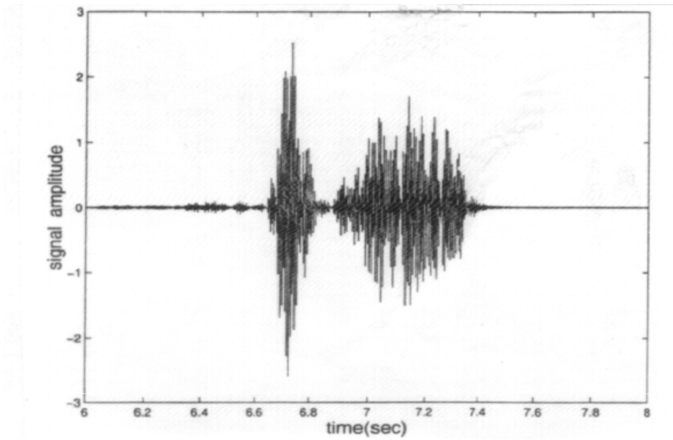
Εισαγωγή στην Ακουστική Ωκεανογραφία

$$f(t) = A\sin(kr - \omega t)$$



$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{i\omega t} d\omega$$

$$\int_{-\infty}^{\infty} x(t)\, dt = \lim_{A\to\infty} \int_{-A}^{A} x(t)dt$$

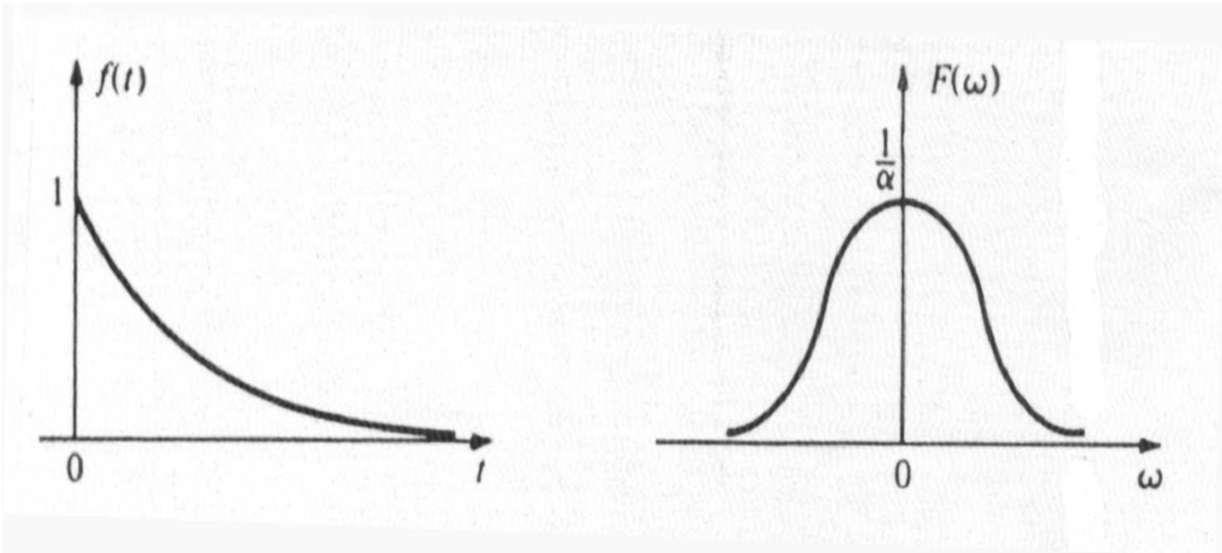
$$F(\omega) = \Im\{ f(t)\}$$

$$f(t) = \Im^{-1}\{F(\omega)\}$$

$$f(t) \leftrightarrow F(\omega)$$

$$f(t) = e^{-at} \quad a > 0 \quad t > 0$$

$$F(\omega) = \int_0^\infty e^{-at} e^{-i\omega t} dt = \int_0^\infty e^{-(a+i\omega)t} dt = \frac{1}{a + i\omega}$$



$$f(t) = f_1(t) + if_2(t) \qquad F(\omega) = R(\omega) + iX(\omega)$$

$$R(\omega) = \int_{-\infty}^{\infty} [f_1(t)\cos\omega t + f_2(t)\sin\omega t]dt$$

