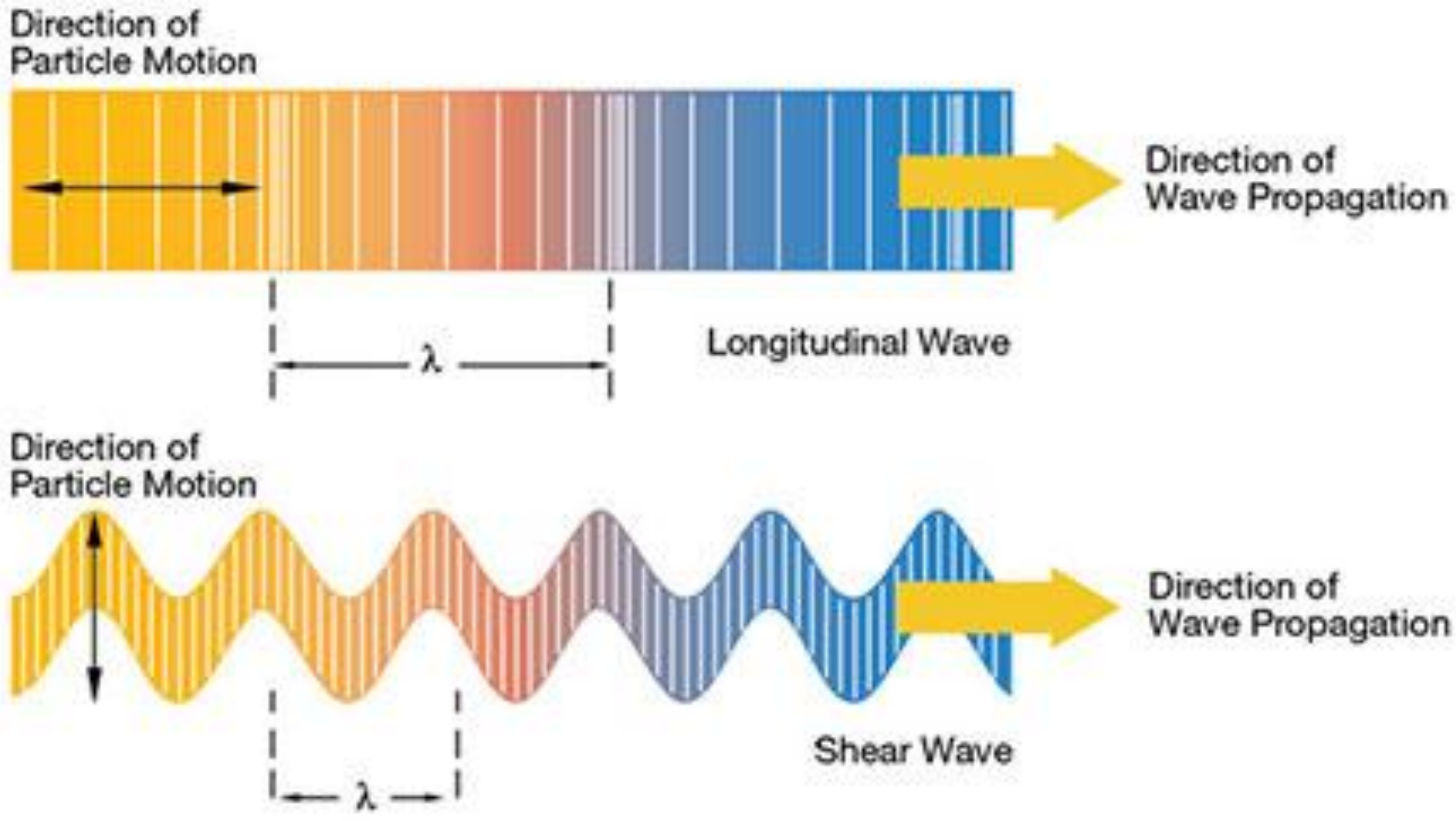
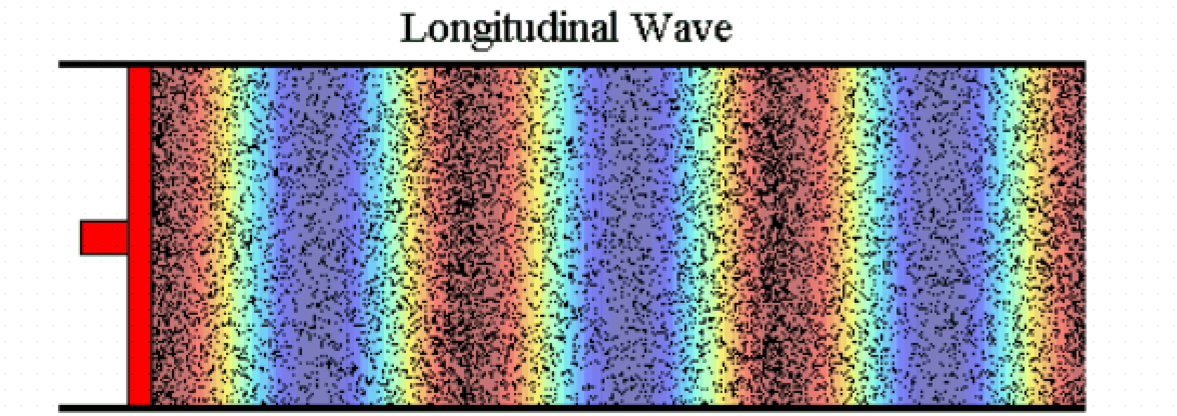
Διάδοση σε ελαστικούς χώρους

Φαινόμενα ανάκλασης και διάθλασης

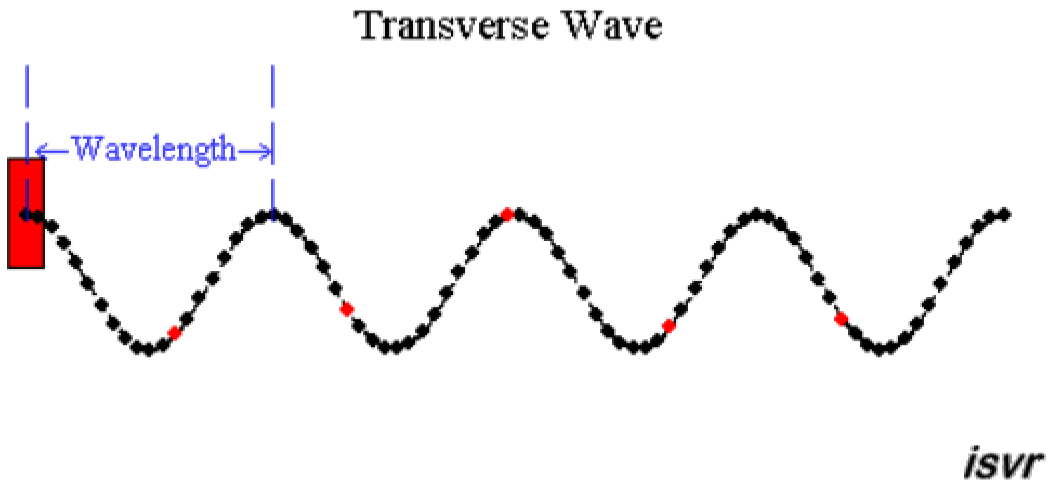
Εισαγωγή στην Ακουστική Ωκεανογραφία


Direction of
Particle Motion
Direction of
Wave Propagation
λ
Longitudinal Wave
Direction of
Particle Motion
Direction of
Wave Propagation
Shear Wave
λ

