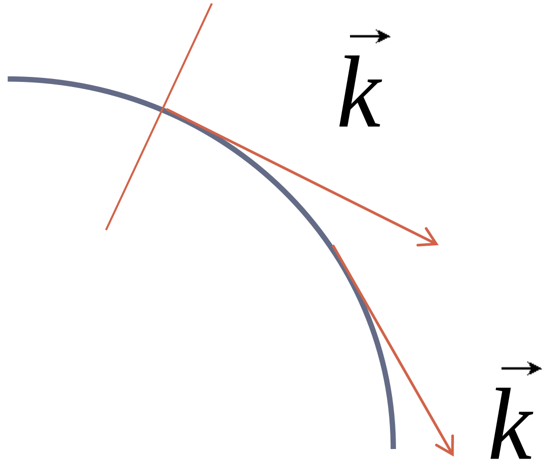
Γεωμετρική Ακουστική

Ακουστικές Ακτίνες

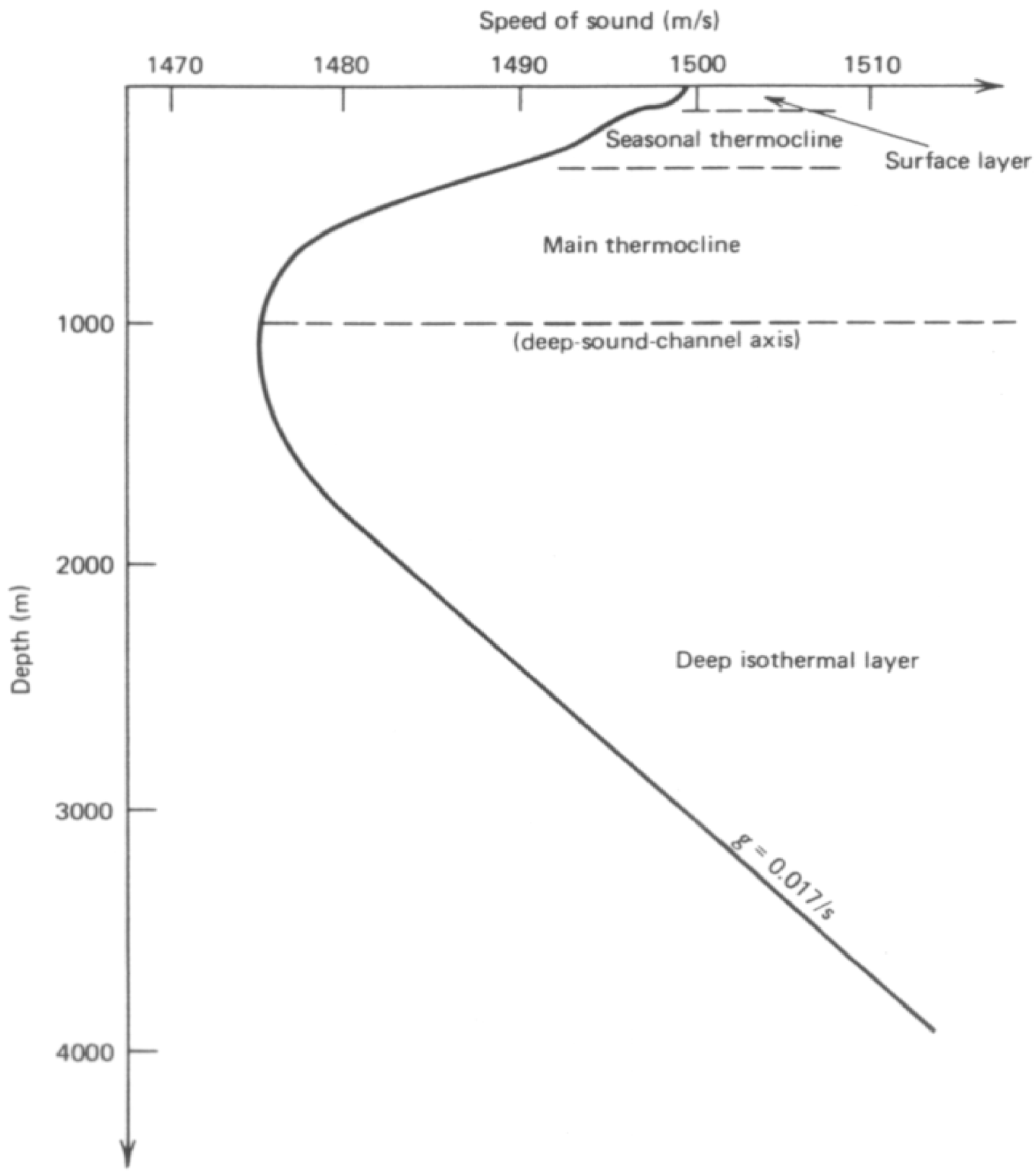
# Εισαγωγή στην Ακουστική Ωκεανογραφία

Ηχητική ακτίνα : Καμπύλη σε κάθε σημείο της οποίας ο αριθμός κύματος είναι εφαπτόμενο διάνυσμα.




Υπόθεση: Επίπεδα κύματα.