$$X(\omega) = \int_{-\infty}^{\infty} [f_2(t)\cos\omega t - f_1(t)\sin\omega t]dt$$

$$f_1(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} [R(\omega)\cos\omega t - X(\omega)\sin\omega t]d\omega$$

$$f_2(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} [R(\omega)\sin\omega t + X(\omega)\cos\omega t]d\omega$$

$$f(t) \in R, \qquad f_1(t) = f(t), \qquad f_2(t) = 0$$

$$R(\omega) = \int_{-\infty}^{\infty} f(t)\cos\omega t dt \qquad X(\omega) = -\int_{-\infty}^{\infty} f(t)\sin\omega t dt$$

$$R(-\omega) = R(\omega) \qquad X(-\omega) = -X(\omega)$$

$$F^*(\omega) = F(-\omega)$$

$$f(-t) = f(t) \qquad X(\omega) = 0$$

$$f(-t) = -f(t) \qquad R(\omega) = 0$$

$$f(t) \leftrightarrow F(\omega)$$

$$F(t) \leftrightarrow 2\pi f(-\omega)$$ Συμμετρία

$$f^*(t) \leftrightarrow F^*(-\omega)$$ Συζυγείς συναρτήσεις

$$f(at) \leftrightarrow \frac{1}{|a|} F(\frac{\omega}{a})$$ Κλίμακα

$$f(t-a) \leftrightarrow e^{-ia\omega} F(\omega)$$ Μετατόπιση

$$f(t)e^{iat} \leftrightarrow F(\omega - a)$$

$$f(t)\cos\omega_0 t \leftrightarrow \frac{1}{2}[F(\omega + \omega_0) + F(\omega - \omega_0)] \quad \text{Διαμόρφωση}$$

Συνέλιξη

$$(f_1 * f_2)(t) = \int_{-\infty}^{\infty} f_1(\tau) f_2(t-\tau) d\tau = \int_{-\infty}^{\infty} f_1(t-\tau) f_2(\tau) d\tau$$

$$(f_1 * f_2)(t) \qquad\qquad t = \tau + x$$

$$\int_{-\infty}^{\infty} e^{-i\omega t}\left[\int_{-\infty}^{\infty} f_1(\tau) f_2(t-\tau) d\tau\right] dt = \int_{-\infty}^{\infty} f_1(\tau) \int_{-\infty}^{\infty} e^{-i\omega(\tau+x)} f_2(x) dx d\tau =$$

$$= \int_{-\infty}^{\infty} f_1(\tau) e^{-i\omega\tau} d\tau \int_{-\infty}^{\infty} f_2(x) e^{-i\omega x} dx = F_1(\omega) F_2(\omega)$$

