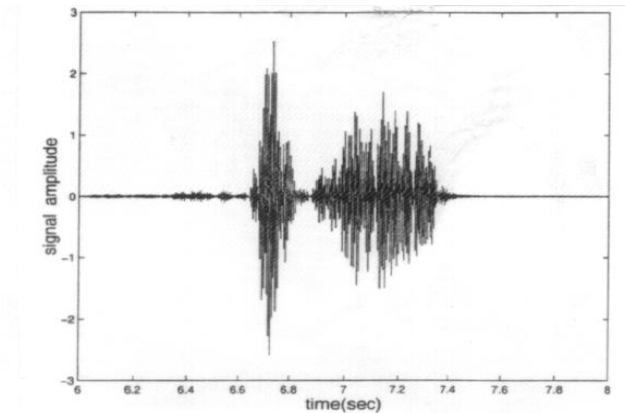
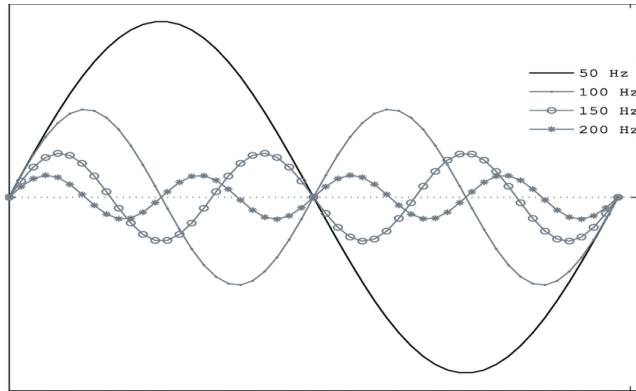


Εισαγωγή στην  
Επεξεργασία Σημάτων

## Εισαγωγή στην Ακουστική Ωκεανογραφία

$$f(t) = A \sin(kr - \omega t)$$



$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{i\omega t} d\omega$$

$$\int_{-\infty}^{\infty} x(t) dt = \lim_{A \rightarrow \infty} \int_{-A}^A x(t) dt$$

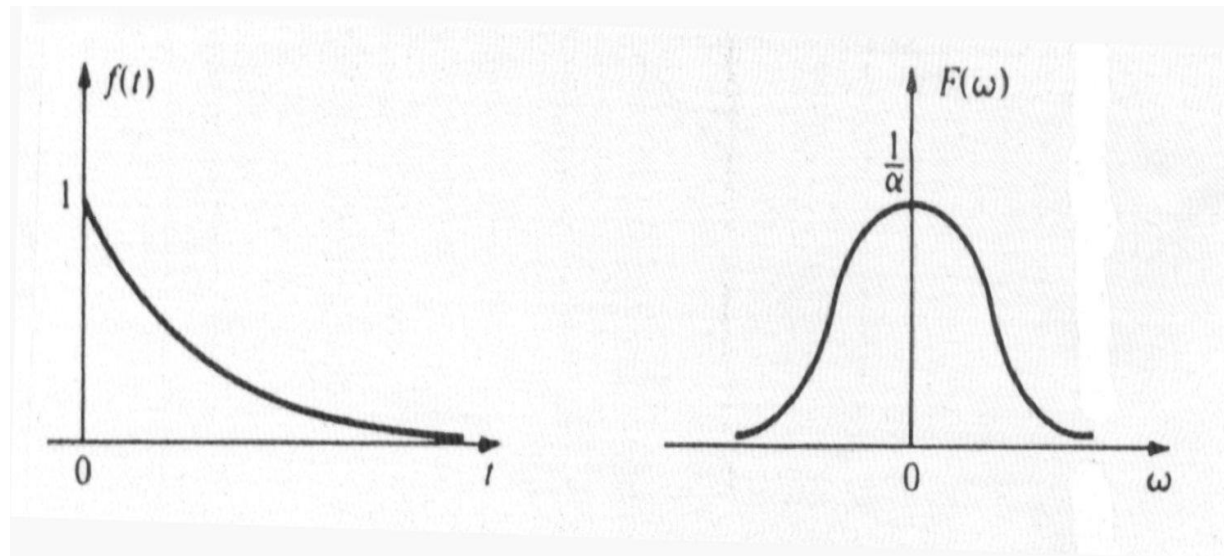
$$F(\omega) = \mathfrak{F}\{f(t)\}$$

$$f(t) = \mathfrak{F}^{-1}\{F(\omega)\}$$

$$f(t) \leftrightarrow \{F(\omega)\}$$

$$f(t) = e^{-at} \quad a > 0 \quad t > 0$$

$$F(\omega) = \int_0^{\infty} e^{-at} e^{-i\omega t} dt = \int_0^{\infty} e^{-(a+i\omega)t} dt = \frac{1}{a+i\omega}$$



$$f(t) = f_1(t) + if_2(t) \quad F(\omega) = R(\omega) + iX(\omega)$$

$$R(\omega) = \int_{-\infty}^{\infty} [f_1(t) \cos \omega t + f_2(t) \sin \omega t] dt$$

