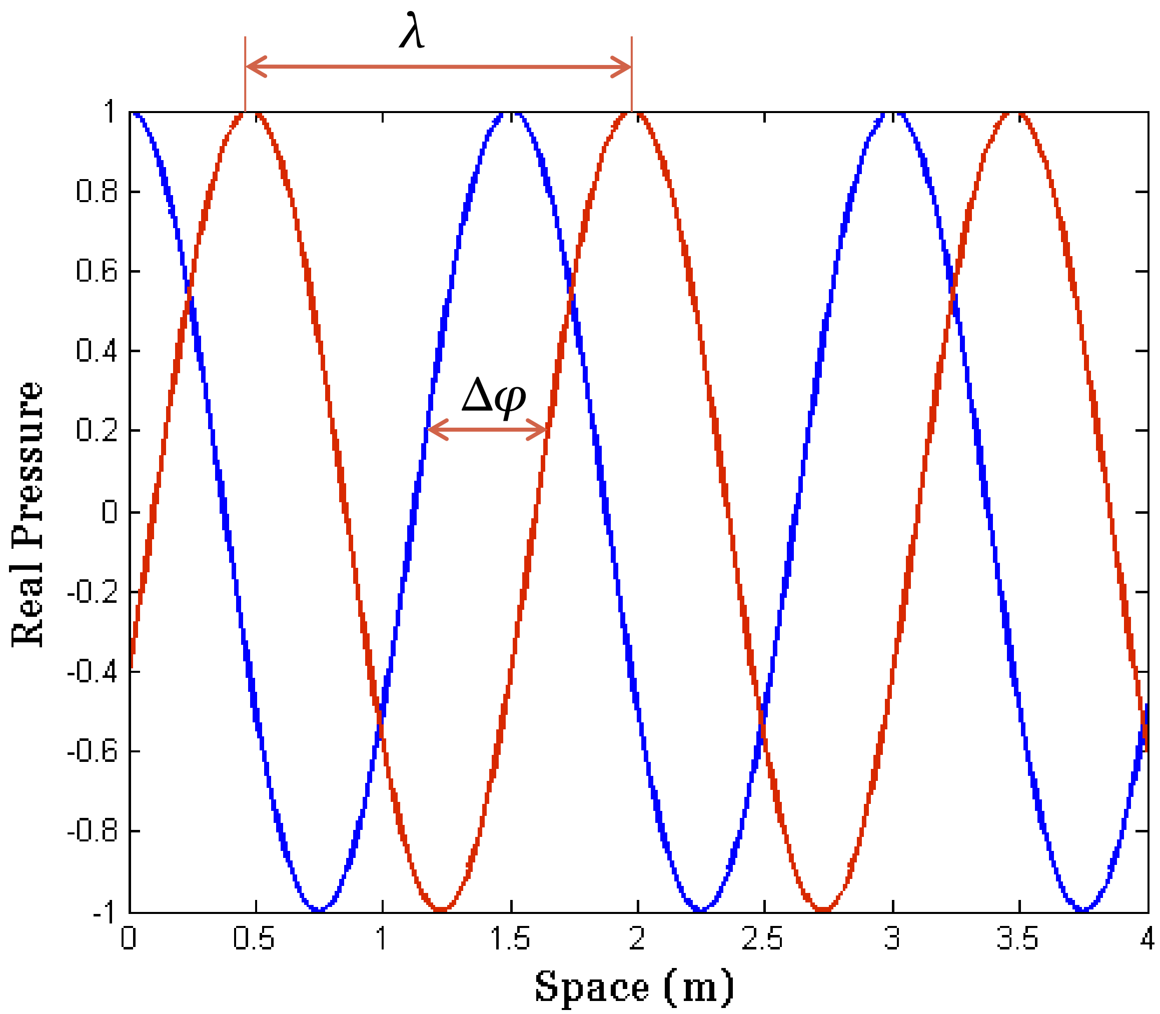
$$f = \frac{1}{T}$$ T : περίοδος κύματος

$\lambda = cT$ $\quad \lambda$: μήκος κύματος



f=1000 Hz, c=1500 m/sec, A_1 =1 T=0,001 sec

$$\tilde{p}_1(x,t) = A_1 \cos(k_x x - \omega t)$$



f=1000 Hz, c=1500 m/sec, t=1 sec, A_1 =1 λ=1,5 m

$$\tilde{p}_1(x,t) = A_1 \cos(k_x x - \omega t + \Delta\varphi)$$