$$f_1(t) \leftrightarrow F_1(\omega) \qquad f_2(t) \leftrightarrow F_2(\omega)$$

$$f_1(t) * f_2(t) \leftrightarrow F_1(\omega) F_2(\omega)$$

$$f_1(t)\, f_2(t) \leftrightarrow \frac{1}{2\pi} F_1(\omega) * F_2(\omega)$$

Ο Παλμός δ

$$\int_{-\infty}^{\infty} \delta(t)\phi(t)dt = \phi(0)$$

$$\int_{-\infty}^{\infty} \delta(t)e^{-i\omega t}dt = 1$$

$$\delta(t) \leftrightarrow 1$$

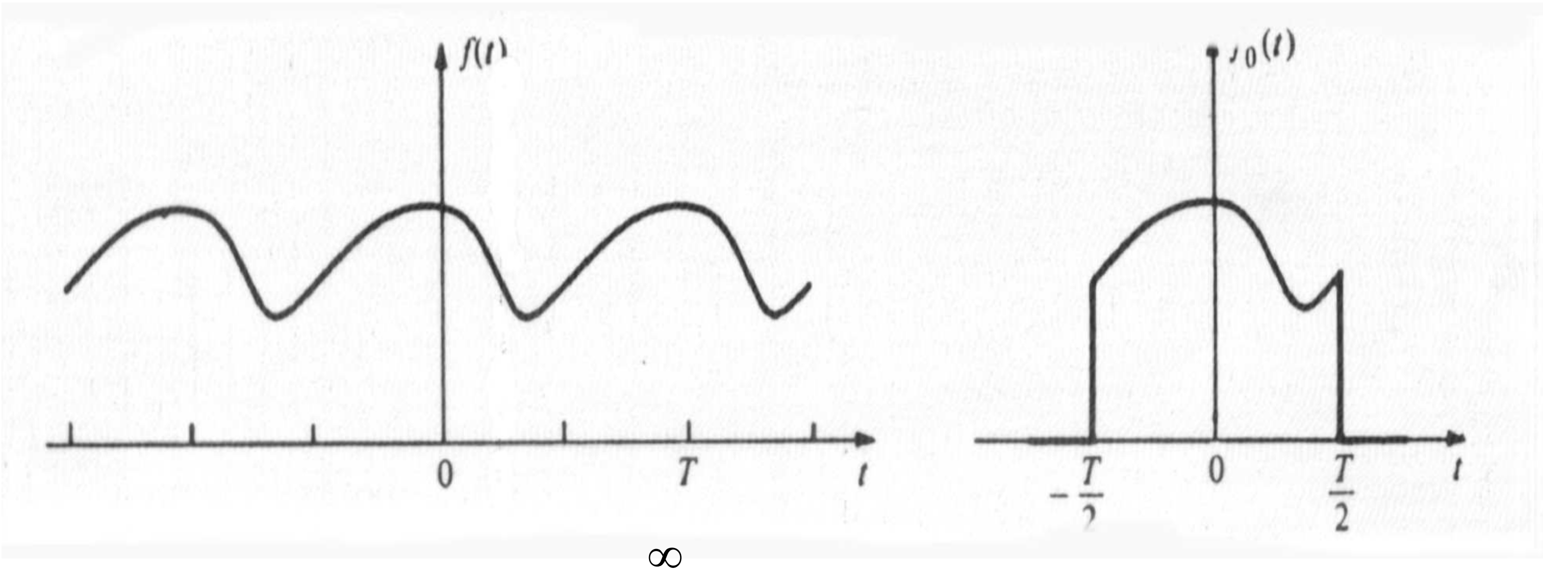
$$\delta(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\omega t}d\omega$$

$$f(t) = 1$$

$$F(\omega) = \int_{-\infty}^{\infty} e^{-i\omega t} dt = 2\pi\delta(\omega)$$

$$1 \leftrightarrow 2\pi\delta(\omega)$$

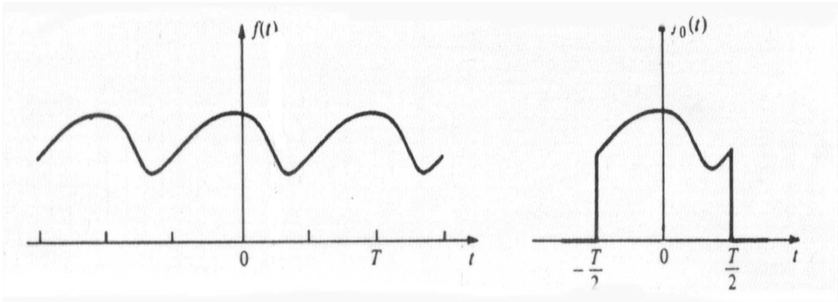
Περιοδικά Σήματα – Σειρές Fourier



$$f(t)=\sum_{n=-\infty}^{\infty} f_0(t+nT)$$

$$f(t)=\sum_{n=-\infty}^{\infty} f_0(t+nT)$$

$$f_0(t)=\begin{cases} f(t) & |t|<\dfrac{T}{2} \\ 0 & |t|>\dfrac{T}{2} \end{cases}$$



$$\bar{\delta}(t)=\sum_{n=-\infty}^{\infty}\delta(t+nT)$$

$$f(t)=\bar{\delta}(t) * f_0(t)$$

$$\bar{\delta}(t) = \sum_{n=-\infty}^{\infty} \delta(t + nT) \qquad \bar{\delta}(\omega) = \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0)$$