Διάμηκες Κύμα



Εγκάρσιο Κύμα



Από τη Μηχανική των σωμάτων

Τάση *σ* - παραμόρφωση ε

$$\mathrm{E} \equiv \frac{\sigma}{\varepsilon}$$

Μέτρα ελαστικότητας

$$K = -V\,\frac{\partial p}{\partial V}$$

Μέτρο διόγκωσης

$$K = \rho\,\frac{\partial p}{\partial \rho}$$

$$c = \sqrt{\frac{\partial p}{\partial \rho}} = \sqrt{\frac{K}{\rho}}$$

Ταχύτητα ήχου στα ρευστά

$$\nabla \times \nabla f = 0$$

$$-\nabla p_1 = \rho_0 \frac{\partial \vec{u}_1}{\partial t}$$

$$\nabla \times \vec{u} = 0$$

Η ταχύτητα των στοιχειωδών σωματιδίων είναι «αστρόβιλο» μέγεθος

$$\vec{u} = \nabla \Phi_u$$

$$\vec{d} = \nabla \Phi$$

Δυναμικό μετατόπισης

$$\vec{u} = (u_x, u_y, u_z)$$

$$\vec{d} = (d_x, d_y, d_z)$$

$$\vec{d} = \nabla \Phi$$

$$p_1 = -K \nabla \cdot \vec{d}$$ Νόμος Hooke

$$p_1 = -K \nabla^2 \Phi$$

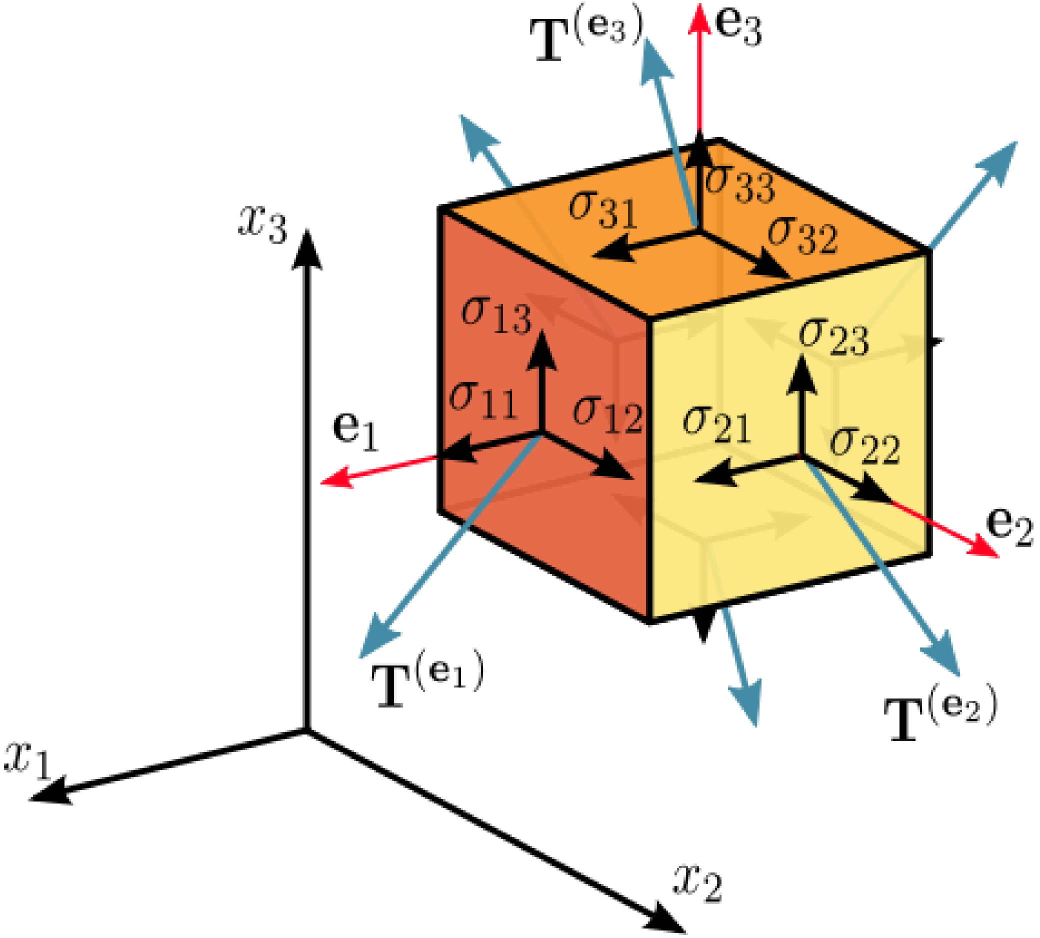
$$\nabla^2 \Phi = \frac{1}{c^2} \frac{\partial^2 \Phi}{\partial t^2}$$

$$p_1 = -K\nabla \cdot \vec{d}$$

$$\nabla^2 \Phi = \frac{1}{c^2}\frac{\partial^2 \Phi}{\partial t^2}$$

$$K = \rho c^2$$

$$p_1 = -\rho \frac{\partial^2 \Phi}{\partial t^2}$$



$\mathbf{T}^{(\mathbf{e}_3)}$
$\mathbf{e}_3$
σ_{33}
σ_{31}
σ_{32}
x_3
σ_{13}
σ_{23}
σ_{11}
σ_{12}
σ_{21}
σ_{22}
$\mathbf{e}_1$
$\mathbf{e}_2$
$\mathbf{T}^{(\mathbf{e}_1)}$
$\mathbf{T}^{(\mathbf{e}_2)}$
x_1
x_2

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial d_i}{\partial x_j} + \frac{\partial d_j}{\partial x_i}\right)$$ Παραμόρφωση (strain)

$$\sigma_{ij} = \lambda \delta_{ij} \varepsilon_{kk} + 2\mu \varepsilon_{ij}$$ Νόμος Hooke

$$\varepsilon_{kk} = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}$$ Ανηγμένη διόγκωση

$$\sigma_{ij} = \lambda \delta_{ij} \varepsilon_{kk} + \mu\left(\frac{\partial d_i}{\partial x_j} + \frac{\partial d_j}{\partial x_i}\right)$$

2ος Νόμος Newton ➡ $$\rho \frac{\partial^2 d_i}{\partial t^2} = \frac{\partial \sigma_{ij}}{\partial x_j}$$

$$-\nabla p_1 = \rho_0 \frac{\partial \vec{u}_1}{\partial t}$$

$$\rho \frac{\partial^2 \vec{d}}{\partial t^2} = (\lambda + 2\mu)\nabla(\nabla \cdot \vec{d}) - \mu \nabla \times (\nabla \times \vec{d})$$

Περιστροφή : ⟹

$$\nabla^2 (\nabla \times \vec{d}) - \frac{1}{c_s^2} \frac{\partial^2 (\nabla \times \vec{d})}{\partial t^2} = 0$$

$$c_s^2 = \frac{\mu}{\rho}$$

Απόκλιση : ⟹

$$\nabla^2 (\nabla \cdot \vec{d}) - \frac{1}{c_p^2} \frac{\partial^2 (\nabla \cdot \vec{d})}{\partial t^2} = 0$$

$$c_p^2 = \frac{\lambda + 2\mu}{\rho}.$$

$$\nabla^2 \Phi = \frac{1}{c_p^2} \frac{\partial^2 \Phi}{\partial t^2}$$

$$\nabla^2 \mathbf{\Psi} = \frac{1}{c_s^2} \frac{\partial^2 \mathbf{\Psi}}{\partial t^2}$$

$$\vec{d} = \nabla \Phi + \nabla \times \mathbf{\Psi}$$

$$c_s = \sqrt{\frac{G}{\rho}}$$

$$\mu \equiv G$$

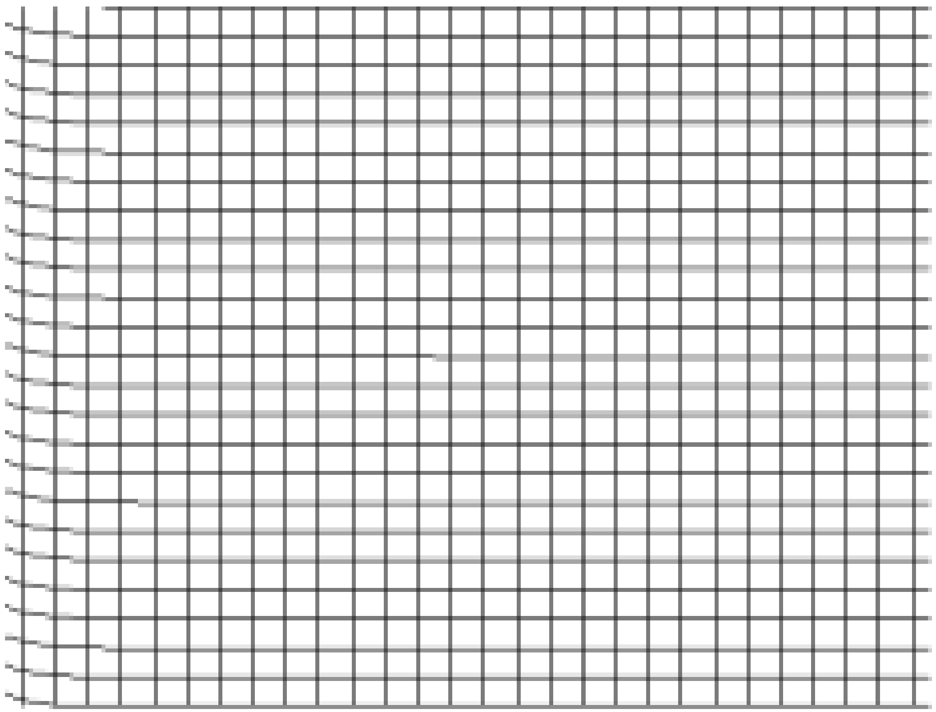
Μέτρο ελαστικότητας
(shear modulus)

$$K = \lambda + (2/3)\mu$$

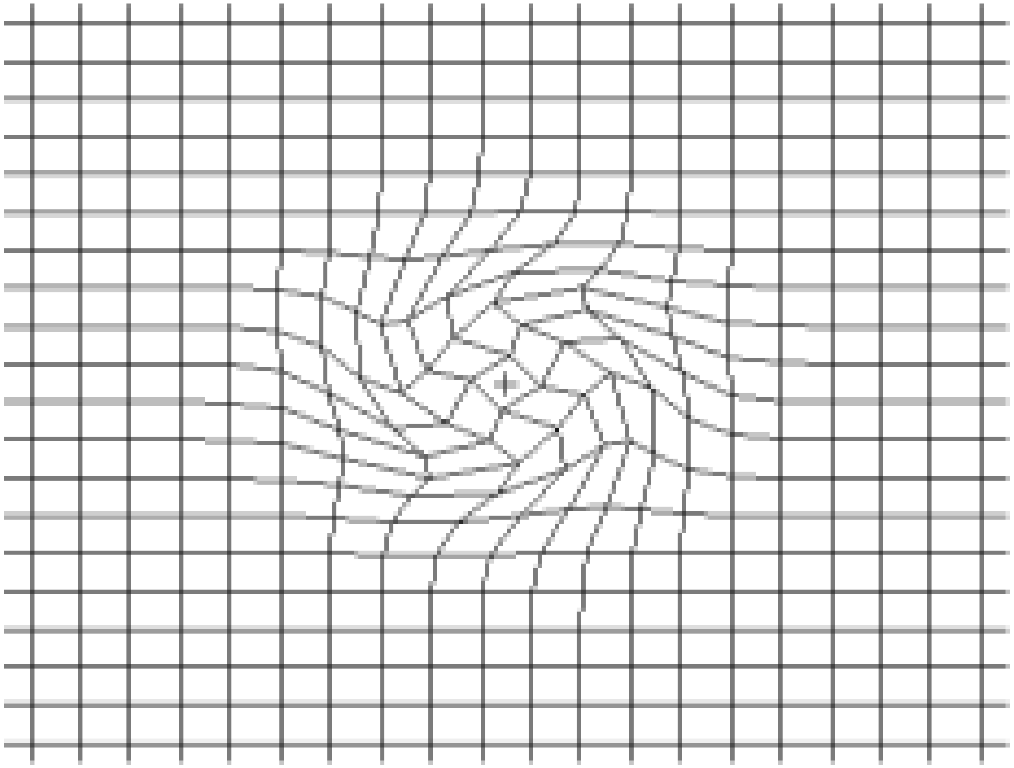
$$c_p = \sqrt{\frac{\lambda + 2\mu}{\rho}}.$$

$$\mu = 0, \quad K = \lambda, \quad c_p = \sqrt{\frac{K}{\rho}} = \sqrt{\frac{\lambda}{\rho}}$$