Υπόθεση: Μεταβολή της ταχύτητας διάδοσης του ήχου μόνο με το βάθος

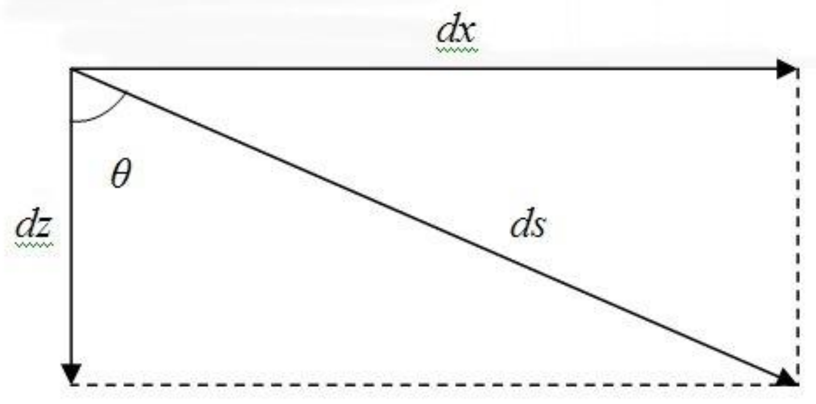

Speed of sound (m/s)
1470
1480
1490
1500
1510
Seasonal thermocline
Surface layer
Main thermocline
1000
(deep-sound-channel axis)
2000
Depth (m)
Deep isothermal layer
3000
g = 0.017/s
4000