$$\omega_0 = \frac{2\pi}{T}$$

$$\bar{\delta}(t) \leftrightarrow \omega_0 \bar{\delta}(\omega)$$

$$f(t) = \bar{\delta}(t) * f_0(t)$$

Μετασχηματισμός Fourier των δύο μερών

$$F(\omega) = \omega_0 \bar{\delta}(\omega) \cdot F_0(\omega) = \omega_0 F_0(\omega) \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0) =$$

$$= \omega_0 \sum_{n=-\infty}^{\infty} F_0(n\omega_0) \delta(\omega - n\omega_0)$$

$$f(t)\delta(t-a) = f(a)\delta(t-a).$$

$$a_n = \frac{1}{T} F_0(n\omega_0) = \frac{1}{T} \int_{-\mathrm{T}/2}^{\mathrm{T}/2} f(t) e^{-in\omega_0 t} dt$$

$$F(\omega) = \omega_0 \sum_{n=-\infty}^{\infty} F_0(n\omega_0)\delta(\omega - n\omega_0)$$

Αντίστροφος μετασχηματισμός των δύο μερών

$$\omega_0 = \frac{2\pi}{\mathrm{T}} \qquad \omega_0 \delta(\omega - n\omega_0) \leftrightarrow e^{in\omega_0 t} / T$$

$$f(t) = \sum_{n=-\infty}^{\infty} a_n e^{in\omega_0 t}$$

$$f(t) = \sum_{n=-\infty}^{\infty} a_n e^{in\omega_0 t}$$

Θεώρημα Συνέλιξης

$$f_1(t) = \sum_{n=-\infty}^{\infty} a_n e^{in\omega_0 t} \qquad f_2(t) = \sum_{n=-\infty}^{\infty} b_n e^{in\omega_0 t}$$

$$\frac{1}{T}\int_{-T/2}^{T/2} f_1(t-\tau) f_2(\tau) d\tau = \sum_{n=-\infty}^{\infty} a_n b_n e^{in\omega_0 t}$$

$$\frac{1}{T}\int_{-T/2}^{T/2} f_1(t)\, f_2(t) e^{-in\omega_0 t} dt = \sum_{m=-\infty}^{\infty} a_m b_{n-m}$$

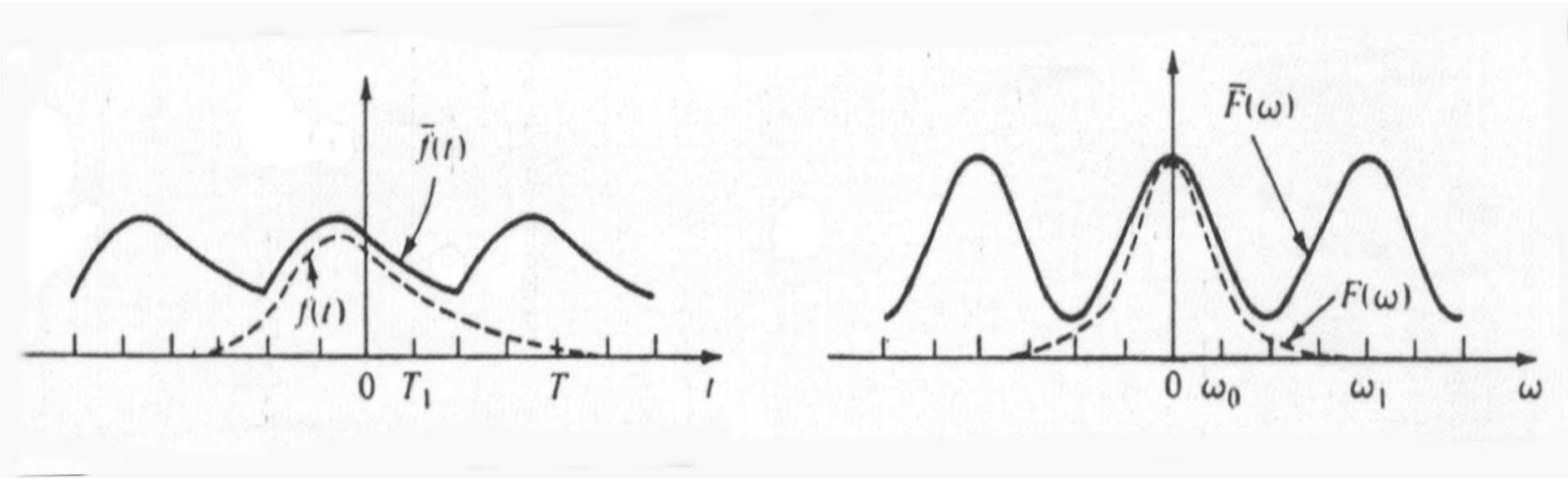
$$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{i2\pi n f_0 t} \qquad X_n = \frac{1}{T}\int_{T_0}^{T_0+T} x(t) e^{-i2\pi n f_0 t} dt$$