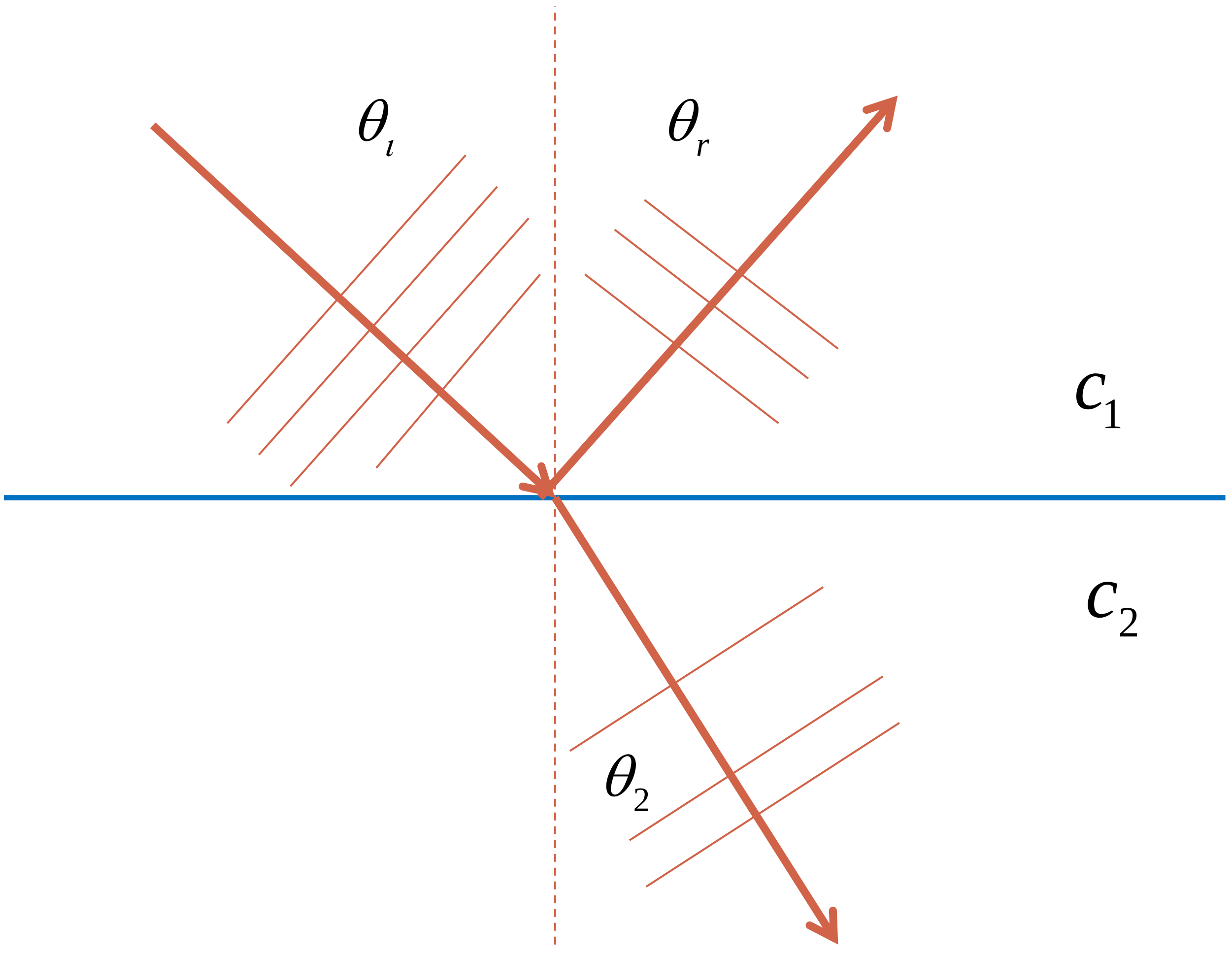
Διάδοση επίπεδου διατμητικού κύματος

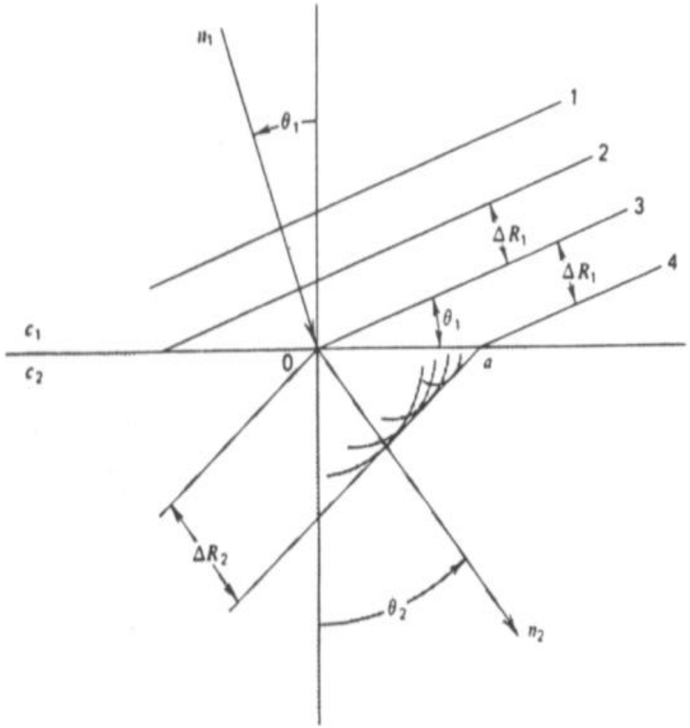


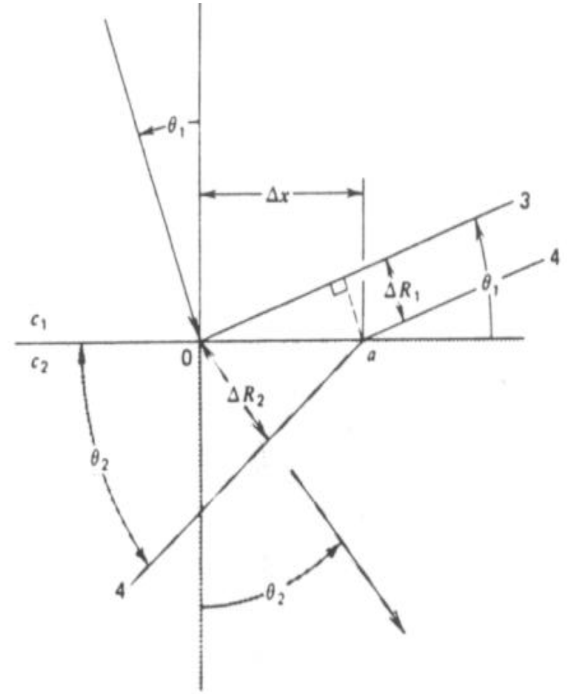
Προβολή διάδοσης σφαιρικού διατμητικού κύματος στο επίπεδο



Φαινόμενα Ανάκλασης και Διάδοσης σε διεπιφάνειες


θι
θr
c1
c2
θ2

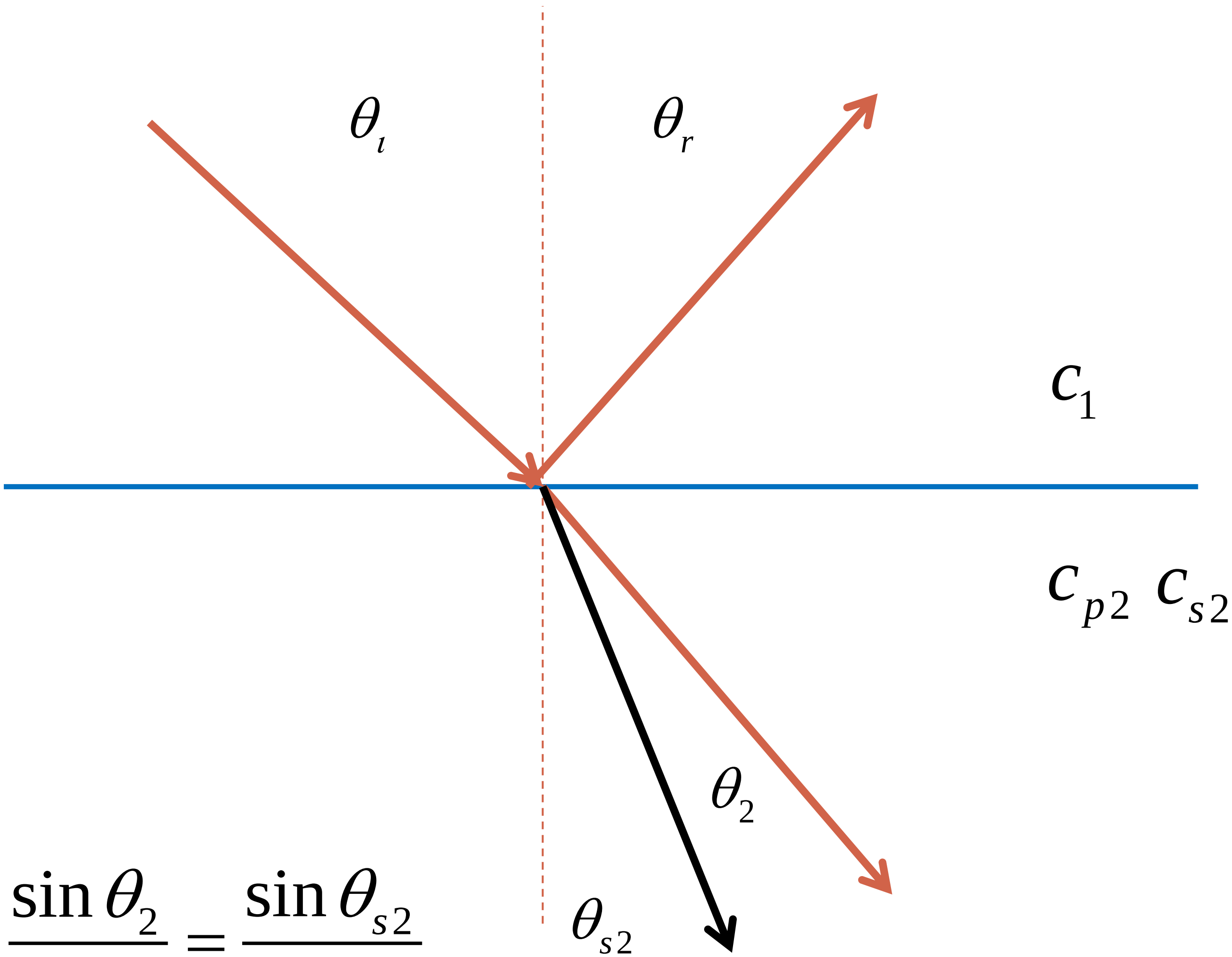




$$\Delta R_2 = \Delta x \sin\theta_2 \qquad \Delta R_1 = \Delta x \sin\theta_1$$

$$\Delta R_1 / \sin\theta_1 = \Delta R_2 / \sin\theta_2$$

$$c_1 = \frac{\Delta R_1}{\Delta t} \qquad c_2 = \frac{\Delta R_2}{\Delta t} \quad \Longrightarrow \quad \frac{\sin\theta_1}{c_1} = \frac{\sin\theta_2}{c_2}$$