$$X(\omega) = \int_{-\infty}^{\infty} [f_2(t) \cos \omega t - f_1(t) \sin \omega t] dt$$

$$f_1(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} [R(\omega) \cos \omega t - X(\omega) \sin \omega t] d\omega$$

$$f_2(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} [R(\omega) \sin \omega t + X(\omega) \cos \omega t] d\omega$$

$$f(t) \in R, \quad f_1(t) = f(t), \quad f_2(t) = 0$$

$$R(\omega) = \int_{-\infty}^{\infty} f(t) \cos \omega t dt \quad X(\omega) = - \int_{-\infty}^{\infty} f(t) \sin \omega t dt$$

$$R(-\omega) = R(\omega) \quad X(-\omega) = -X(\omega)$$

$$F^*(\omega) = F(-\omega)$$

$$f(-t) = f(t) \quad X(\omega) = 0$$

$$f(-t) = -f(t) \quad R(\omega) = 0$$

$$f(t) \leftrightarrow F(\omega)$$

$$F(t) \leftrightarrow 2\pi f(-\omega)$$

Συμμετρία

$$f^*(t) \leftrightarrow F^*(-\omega)$$

Συζυγείς συναρτήσεις

$$f(at) \leftrightarrow \frac{1}{|a|} F\left(\frac{\omega}{a}\right)$$

Κλίμακα

$$f(t-a) \leftrightarrow e^{-ia\omega} F(\omega)$$

$$e^{iat} f(t) \leftrightarrow F(\omega - \alpha)$$

Μετατόπιση

$$f(t)e^{iat} \leftrightarrow F(\omega - a)$$

$$f(t) \cos \omega_0 t \leftrightarrow \frac{1}{2} [F(\omega + \omega_0) + F(\omega - \omega_0)] \quad \text{Διαμόρφωση}$$



# Συνέλιξη

$$(f_1 * f_2)(t) = \int_{-\infty}^{\infty} f_1(\tau) f_2(t - \tau) d\tau = \int_{-\infty}^{\infty} f_1(t - \tau) f_2(\tau) d\tau$$

$$(f_1 * f_2)(t) \quad t = \tau + x$$

$$\begin{aligned} \int_{-\infty}^{\infty} e^{-i\omega t} \left[ \int_{-\infty}^{\infty} f_1(\tau) f_2(t - \tau) d\tau \right] dt &= \int_{-\infty}^{\infty} f_1(\tau) \int_{-\infty}^{\infty} e^{-i\omega(\tau+x)} f_2(x) dx d\tau = \\ &= \int_{-\infty}^{\infty} f_1(\tau) e^{-i\omega\tau} d\tau \int_{-\infty}^{\infty} f_2(x) e^{-i\omega x} dx = F_1(\omega) F_2(\omega) \end{aligned}$$

$$f_1(t) \leftrightarrow F_1(\omega)$$

$$f_2(t) \leftrightarrow F_2(\omega)$$

$$f_1(t) * f_2(t) \leftrightarrow F_1(\omega)F_2(\omega)$$

$$f_1(t)f_2(t) \leftrightarrow \frac{1}{2\pi} F_1(\omega) * F_2(\omega)$$

$$\int_{-\infty}^{\infty} \delta(t) \phi(t) dt = \phi(0)$$

$$\int_{-\infty}^{\infty} \delta(t) e^{-i\omega t} dt = 1$$

$$\delta(t) \leftrightarrow 1$$

$$\delta(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\omega t} d\omega$$