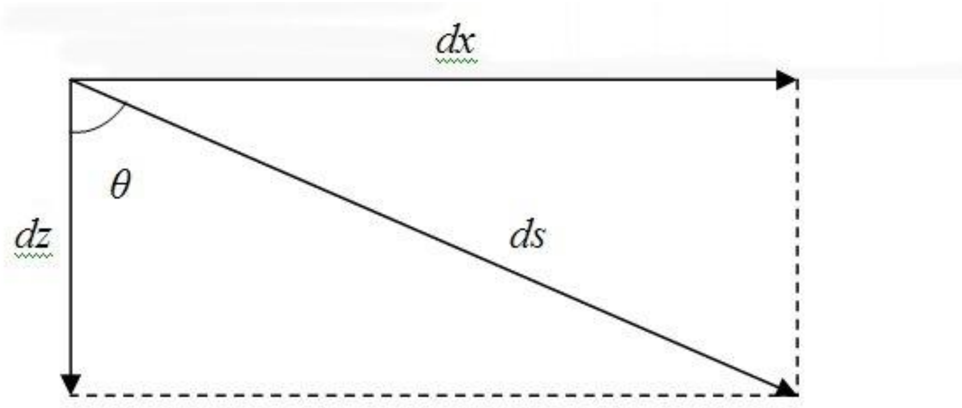
$$\frac{\sin\theta(z)}{c(z)} = a$$

**Νόμος Snell**

$$ds = \frac{dz}{\cos\theta} \qquad dx = \tan\theta\, dz$$

$$dt = \frac{ds}{c(z)} = \frac{dz}{c(z)\cos\theta}$$

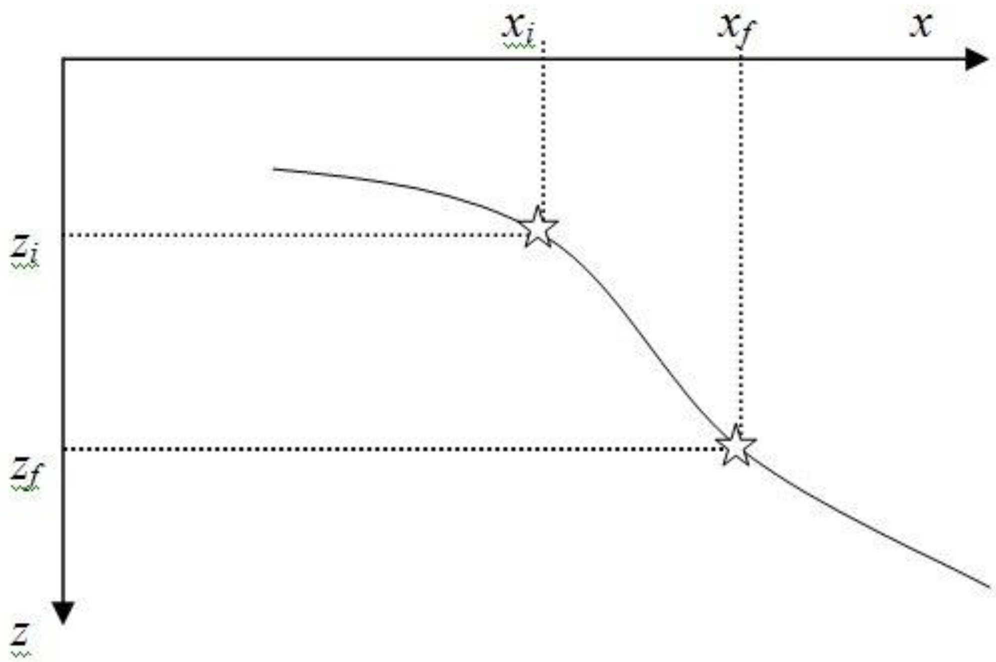




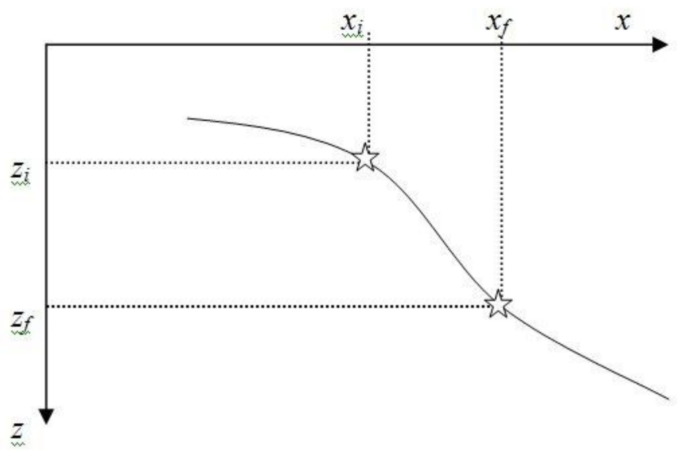

$$\sin\theta = ac(z)$$

$$\cos\theta = [1 - a^2c^2(z)]^{1/2}$$

$$\tan\theta = ac(z)/[1 - a^2c^2(z)]^{1/2}$$



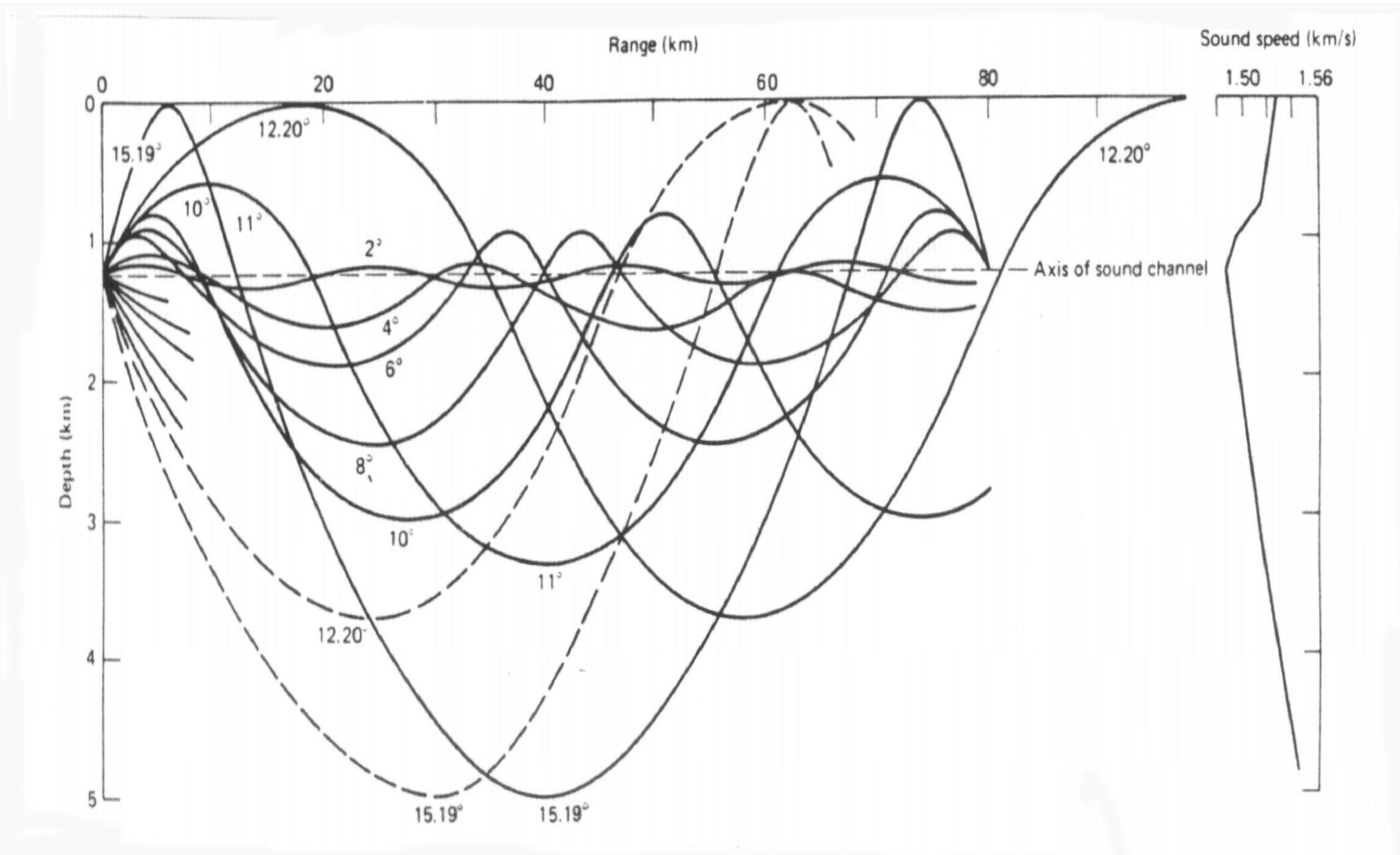
$x_i$
$x_f$
$x$
$z_i$
$z_f$
$z$




$$x_f - x_i = \int_{x_i}^{x_f} dx = \int_{z_i}^{z_f} \frac{ac(z)dz}{[1-a^2c^2(z)]^{1/2}}$$

$$t_f - t_i = \int_{t_i}^{t_f} dt = \int_{z_i}^{z_f} \frac{dz}{c(z)[1-a^2c^2(z)]^{1/2}}$$