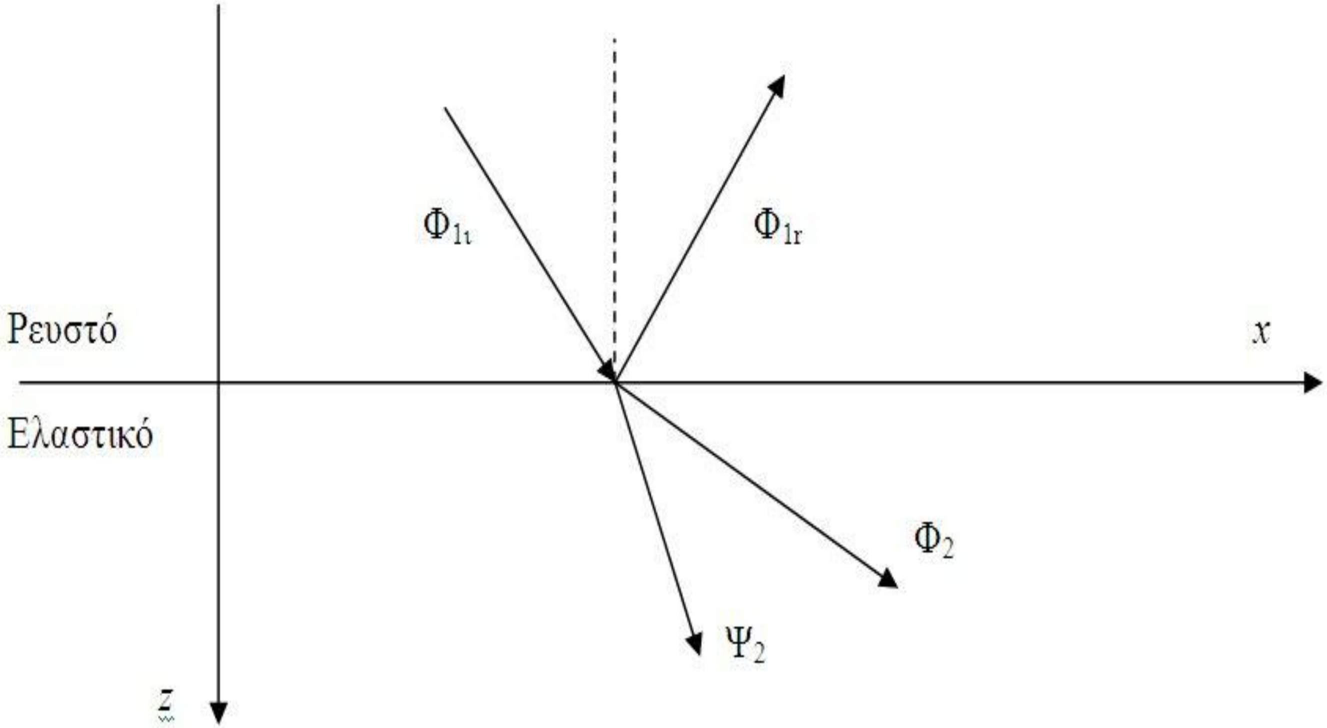


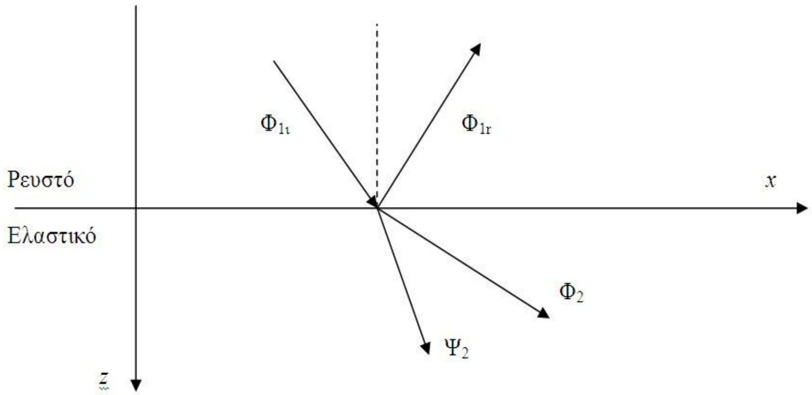
$$\frac{\sin\theta_1}{c_1} = \frac{\sin\theta_2}{c_{p2}} = \frac{\sin\theta_{s2}}{c_{s2}}$$

$$\sigma_{ij} = \lambda \delta_{ij} \varepsilon_{kk} + 2\mu \varepsilon_{ij}$$ Νόμος Hooke

$$\sigma_{zz} = \lambda \nabla^2 \Phi + 2\mu \left(\frac{\partial^2 \Phi}{\partial z^2} - \frac{\partial^2 \Psi}{\partial z \partial x} \right)$$

$$\sigma_{zx} = \mu \left(\frac{\partial^2 \Psi}{\partial z^2} - \frac{\partial^2 \Psi}{\partial x^2} + 2 \frac{\partial^2 \Phi}{\partial z \partial x} \right)$$


Φ1ι
Φ1r
Ρευστό
x
Ελαστικό
Φ2
Ψ2
z

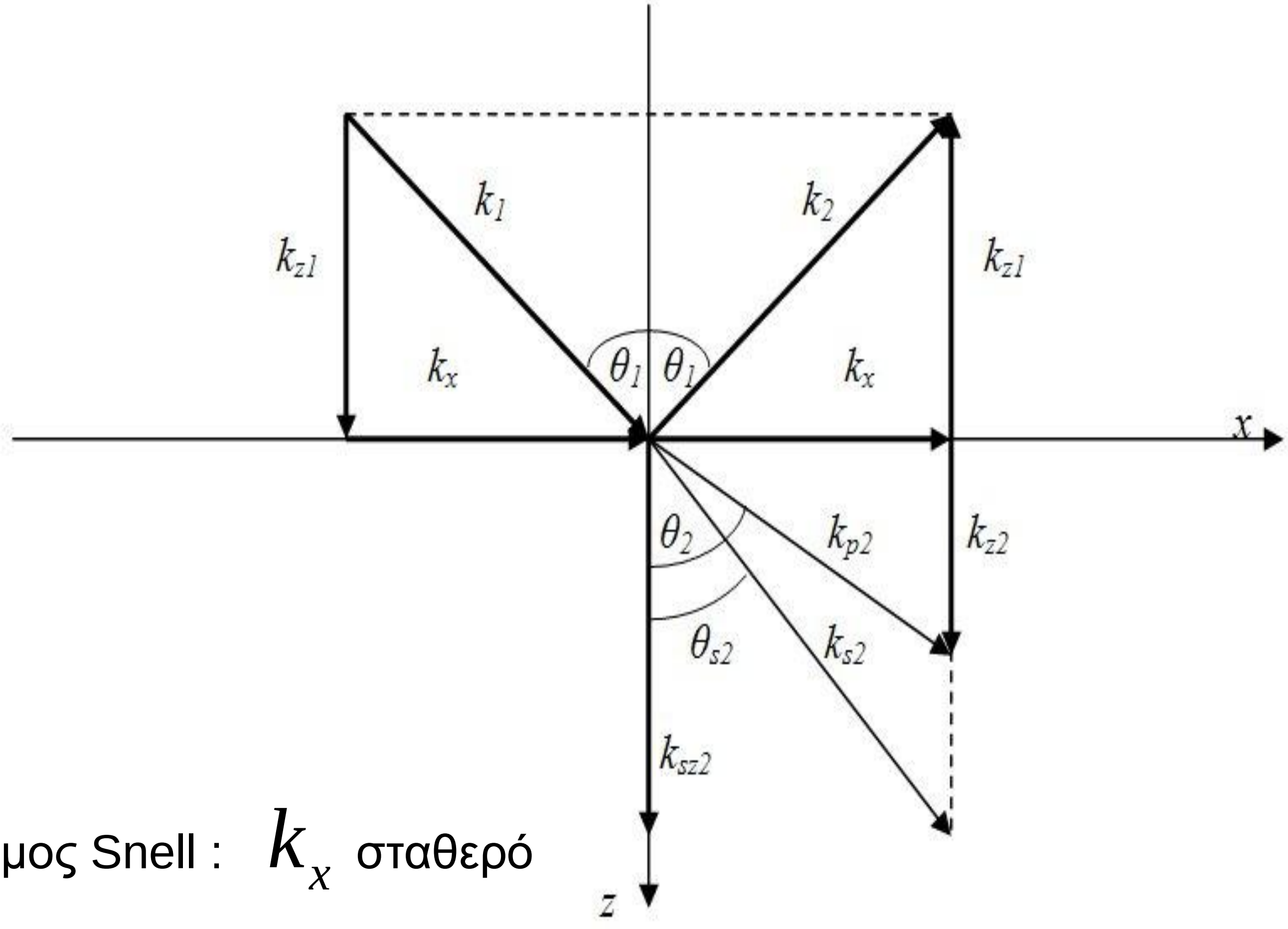


Φ_{1i} Δυναμικό προσπίπτοντος ακουστικού κύματος

Φ_{1r} Δυναμικό ανακλώμενου ακουστικού κύματος

Φ_{2} Δυναμικό διαδιδόμενου ακουστικού κύματος

Ψ_{2} Δυναμικό διαδιδόμενου διατμητικού κύματος



Νόμος Snell : k_x σταθερό

$$\nabla^2 \Phi_* = \frac{1}{{c_*}^2} \frac{\partial^2 \Phi_*}{\partial t^2}, * = 1\imath, 1r, 2$$