$$f_0 = \frac{1}{T}$$

Διακριτός Μετασχηματισμός Fourier

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$$

$$\bar{f}(t) = \sum_{n=-\infty}^{\infty} f(t + nT) \qquad \bar{F}(\omega) = \sum_{n=-\infty}^{\infty} F(\omega + n\omega_1)$$



$$\bar{f}(t) = \sum_{n=-\infty}^{\infty} f(t + nT) \qquad \bar{F}(\omega) = \sum_{n=-\infty}^{\infty} F(\omega + n\omega_1)$$

Poisson Sum Formula

$$\bar{f}(t) = \frac{1}{T} \sum_{n=-\infty}^{\infty} F(n\omega_0) e^{in\omega_0 t} \qquad \omega_0 = \frac{2\pi}{\mathrm{T}}$$

$$\bar{F}(\omega) = \frac{2\pi}{\omega_1} \sum_{n=-\infty}^{\infty} f(nT_1) e^{-inT_1\omega} \qquad T_1 = \frac{2\pi}{\omega_1}$$

$$F(n\omega_0) = \int_{-\mathrm{T}/2}^{\mathrm{T}/2} \bar{f}(t) e^{-in\omega_0 t} dt$$

Συντελεστές Fourier της $T\bar{f}(t)$

Χρειαζόμαστε κάτι ανάλογο για την συνάρτηση $f(t)$

$$y(t) = \sum_{k=-\infty}^{\infty} c_k e^{ik\omega_0 t}$$

$$T_1 = T/N$$

$$y(mT_1) = \sum_{k=-\infty}^{\infty} c_k e^{ik\omega_0 m T_1} = \sum_{k=-\infty}^{\infty} c_k w_N^{\ km} \qquad w_N = e^{i2\pi/N}$$

$$\omega_0 T_1 = \omega_0 T/N = 2\pi/N$$

$$k = n + rN$$

$$n = 0, \ldots\ldots, N-1$$

$$r = \ldots\ldots\ldots, -1, 0, 1, \ldots\ldots\ldots\ldots\ldots$$

$$w_N^N = 1, \quad w_N^{km} = w_N^{(n+rN)m} = w_N^{mN}$$

$$y(mT_1) = \sum_{n=0}^{N-1} \sum_{r=-\infty}^{\infty} c_{n+rN} \, w_N^{\,(n+rN)m} = \sum_{n=0}^{N-1} w_N^{\,mn} \sum_{r=-\infty}^{\infty} c_{n+rN}$$

$$y(mT_1) = \sum_{n=0}^{N-1} \sum_{r=-\infty}^{\infty} c_{n+rN} \, w_N^{(n+rN)m} = \sum_{n=0}^{N-1} w_N^{mn} \sum_{r=-\infty}^{\infty} c_{n+rN}$$

$$\overline{c}_n \equiv \sum_{r=-\infty}^{\infty} c_{n+rN}$$

$$y(mT_1) = \sum_{n=0}^{N-1} \overline{c}_n w_N^{mn}, \qquad m = 0, \ldots., N-1$$

$$T_1 = T / N, \quad \omega_0 = \frac{2\pi}{T}, \quad \omega_1 = \frac{2\pi}{T_1} = N\omega_0$$

$$\bar{f}(mT_1) = \frac{1}{T} \sum_{n=0}^{N-1} \overline{F}(n\omega_0) w_N^{mn}, \quad w_N = e^{i2\pi / N}$$

$$F(\omega) = 0 \qquad |\omega| > \sigma \qquad \omega_1 > 2\sigma$$

$$F(\omega) = \bar{F}(\omega) \qquad |\omega| < \sigma$$

$$A_m = \sum_{n=0}^{N-1} a_n w_N^{\ mn} \qquad m = 0, \ldots\ldots, N-1, \qquad w_N = e^{i2\pi/\mathrm{N}}$$

$$a_n = \frac{1}{N} \sum_{m=0}^{N-1} A_m w_N^{\ -mn} \qquad n = 0, \ldots., N-1$$