Οριζοντιοποίηση ακτίνας $\sin\theta_i = c(z_i)/c(z)$

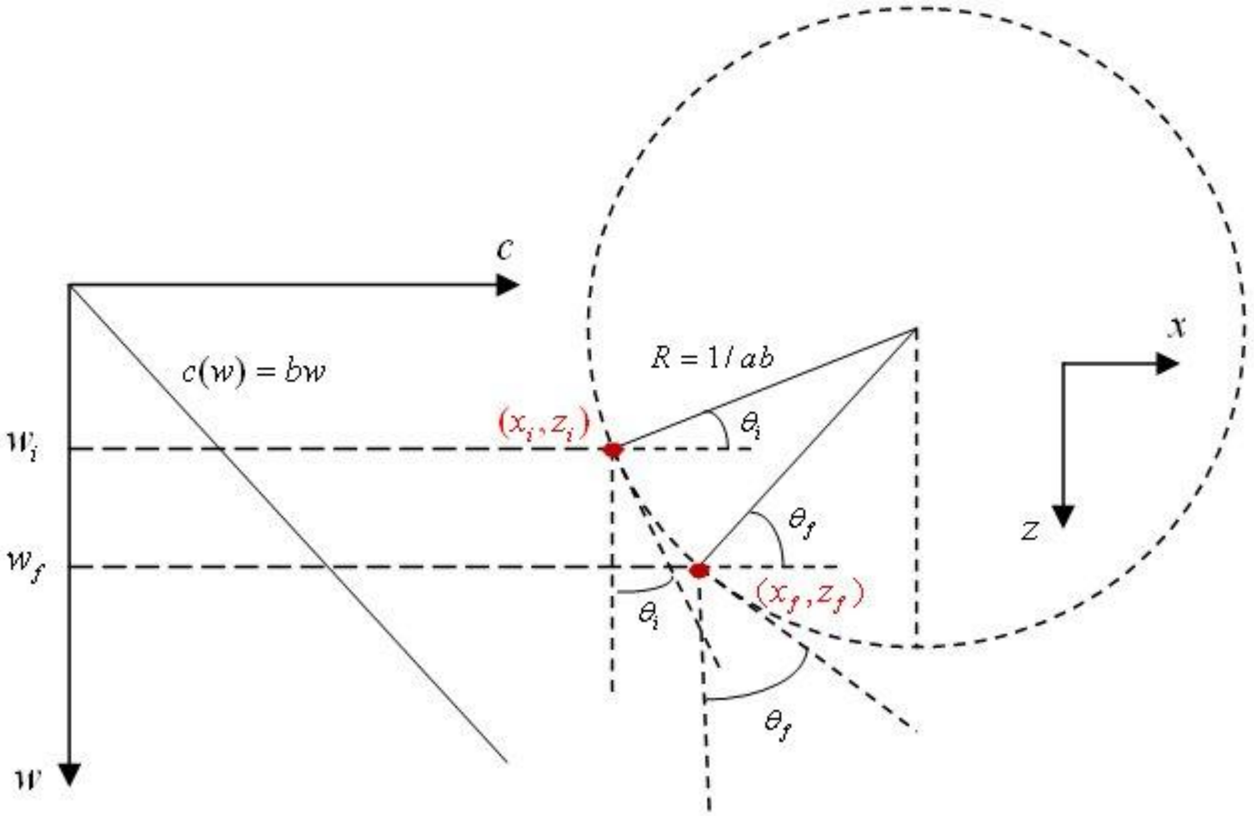

Range (km)
Sound speed (km/s)
0
20
40
60
80
1.50
1.56
Depth (km)
1
2
3
4
5
15.19°
12.20°
10°
11°
2°
4°
6°
8°
Axis of sound channel

$$c(z) = c(z_1) + b(z - z_1) \qquad z_1 \leq z \leq z_2$$

$$w = z - z_1 + \frac{c(z_1)}{b}$$

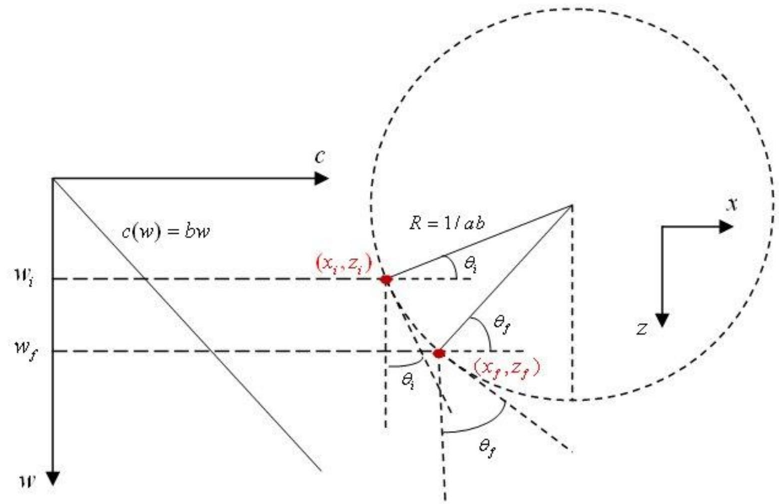
$$dw = dz$$

$$c(z) = bw$$



c
c(w) = bw
R = 1/ab
(x_i, z_i)
θ_i
θ_f
w_i
w_f
(x_f, z_f)
θ_i
θ_f
w
x
z

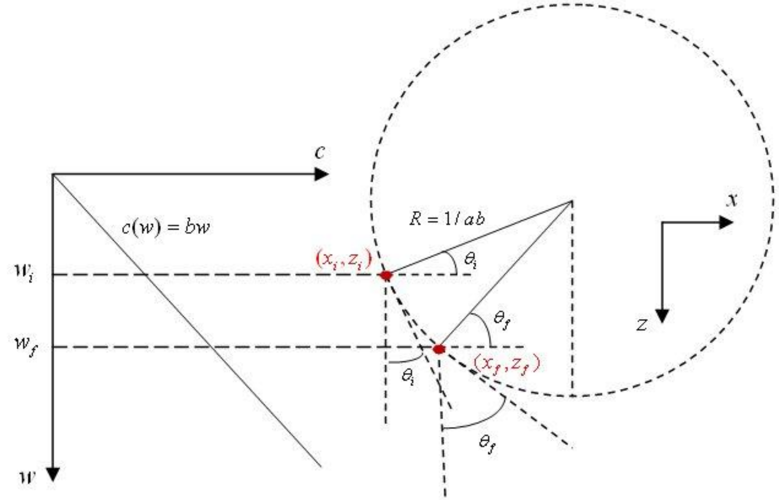
$$x_f - x_i = \int_{w_i}^{w_f} \frac{abw\,dw}{(1 - a^2 b^2 w^2 )}$$




$$t_f - t_i = \int_{w_i}^{w_f} \frac{dw}{bw(1 - a^2 b^2 w^2 )^{1/2}}$$

$$t_f - t_i = \frac{1}{b} \log_e \frac{w_f [1 + (1 - a^2 b^2 w_i^2 )^{1/2} ]}{w_i [1 + (1 - a^2 b^2 w_f^2 )^{1/2} ]}$$