$$c_{1i} = c_{1r} = c_1, \quad c_2 = c_{p2}$$

$$\nabla^2 \mathbf{\Psi}_2 = \frac{1}{c_{s2}^2} \frac{\partial^2 \mathbf{\Psi}_2}{\partial t^2}$$

$$\Phi_{1\iota} = e^{i(k_{x1}x + k_{z1}z - \omega t)}$$

$$\Phi_{1r} = R_{12}e^{i(k_{x1}x - k_{z1}z - \omega t)}$$

$$\Phi_{2} = T_{p}e^{i(k_{x2}x + k_{z2}z - \omega t)}$$

$$\Psi_{2} = T_{s}e^{i(k_{x2}x + k_{sz2}z - \omega t)}$$

$$k_1 = \frac{\omega}{c_1}$$

$$k_{p2} = \frac{\omega}{c_{p2}}$$

$$k_{s2} = \frac{\omega}{c_{s2}}$$

$$k_{x1} = k_{x2} = k_x$$

Οριακές συνθήκες στη διεπιφάνεια

$$\sigma_{1,zz} = -p_1 = \sigma_{2,zz}$$ Κάθετη τάση

$$\sigma_{2,zx} = 0$$ Διατμητική τάση

$$d_{1z} = d_{2z}$$ Μετατόπιση

$$\vec{d} = \nabla\Phi + \nabla \times \mathbf{\Psi}$$

$$\sigma_{zz} = \lambda\nabla^2\Phi + 2\mu\left(\frac{\partial^2\Phi}{\partial z^2} - \frac{\partial^2\Psi}{\partial z\partial x}\right)$$

$$\sigma_{zx} = \mu\left(\frac{\partial^2\Psi}{\partial z^2} - \frac{\partial^2\Psi}{\partial x^2} + 2\frac{\partial^2\Phi}{\partial z\partial x}\right)$$

$$p_1 = -\rho_1 \frac{\partial^2 \Phi_1}{\partial t^2}$$

$$-\rho_1 \frac{\partial^2 \Phi_1}{\partial t^2} + \frac{\lambda}{c_{p2}^2} \frac{\partial^2 \Phi_2}{\partial t^2} + 2\mu\left(\frac{\partial^2 \Phi_2}{\partial z^2} - \frac{\partial^2 \Psi_2}{\partial z \partial x}\right) = 0$$

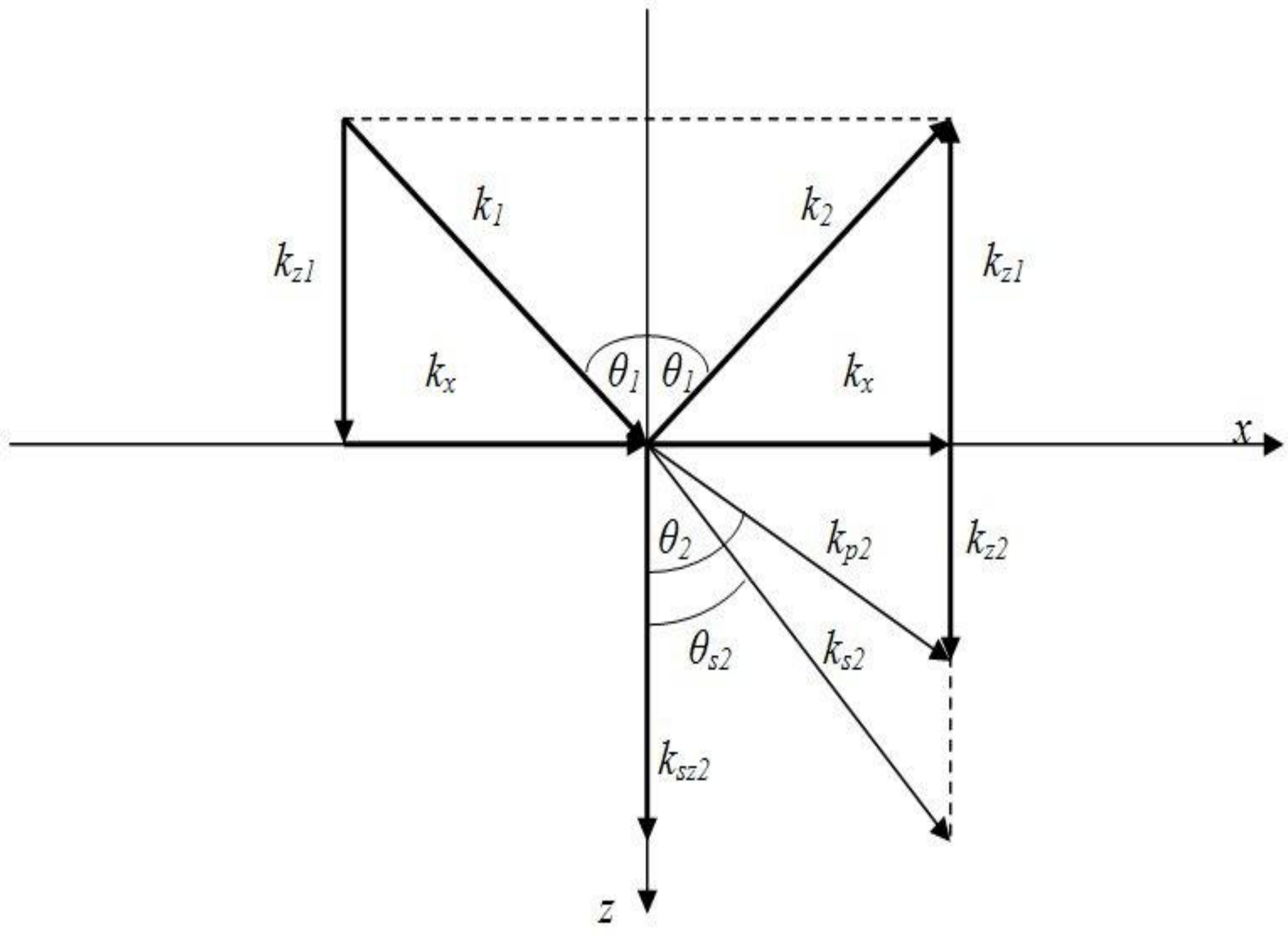
$$\mu\left(\frac{\partial^2 \Psi_2}{\partial z^2} - \frac{\partial^2 \Psi_2}{\partial x^2} + 2\frac{\partial^2 \Phi_2}{\partial z \partial x}\right) = 0$$

$$\frac{\partial \Phi_1}{\partial z} = \frac{\partial \Phi_2}{\partial z} - \frac{\partial \Psi_2}{\partial x}$$

$$\Phi_1 = \Phi_{1i} + \Phi_{1r}$$

$$R_{12} = \frac{4k_{z2}k_{sz2}k_x^2 + (k_{sz2}^2 - k_x^2)^2 - (\rho_1/\rho_2)(k_{z2}/k_{z1})(\omega^4/c_{s2}^4)}{4k_{z2}k_{sz2}k_x^2 + (k_{sz2}^2 - k_x^2)^2 + (\rho_1/\rho_2)(k_{z2}/k_{z1})(\omega^4/c_{s2}^4)}$$

$$u = e^{iax} \qquad a = ia_1,\ a_1 \in R^+ \qquad u = e^{-a_1 x}$$



$$k_{p2} > k_{z2},\ k_{p2} > k_x,\ \ k_{s2} > k_{zs2},\ k_{s2} > k_x$$

$$c_{p2} > c_1$$

$$\sin\theta_2 = \frac{c_{p2}}{c_1}\sin\theta_1$$

$$k_{z2} = ig_2\,, \quad k_{sz2} = id_2$$

$$R_{12} = -e^{i2n}$$

$$n = Arc\tan\{\frac{\rho_2}{\rho_1}\frac{\kappa_{z1}}{g_2}\frac{{c_{s2}}^4}{\omega^4}[-4g_2d_2{k_x}^2 + ({d_2}^2 + {k_x}^2)^2]\}$$