$a_N \leftrightarrow A_m$ Περιοδική $A_{m+N} = A_m, \quad a_{n+N} = a_n$

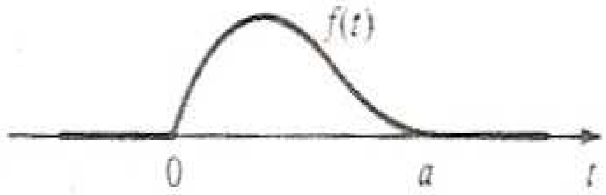
$$A_{m+N} = A_m, \quad a_{n+N} = a_n$$

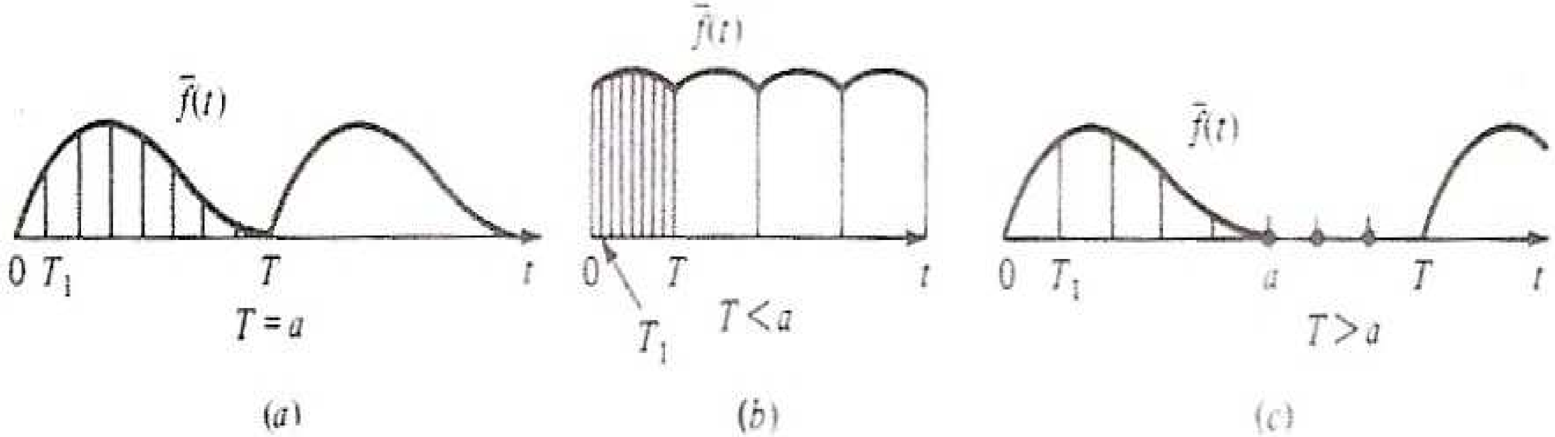
$$\bar{f}(mT_1) = \sum_{k=-\infty}^{\infty} f(mT_1 + kT), \quad \bar{F}(n\omega_0) = \sum_{r=-\infty}^{\infty} F(n\omega_0 + r\omega_1)$$

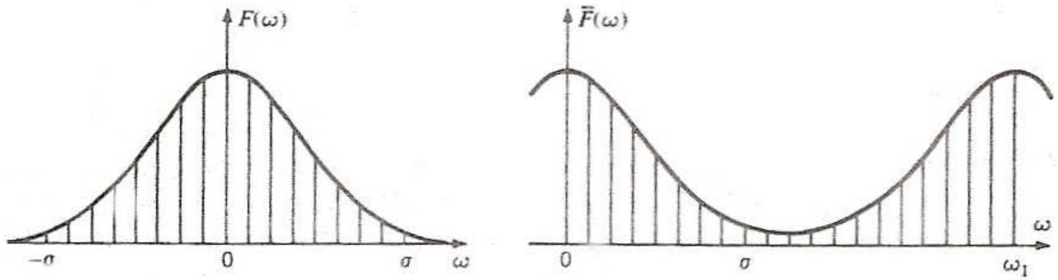
$$\frac{1}{T}\bar{F}(n\omega_0) \leftrightarrow \bar{f}(mT_1)$$