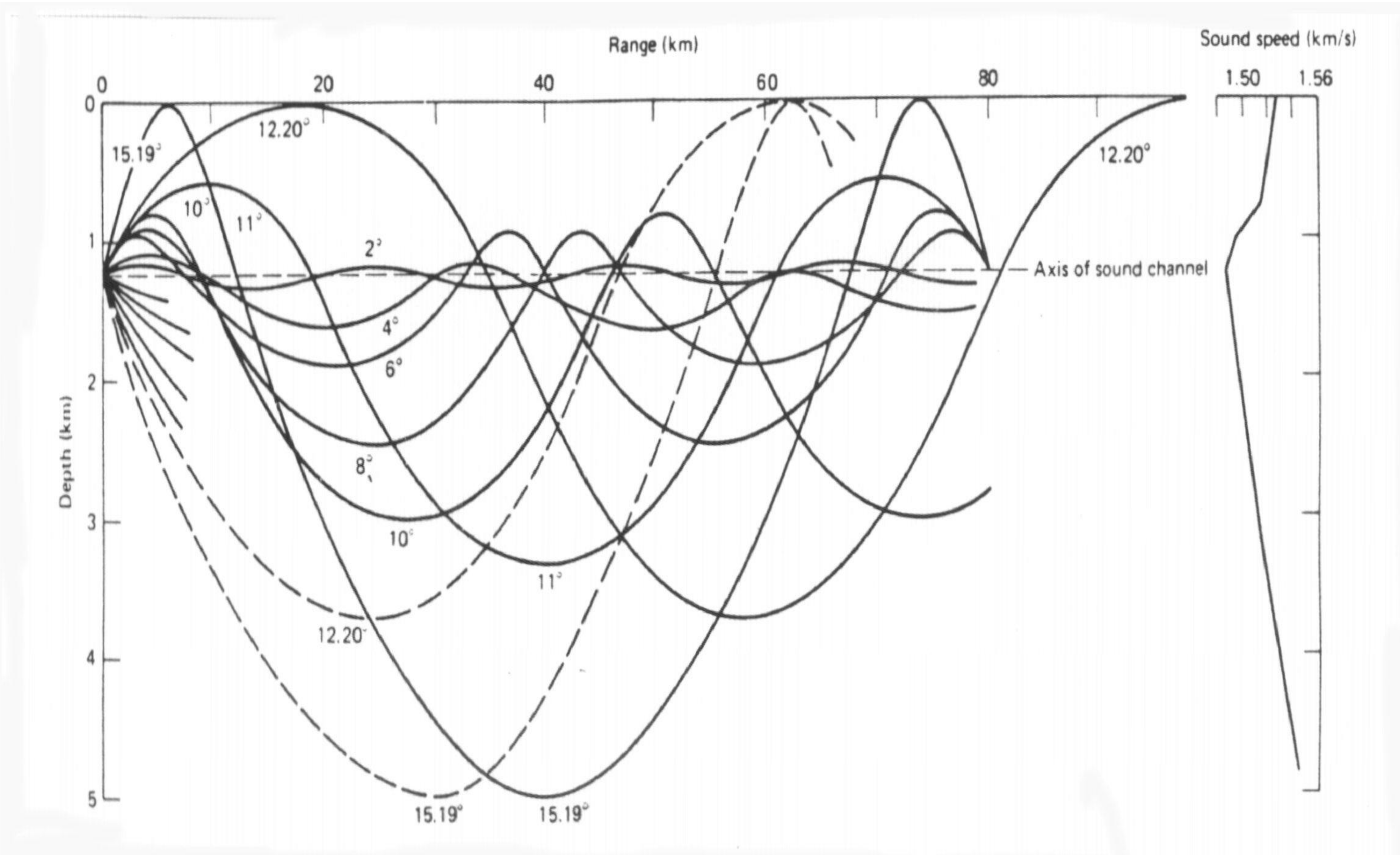
$$t_f - t_i = \frac{1}{b}\log_e \frac{w_f(1+\cos\theta_i)}{w_i(1+\cos\theta_f)}$$