Περίπτωση Ρευστού Πυθμένα

$$\Phi_{1\imath} = e^{i(k_x x + k_{z1} z - \omega t)}$$

$$\Phi_{1r} = R_{12} e^{i(k_x x - k_{z1} z - \omega t)}$$

$$\Phi_2 = T_p e^{i(k_x x + k_{z2} z - \omega t)}$$

$$p_1 = p_2$$

$$d_{1z} = d_{2z}$$

$$\frac{\lambda_1}{{c_1}^2} \frac{\partial^2 \Phi_1}{\partial t^2} = \frac{\lambda_2}{{c_2}^2} \frac{\partial^2 \Phi_2}{\partial t^2}$$

$$\frac{\partial \Phi_1}{\partial z} = \frac{\partial \Phi_2}{\partial z}$$

$$\Phi_1 = \Phi_{1i} + \Phi_{1r}$$

$$-\rho_1\omega^2\{e^{i(k_x x + k_{z1}z - \omega t)} + R_{12}e^{i(k_x x - k_{z1}z - \omega t)}\} = -\rho_2\omega^2 T_p e^{i(k_x x + k_{z2}z - \omega t)}$$

$$ik_{z1}\{e^{i(k_x x + k_{z1}z - \omega t)} - R_{12}e^{i(k_x x - k_{z1}z - \omega t)}\} = ik_{z2}T_p e^{i(k_x x + k_{z2}z - \omega t)}$$

$$\rho_1(1 + R_{12}) = \rho_2 \mathrm{T}_p$$

$$k_{z1}(1 - R_{12}) = k_{z2}T_p$$

$$R_{12} = \frac{k_{z1}\rho_2 - k_{z2}\rho_1}{k_{z1}\rho_2 + k_{z2}\rho_1}$$

$$T_p = \frac{2k_{z1}\rho_1}{k_{z1}\rho_2 + k_{z2}\rho_1}$$

$$k_{z1} = k_1 \cos\theta_1 = \frac{\omega}{c_1}\cos\theta_1$$

$$k_{z2} = k_2 \cos\theta_2 = \frac{\omega}{c_2}\cos\theta_2$$

$$R_{12} = \frac{\rho_2 c_2 \cos\theta_1 - \rho_1 c_1 \cos\theta_2}{\rho_2 c_2 \cos\theta_1 + \rho_1 c_1 \cos\theta_2}$$

$$\sin\theta_2 = \frac{c_2}{c_1}\sin\theta_1$$

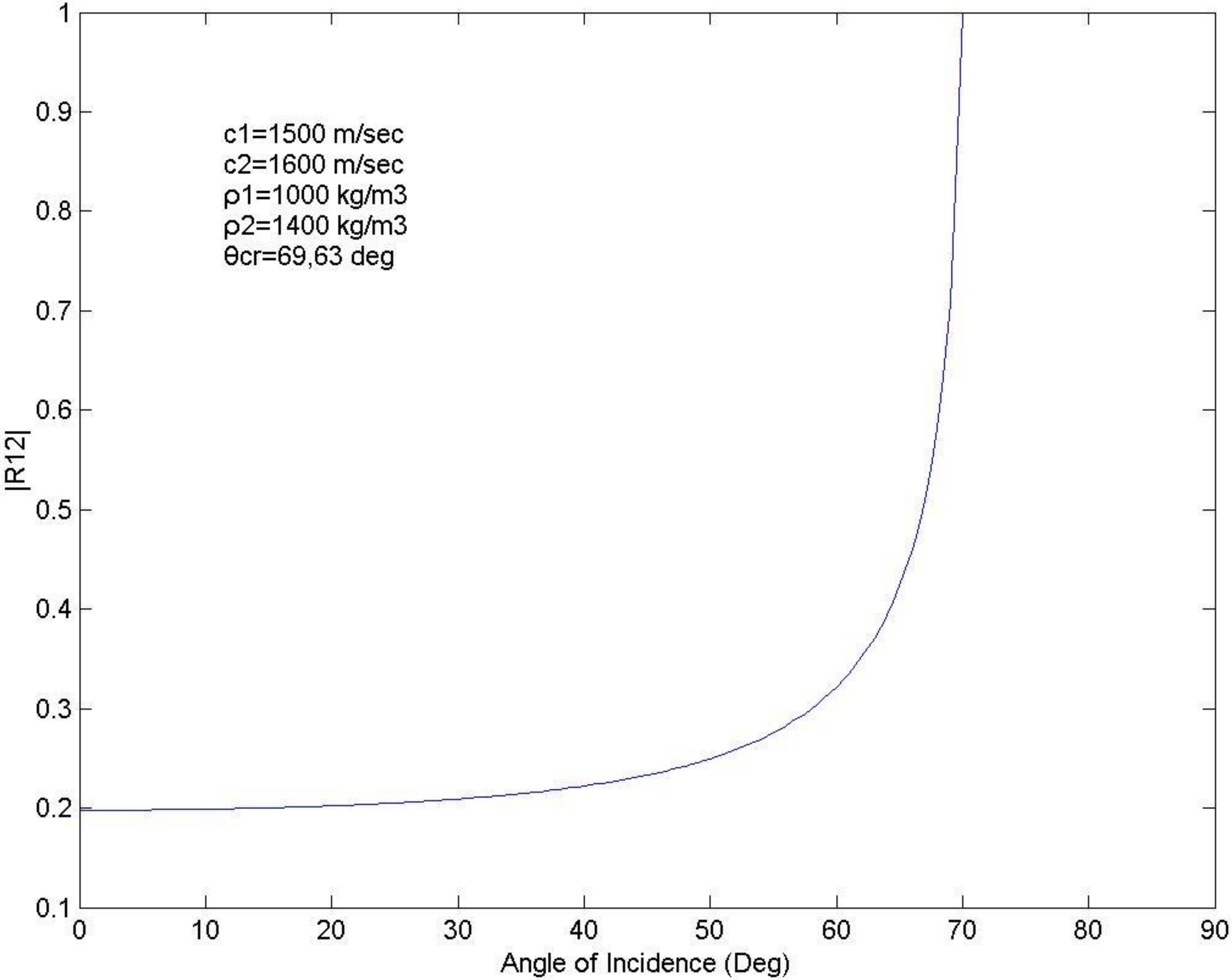
$$\cos\theta_2 = \sqrt{1-(\frac{c_2}{c_1})^2\sin^2\theta_1}$$

$$\theta_{cr} = Arc\sin\frac{c_1}{c_2}$$

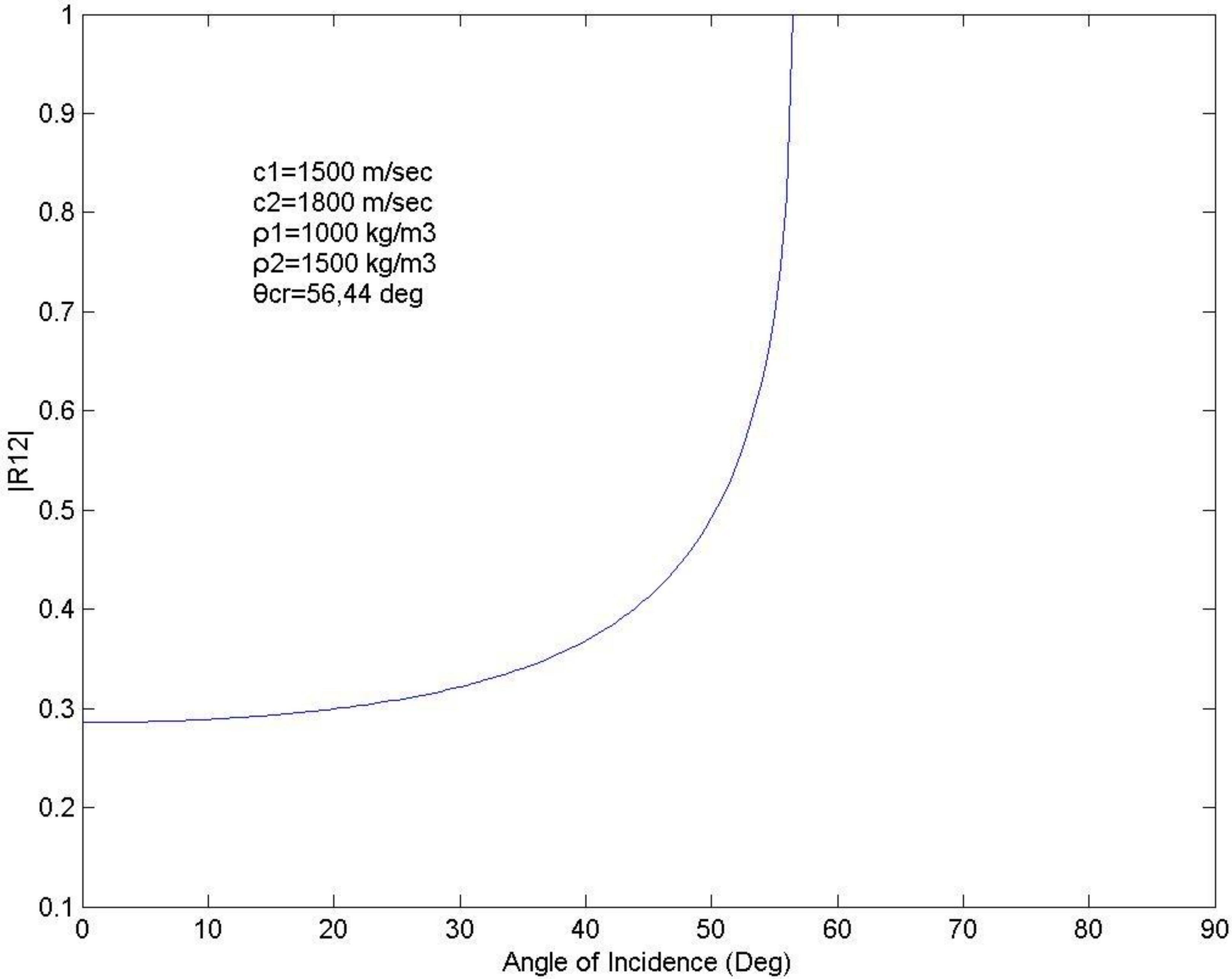
$$R_{12} = \frac{k_{z1}\rho_2 - ig_2\rho_1}{k_{z1}\rho_2 + ig_2\rho_1}$$