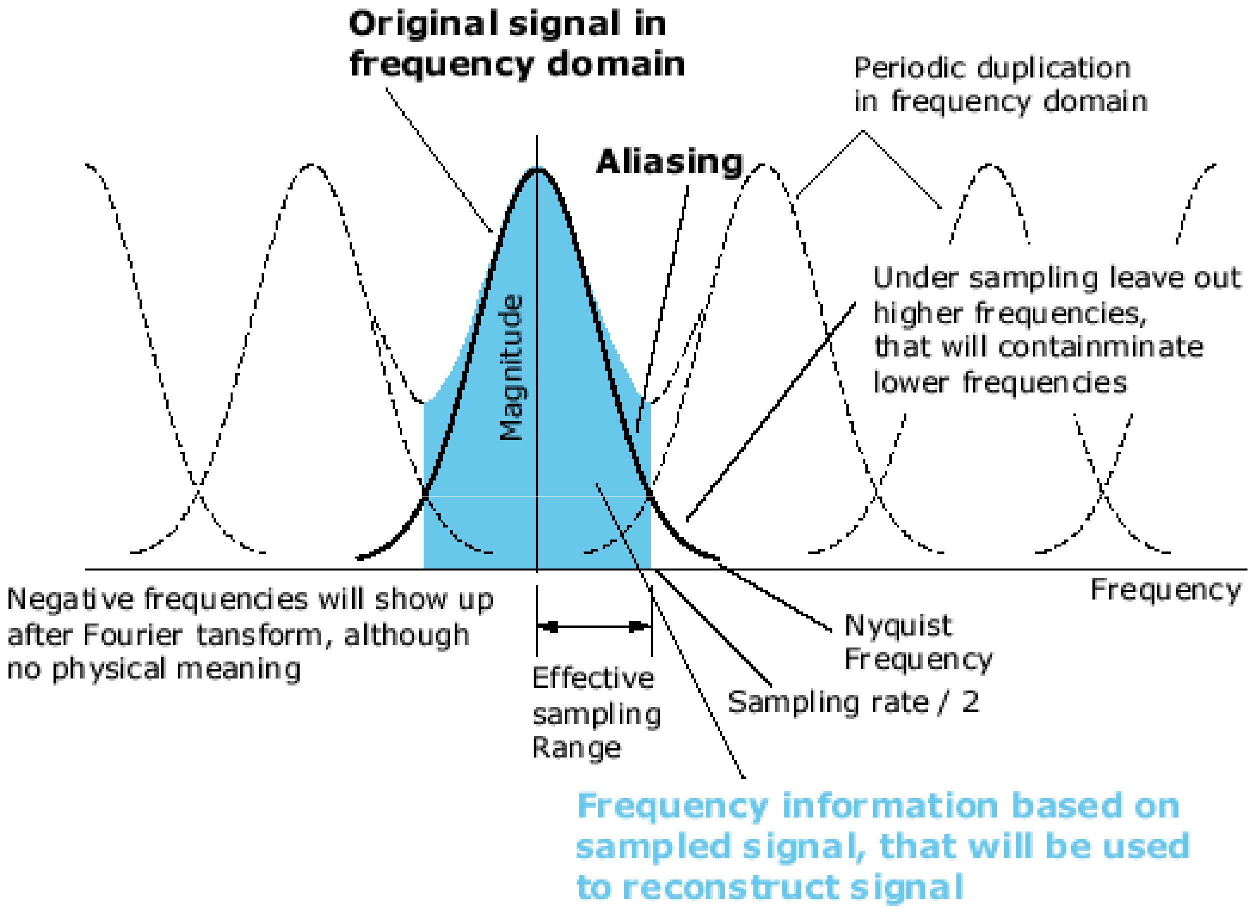
$$a_n = \frac{1}{N}\sum_{m=0}^{N-1} A_m w_N^{-mn} \quad n = 0,\ldots.,N-1$$