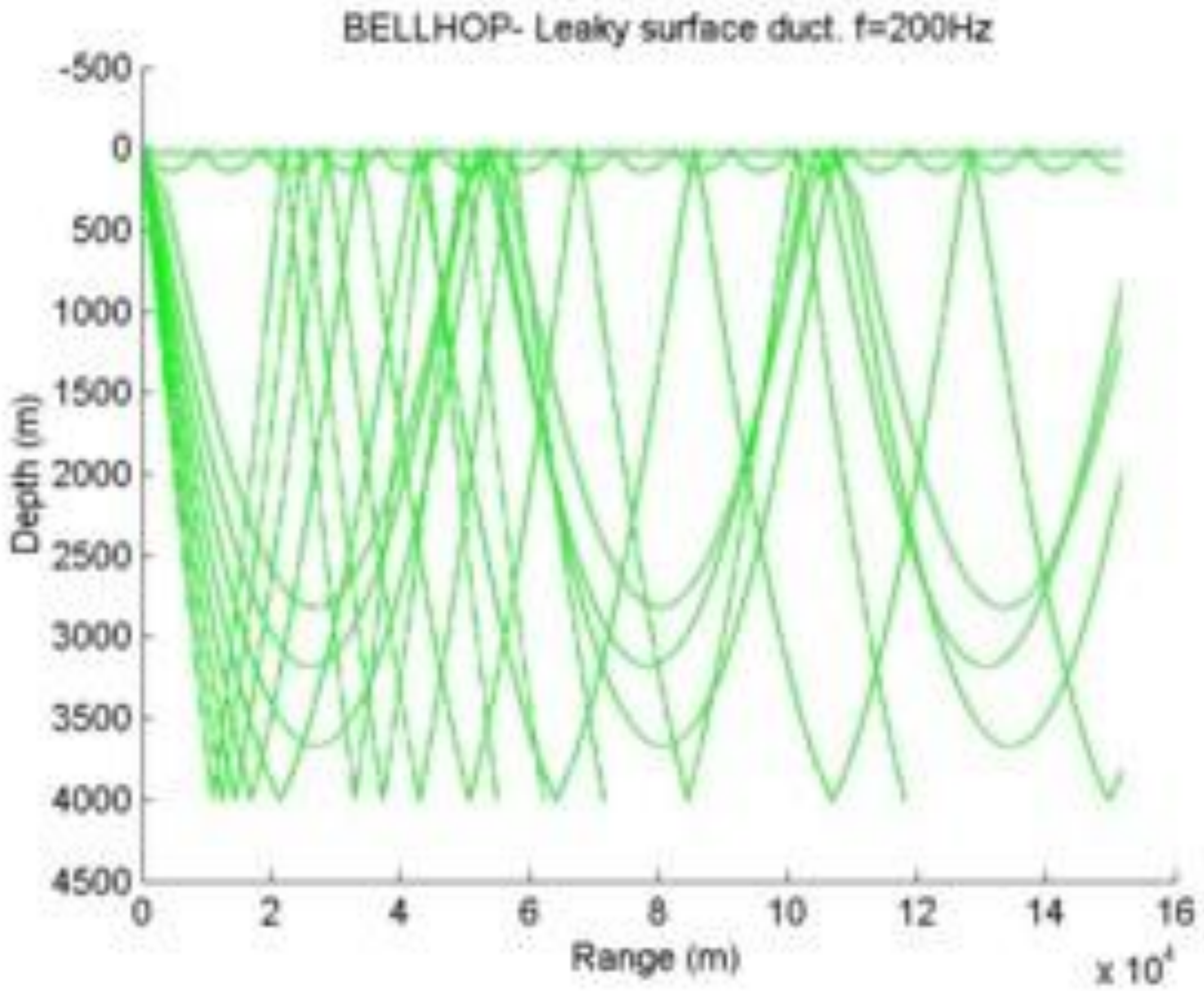

$$x_f - x_i = \frac{1}{ab}[(1-a^2b^2w_i^2)^{1/2} - (1-a^2b^2w_f^2)^{1/2}]$$

$$x_f - x_i = \frac{1}{ab}(\cos\theta_i - \cos\theta_f) = R(\cos\theta_i - \cos\theta_f)$$

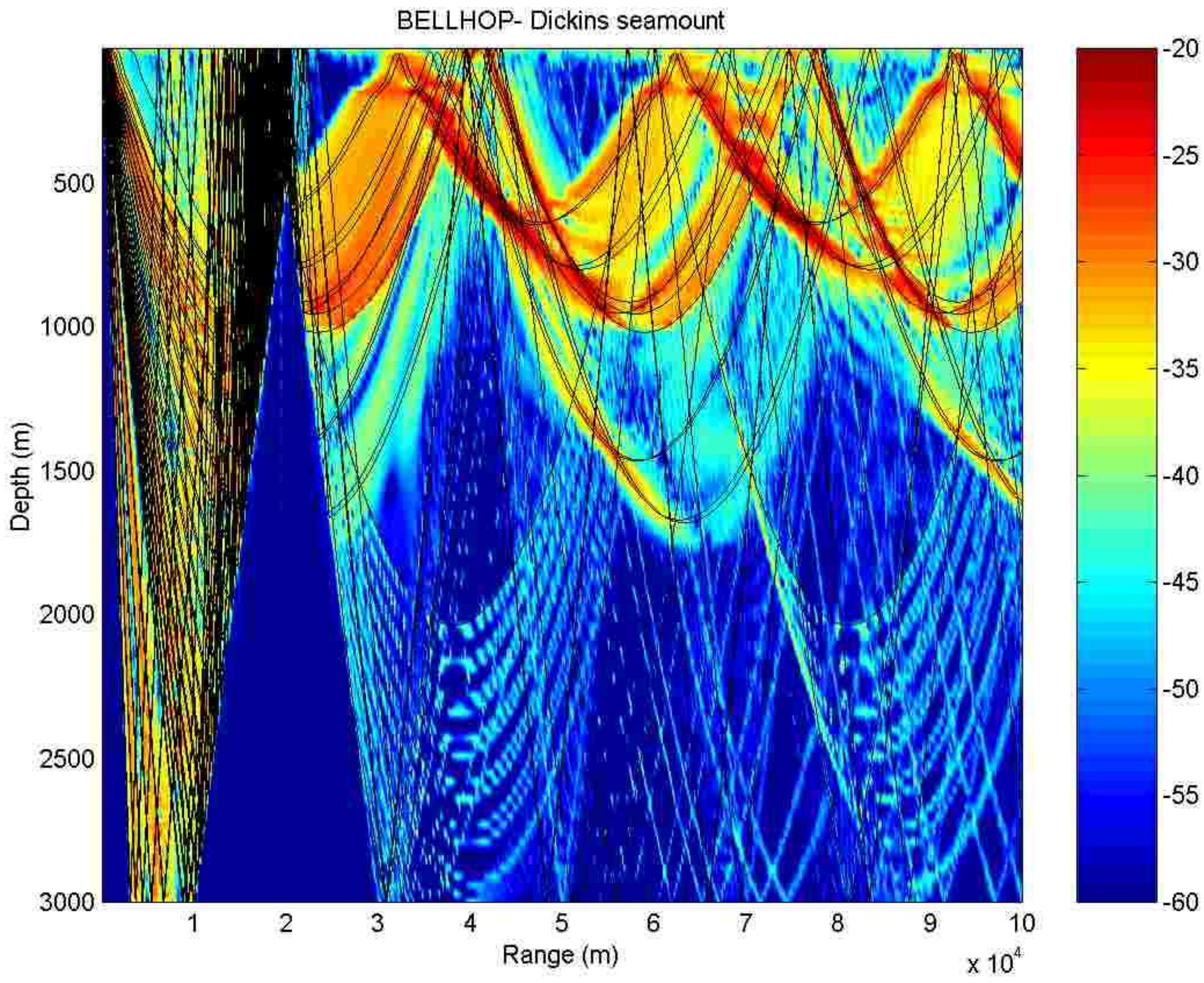
$$z_f - z_i = R(\sin\theta_f - \sin\theta_i)$$