$$R_{12} = -e^{i2\chi}$$

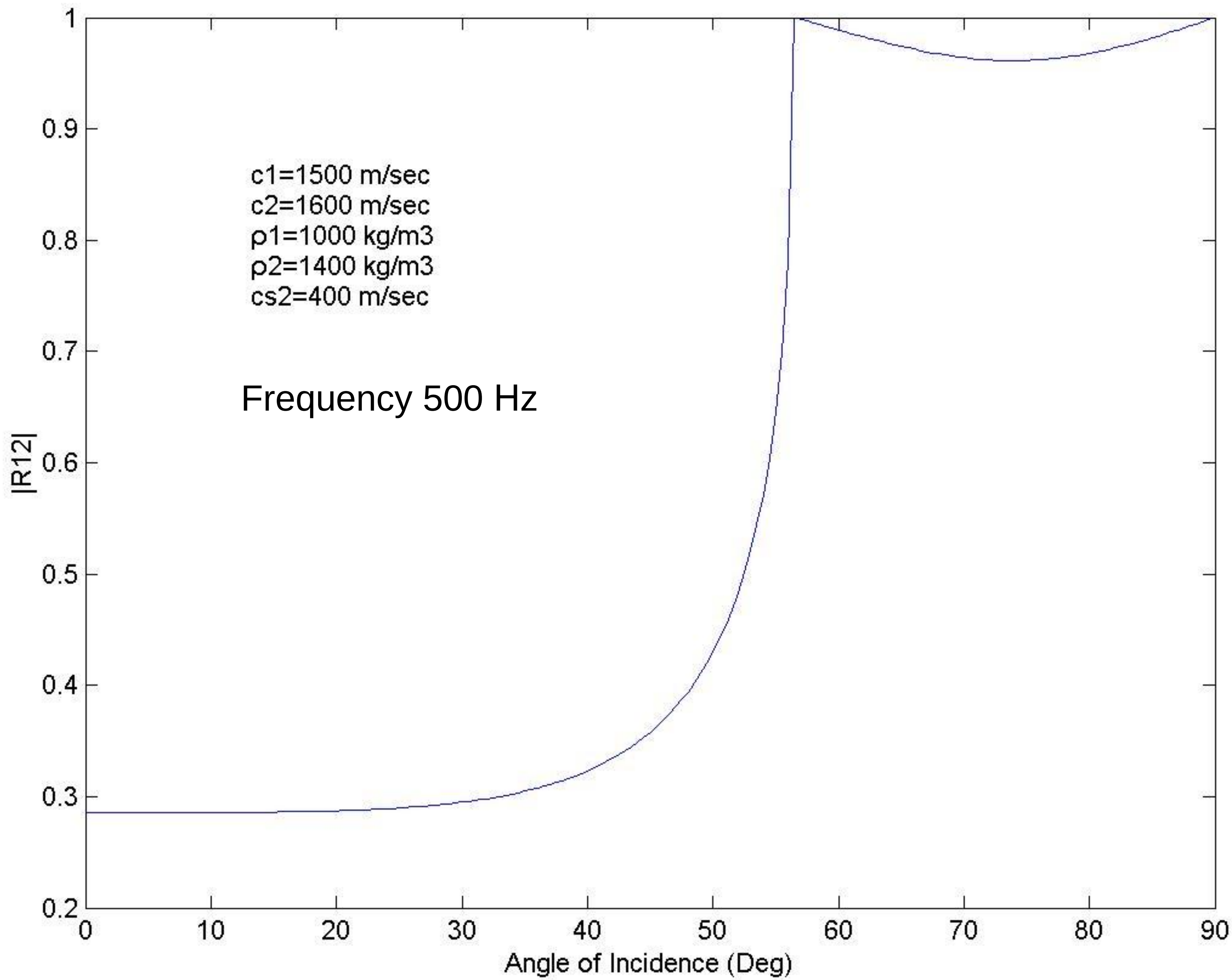
$$\chi = Arc\tan(\frac{\rho_2}{\rho_1}\frac{k_{z1}}{g_2})$$

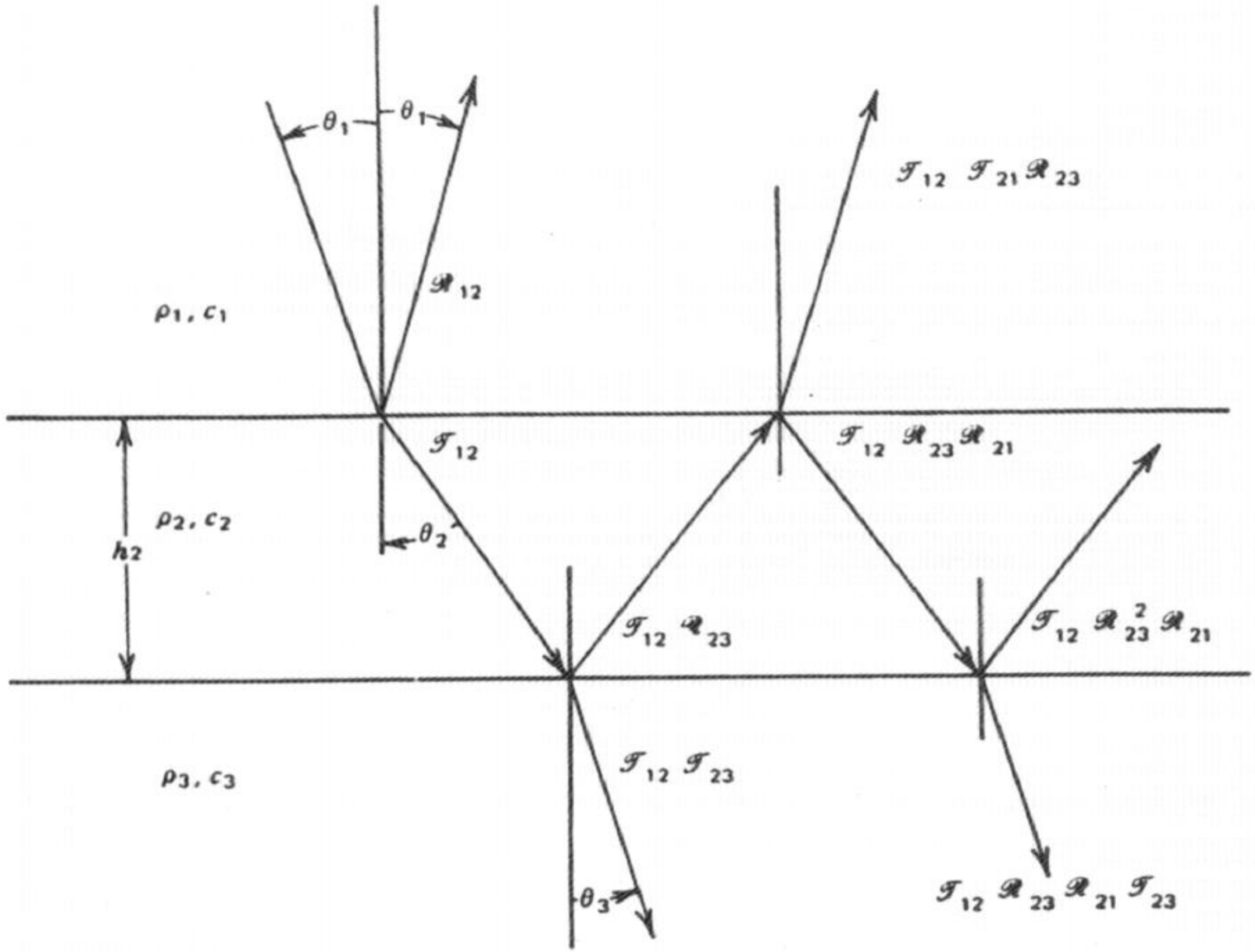


c1=1500 m/sec
c2=1600 m/sec
ρ1=1000 kg/m3
ρ2=1400 kg/m3
θcr=69,63 deg
|R12|
Angle of Incidence (Deg)
0
10
20
30
40
50
60
70
80
90
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1



c1=1500 m/sec
c2=1800 m/sec
ρ1=1000 kg/m3
ρ2=1500 kg/m3
θcr=56,44 deg
|R12|
Angle of Incidence (Deg)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
10
20
30
40
50
60
70
80
90


c1=1500 m/sec
c2=1600 m/sec
ρ1=1000 kg/m3
ρ2=1400 kg/m3
cs2=400 m/sec
Frequency 500 Hz
|R12|
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0
10
20
30
40
50
60
70
80
90
Angle of Incidence (Deg)

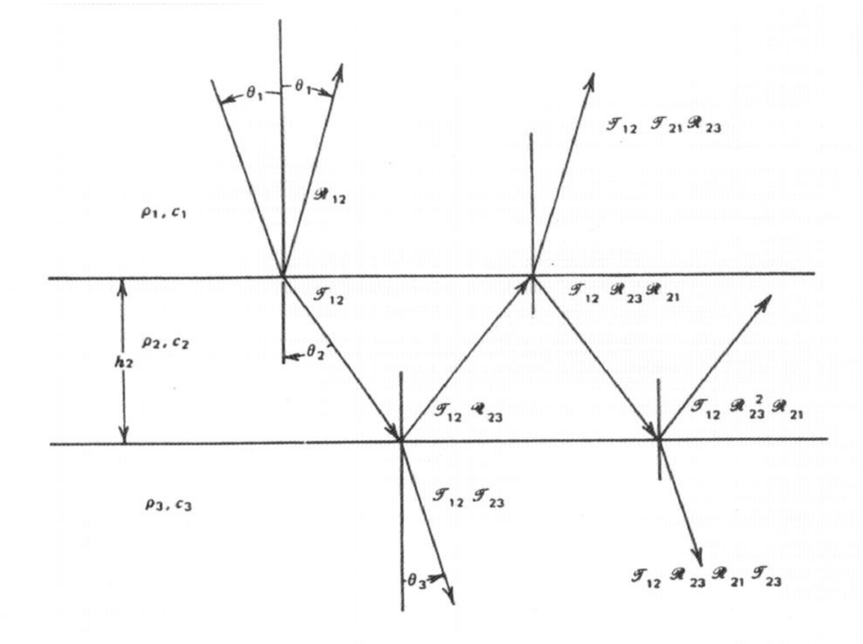

θ1
θ1
ℛ12
ρ1, c1
𝒯12 𝒯21 ℛ23
𝒯12
𝒯12 ℛ23 ℛ21
ρ2, c2
h2
θ2
𝒯12 ℛ23
𝒯12 ℛ23² ℛ21
ρ3, c3
𝒯12 𝒯23
θ3
𝒯12 ℛ23 ℛ21 𝒯23

$$R_{12} = \frac{\rho_2 c_2 \cos\theta_1 - \rho_1 c_1 \cos\theta_2}{\rho_2 c_2 \cos\theta_1 + \rho_1 c_1 \cos\theta_2}$$

$$R_{23} = \frac{\rho_3 c_3 \cos\theta_2 - \rho_2 c_2 \cos\theta_3}{\rho_3 c_3 \cos\theta_2 + \rho_2 c_2 \cos\theta_3}$$

$$T_{12} = \frac{2\rho_1 c_2 \cos\theta_1}{\rho_2 c_2 \cos\theta_1 + \rho_1 c_1 \cos\theta_2}$$

$$T_{23} = \frac{2\rho_2 c_3 \cos\theta_2}{\rho_3 c_3 \cos\theta_2 + \rho_2 c_2 \cos\theta_3}$$



$$2k_2h_2\cos\theta_2 = 2k_{z2}h_2$$

$$R_{12} = -R_{21}$$

$$T_{12}T_{21} = 1 - R_{12}{}^2$$

$$R_{13} = R_{12} + T_{12}T_{21}R_{23}\exp(2i\phi_2) + T_{12}T_{21}R_{23}{}^2R_{21}\exp(4i\phi_2) + \ldots$$

$$\phi_2 = k_2h_2\cos\theta_2$$

$$S=\sum_{n=0}^{\infty} r^{n}=(1-r)^{-1} \qquad |r|<1$$

$$R_{13}=R_{12}+T_{12}T_{21}R_{23}\exp(2i\phi_2)\sum_{n=0}^{\infty}[R_{23}R_{21}\exp(2i\phi_2)]^{n}$$