$$\bar{F}(n\omega_0) = T_1 \sum_{m=0}^{N-1} \bar{f}(mT_1) w_N^{-mn}$$

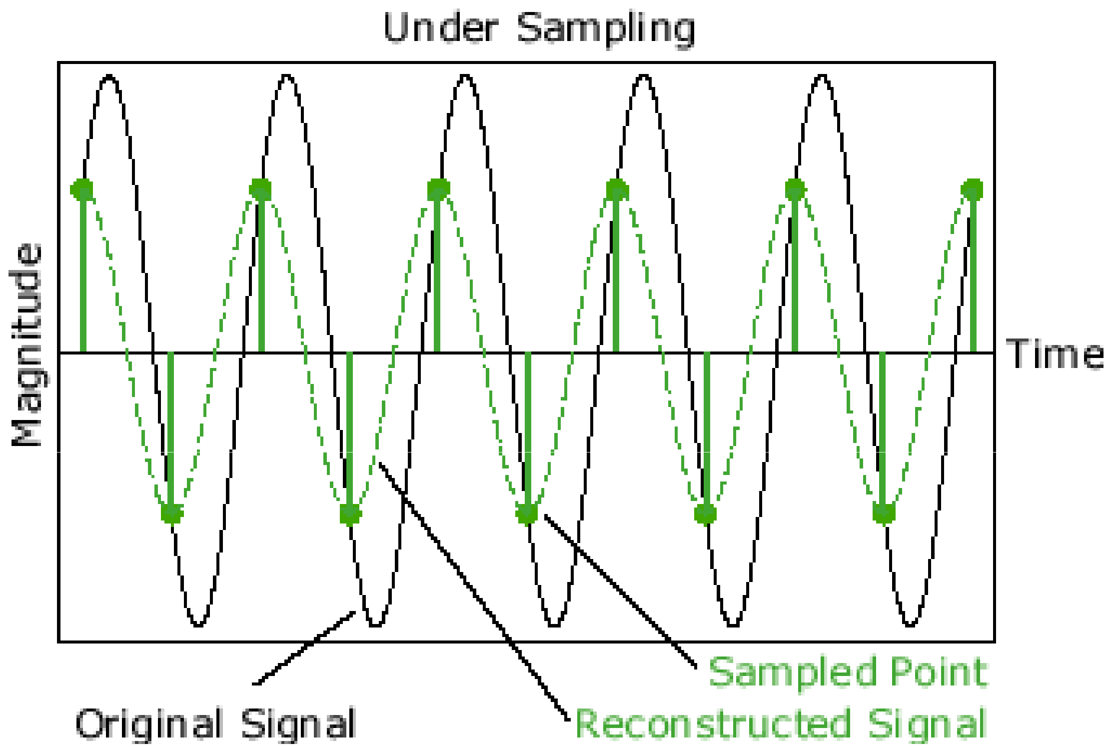

f(t)
0
a
t


$\bar{f}(t)$
0 T_1
T
t
T = a
(a)
$\bar{f}(t)$
0
T
t
T_1
T < a
(b)
$\bar{f}(t)$
0 T_1
a
T
t
T > a
(c)


F(ω)
−σ
0
σ
ω
$\bar{F}(\omega)$
ω
0
σ
ω_1



Original signal in frequency domain
Periodic duplication in frequency domain
Aliasing
Magnitude
Under sampling leave out higher frequencies, that will containminate lower frequencies
Frequency
Negative frequencies will show up after Fourier tansform, although no physical meaning
Nyquist Frequency
Effective sampling Range
Sampling rate / 2
Frequency information based on sampled signal, that will be used to reconstruct signal


Under Sampling
Magnitude
Time
Original Signal
Sampled Point
Reconstructed Signal

$$F(n\omega_0) \simeq \begin{cases} \bar{F}(n\omega_0) & \text{για} \left|n\right| \leq \dfrac{N}{2} \\ 0 & \text{για} \left|n\right| < \dfrac{N}{2} \end{cases}$$

$$F(n\omega_0) \simeq \begin{cases} \bar{F}(n\omega_0) & \text{για} \quad 0 \leq n \leq \dfrac{N}{2} \\ \bar{F}(N\omega_0 + n\omega_0) & \text{για} \quad \left|n\right| < \dfrac{N}{2} \end{cases}$$