Range (km)
Sound speed (km/s)
0
20
40
60
80
1.50
1.56
0
1
2
3
4
5
Depth (km)
15.19°
12.20°
10°
11°
2°
4°
6°
8°
10°
11°
12.20°
15.19°
15.19°
12.20°
Axis of sound channel



BELLHOP- Leaky surface duct. f=200Hz
Depth (m)
-500
0
500
1000
1500
2000
2500
3000
3500
4000
4500
0
2
4
6
8
10
12
14
16
Range (m)
x 10^4


BELLHOP- Dickins seamount
Depth (m)
500
1000
1500
2000
2500
3000
1
2
3
4
5
6
7
8
9
10
Range (m)
x 10^4
-20
-25
-30
-35
-40
-45
-50
-55
-60

# Υπολογισμός της ακουστικής πίεσης

$$\frac{1}{r^2}\frac{\partial}{\partial r}[r^2\frac{\partial}{\partial r}p(r,t)] = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}p(r,t)$$

$$p(r,t) = \frac{b_c}{r}\exp[i(kr-\omega t)]$$

$$\frac{\partial p}{\partial r} = -\rho\frac{\partial u}{\partial t}$$

Για $r$ >> $$p(r,t) \approx (\rho c)u(r,t)$$

# Υπολογισμός της ακουστικής πίεσης

$$p(r,t) \approx (\rho c)u(r,t)$$

Στιγμιαία ένταση : Ισχύς που διαδίδεται από την μονάδα της επιφάνειας.

$$I(t) = p(r,t)u(r,t)$$

Μέση ένταση :

$$< I >= \text{Real}\,\left(\frac{1}{T}\int_0^T p(r,t)u^*(r,t)dt\right)$$

$$< I >= \text{Real}\left(\frac{1}{T}\int_0^T p(r,t)u^*(r,t)dt\right)$$

$$< I > \approx \text{Real}\left(\frac{1}{\rho c}\frac{1}{T}\int_0^T p(r,t)^2 dt\right)$$

$$|p|^2 = \frac{1}{T}\int_0^T p(r,t)^2 dt$$

$$< I > \approx \frac{|p|^2}{\rho c}$$

Ισχύς

$$\Pi = \int_S < I > dS = \int_{4\pi} < I > r^2 d\Omega$$

$$(d\Omega = dS / r^2)$$

$$\Pi =< I > r^2 \int d\Omega = 4\pi |p|^2 \frac{r^2}{\rho c}$$

$$|p| = \left( \frac{\Pi \rho c}{4\pi r^2} \right)^{\frac{1}{2}}$$

1 Pa = $1N / m^2$

Επίπεδο Έντασης $$SIL = 10\log_{10}\frac{I}{I_{ref}} \qquad dB\, re\, I_{ref}$$

Επίπεδο πίεσης $$SPL = 20\log_{10}\left|\frac{p}{p_{ref}}\right| \qquad dB\, re\, p_{ref}$$

$$p_{ref} = 1\mu Pa\ (10^{-6}\ \mathrm{N/m^2})$$

$$|p| = \frac{|p_0| r_0}{r}$$

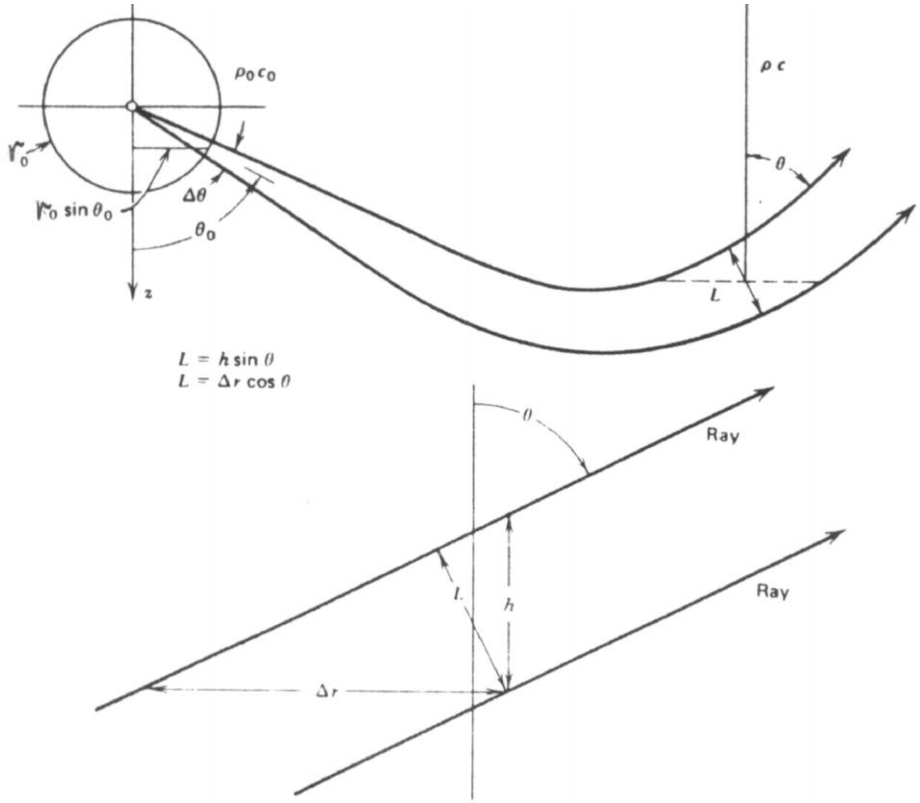
Απώλεια διάδοσης $TL_{12} = SPL_1 - SPL_2$