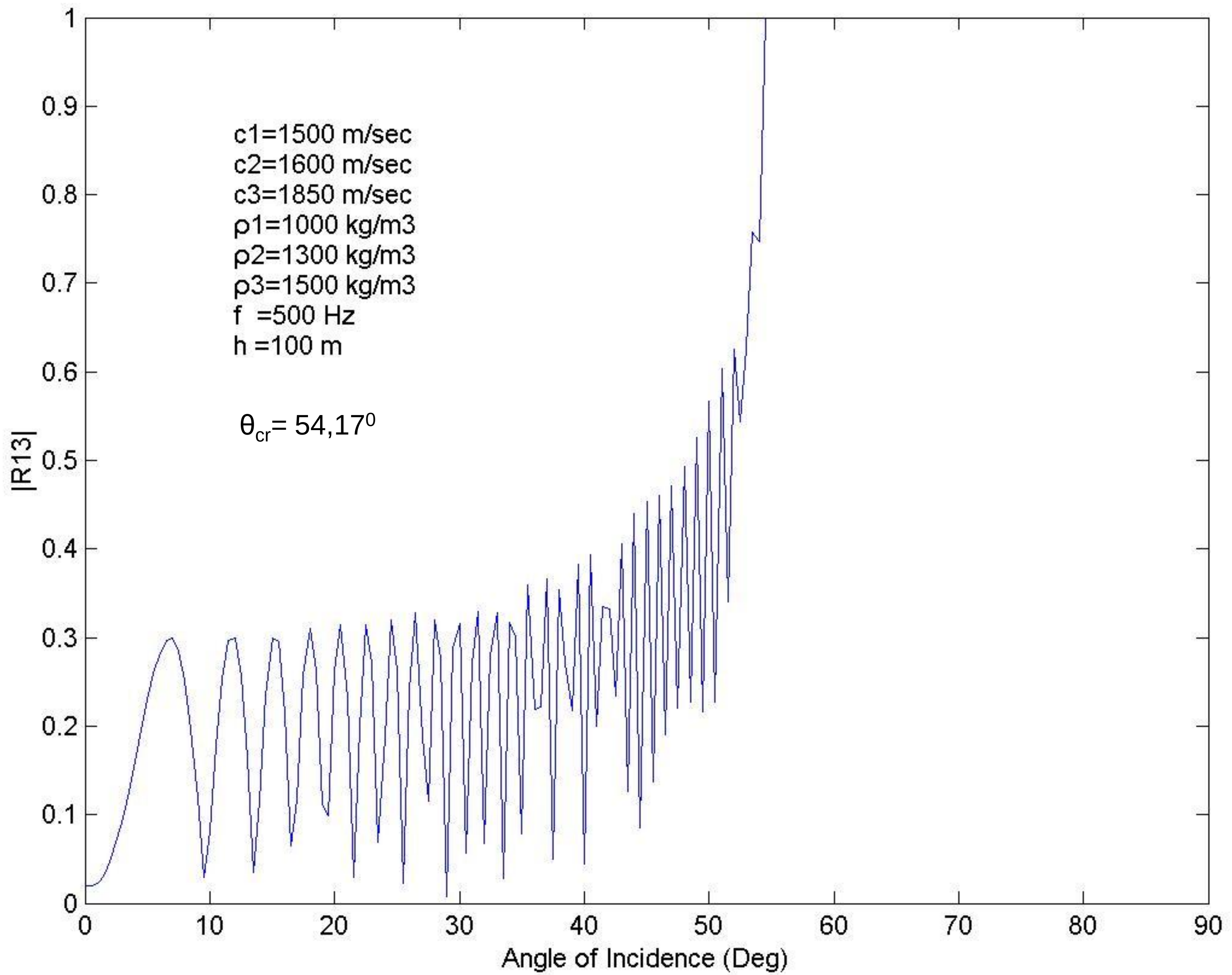
$$R_{13}=\frac{R_{12}+R_{23}\exp(2i\phi_2)}{1+R_{12}R_{23}\exp(2i\phi_2)}$$

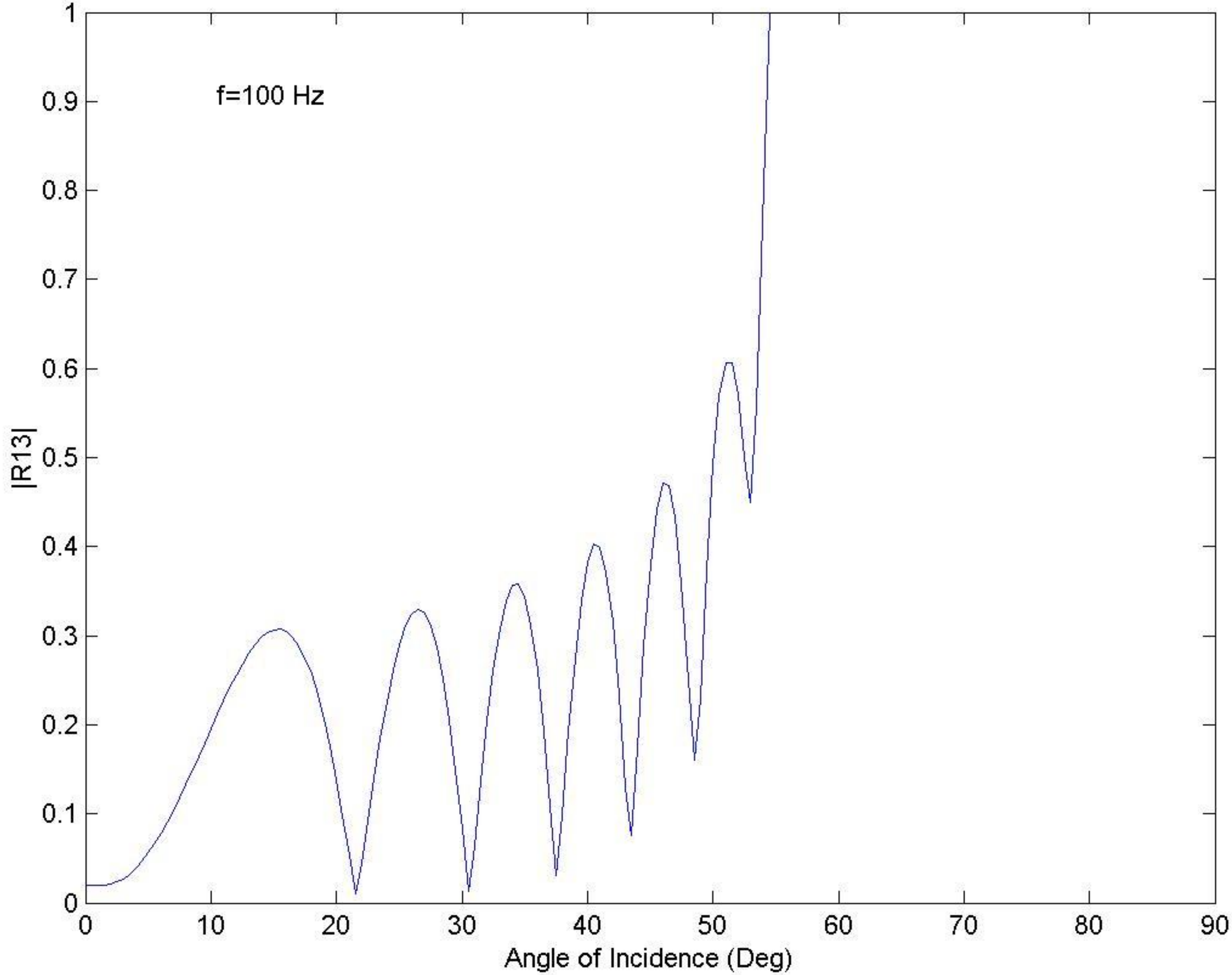
$$T_{13}=\frac{T_{12}T_{23}\exp(i\phi_2)}{1+R_{12}R_{23}\exp(2i\phi_2)}$$

$$R_{(n-2)n} = \frac{R_{(n-2)(n-1)} + R_{(n-1)n} \exp(2i\phi_{n-1})}{1 + R_{(n-2)(n-1)} R_{(n-1)n} \exp(2i\phi_{n-1})}$$

$$R_{(n-3)n} = \frac{R_{(n-3)(n-2)} + R_{(n-2)n} \exp(2i\phi_{n-2})}{1 + R_{(n-3)(n-2)} R_{(n-2)n} \exp(2i\phi_{n-2})}$$

$$R_{1n} = \frac{R_{12} + R_{2n} \exp(2i\phi_{2})}{1 + R_{12} R_{2n} \exp(2i\phi_{2})}$$


c1=1500 m/sec
c2=1600 m/sec
c3=1850 m/sec
ρ1=1000 kg/m3
ρ2=1300 kg/m3
ρ3=1500 kg/m3
f =500 Hz
h =100 m
θcr= 54,17⁰
|R13|
Angle of Incidence (Deg)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
10
20
30
40
50
60
70
80
90


f=100 Hz
|R13|
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
10
20
30
40
50
60
70
80
90
Angle of Incidence (Deg)