$$TL_{12} = 20\log_{10}\left|\frac{p_1}{p_{ref}}\right| - 20\log_{10}\left|\frac{p_2}{p_{ref}}\right| = 20\log_{10}\left|\frac{p_1}{p_2}\right| = 20\log_{10}\frac{r_2}{r_1}$$

$$TL = 20\log_{10}\frac{r}{r_0}$$

Απώλεια πυθμένα $BL = -20\log_{10}\left|\frac{p_r}{p_i}\right| = -20\log_{10}\left|R_{12}\right|$

# Υπολογισμός της ακουστικής πίεσης




$$\Delta\Pi = \frac{|p_0|^2}{\rho_0 c_0}(2\pi r_0 \sin\theta_0) r_0 \Delta\theta$$

$$|p_0|^2 = \frac{\rho_0 c_0 \Pi}{4\pi r_0^2}$$

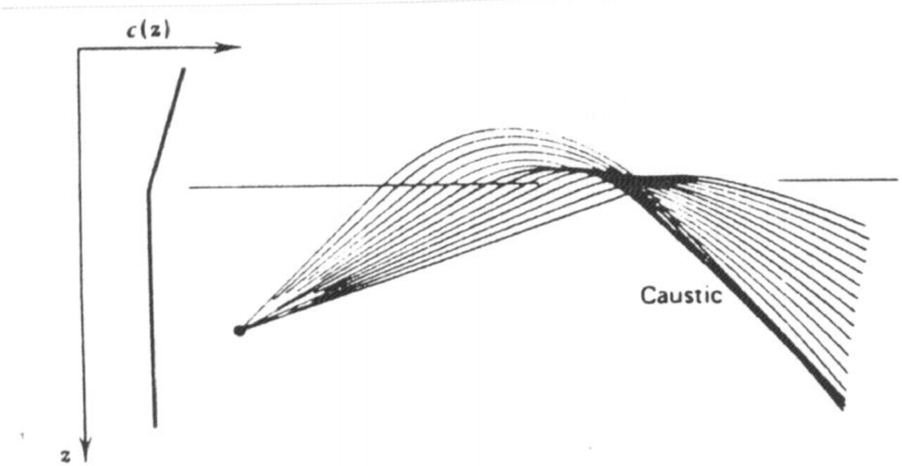
$$\Delta\Pi = \frac{2\pi r_0^2}{\rho_0 c_0}|p_0|^2 \sin\theta_0 \Delta\theta = \frac{2\pi r L |p|^2}{\rho c}$$

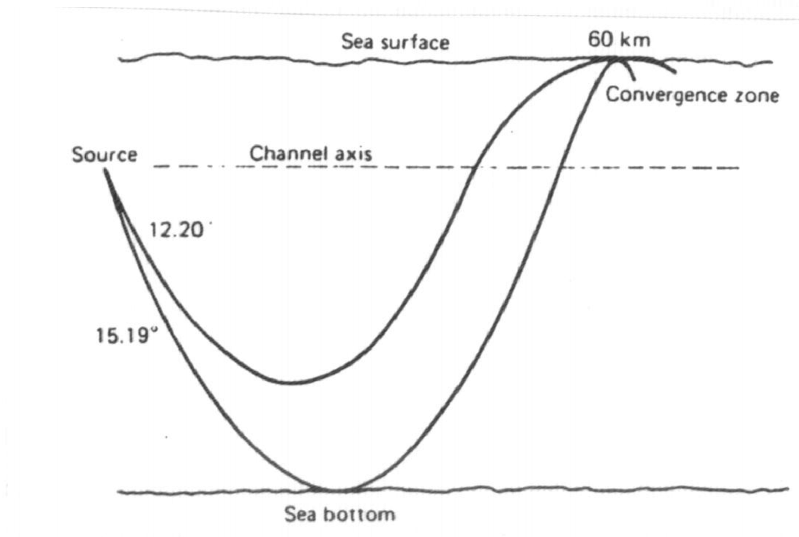
$$|p|^2 = \frac{|p_0|^2 r_0^2 \rho c \sin\theta_0 \Delta\theta}{\rho_0 c_0 r L}$$

$$TL = -20\log_{10}\left|\frac{p}{p_0}\right|$$

$$TL = 10\log_{10}\frac{r}{r_0} - 10\log_{10}\frac{\rho c}{\rho_0 c_0} + 10\log_{10}\frac{L}{r_0 \sin\theta_0 \Delta\theta}$$

# Υπολογισμός της ακουστικής πίεσης




$$\Delta\Pi = \frac{|p_0|^2}{\rho_0 c_0}(2\pi r_0 \sin\theta_0) r_0 \Delta\theta$$

$$|p_0|^2 = \frac{\rho_0 c_0 \Pi}{4\pi r_0^2}$$

$$\Delta\Pi = \frac{2\pi r_0^2}{\rho_0 c_0}|p_0|^2 \sin\theta_0 \Delta\theta = \frac{2\pi r L |p|^2}{\rho c}$$

$$|p|^2 = \frac{|p_0|^2 r_0^2 \rho c \sin\theta_0 \Delta\theta}{\rho_0 c_0 r L}$$

$$TL = -20\log_{10}\left|\frac{p}{p_0}\right|$$

$$TL = 10\log_{10}\frac{r}{r_0} - 10\log_{10}\frac{\rho c}{\rho_0 c_0} + 10\log_{10}\frac{L}{r_0 \sin\theta_0 \Delta\theta}$$