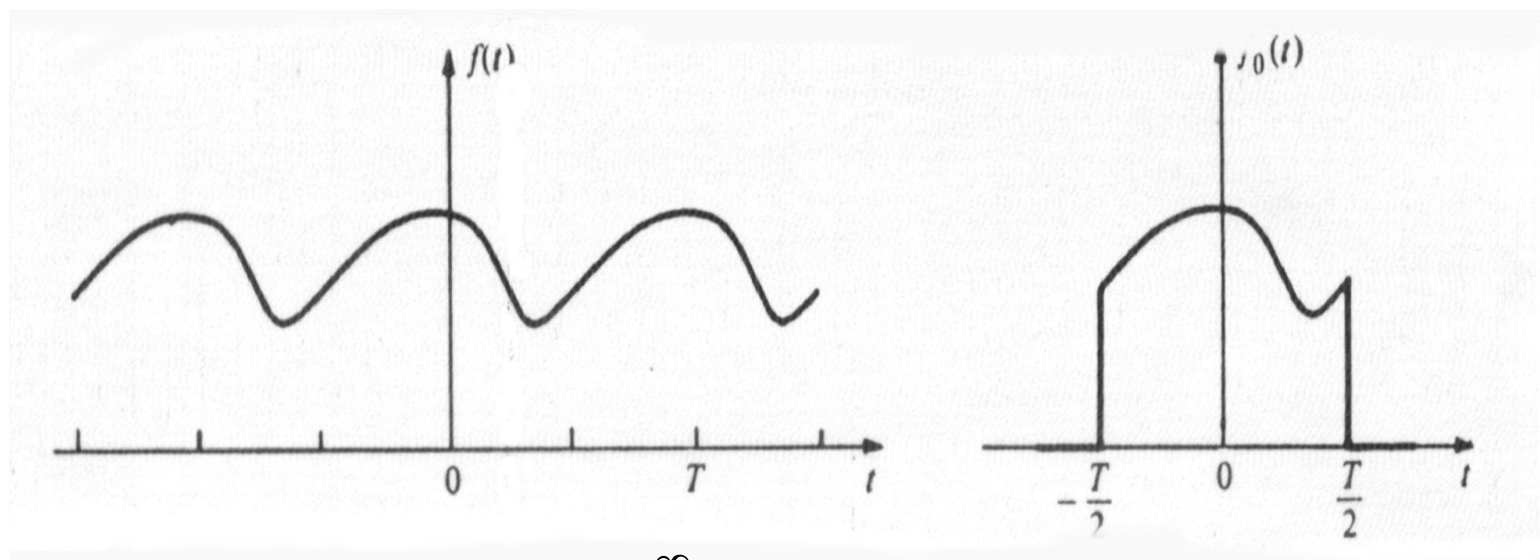
$$f(t) = 1$$

$$F(\omega) = \int_{-\infty}^{\infty} e^{-i\omega t} dt = 2\pi\delta(\omega)$$

$$1 \leftrightarrow 2\pi\delta(\omega)$$

$$1 \leftrightarrow 2\pi\delta(\omega)$$

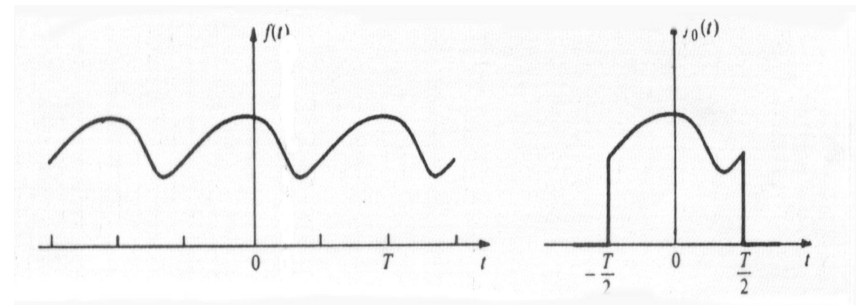
# Περιοδικά Σήματα – Σειρές Fourier



$$f(t) = \sum_{n=-\infty}^{\infty} f_0(t + nT)$$

$$f(t) = \sum_{n=-\infty}^{\infty} f_0(t + nT)$$

$$f_0(t) = \begin{cases} f(t) & |t| < \frac{T}{2} \\ 0 & |t| > \frac{T}{2} \end{cases}$$



$$\bar{\delta}(t) = \sum_{n=-\infty}^{\infty} \delta(t + nT)$$

$$f(t) = \bar{\delta}(t) * f_0(t)$$

$$\bar{\delta}(\omega) = \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0)$$

$$\omega_0 = \frac{2\pi}{T}$$

$$\bar{\delta}(t) \leftrightarrow \omega_0 \bar{\delta}(\omega)$$

Μετασχηματισμός Fourier των δύο μερών  $f(t) = \bar{\delta}(t) * f_0(t)$

$$F(\omega) = \omega_0 \bar{\delta}(\omega) \cdot F_0(\omega) = \omega_0 F_0(\omega) \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0) =$$

$$= \omega_0 \sum_{n=-\infty}^{\infty} F_0(n\omega_0) \delta(\omega - n\omega_0) \quad f(t)\delta(t-a) = f(a)\delta(t-a).$$

$$a_n = \frac{1}{T} F_0(n\omega_0) = \frac{1}{T} \int_{-T/2}^{T/2} f(t) e^{-in\omega_0 t} dt$$

$$F(\omega) = \omega_0 \sum_{n=-\infty}^{\infty} F_0(n\omega_0) \delta(\omega - n\omega_0)$$

$$\mathfrak{F}^{-1}(F(\omega)) = \mathfrak{F}^{-1}\left[\omega_0 \sum_{n=-\infty}^{\infty} F_0(n\omega_0) \delta(\omega - n\omega_0)\right]$$



$$e^{iat} f(t) \leftrightarrow F(\omega - \alpha)$$

$$1 \leftrightarrow 2\pi\delta(\omega)$$

$$e^{iat} \leftrightarrow 2\pi\delta(\omega - a)$$

$$e^{in\omega_0 t} \leftrightarrow 2\pi\delta(\omega - n\omega_0)$$

$$\mathfrak{F}^{-1}(F(\omega)) = \mathfrak{F}^{-1}\left[\omega_0 \sum_{n=-\infty}^{\infty} F_0(n\omega_0)\delta(\omega - n\omega_0)\right]$$

$$f(t) = \mathfrak{F}^{-1}(F(\omega)) = \mathfrak{F}^{-1}\left[\omega_0 \sum_{n=-\infty}^{\infty} F_0(n\omega_0)\delta(\omega - n\omega_0)\right] =$$

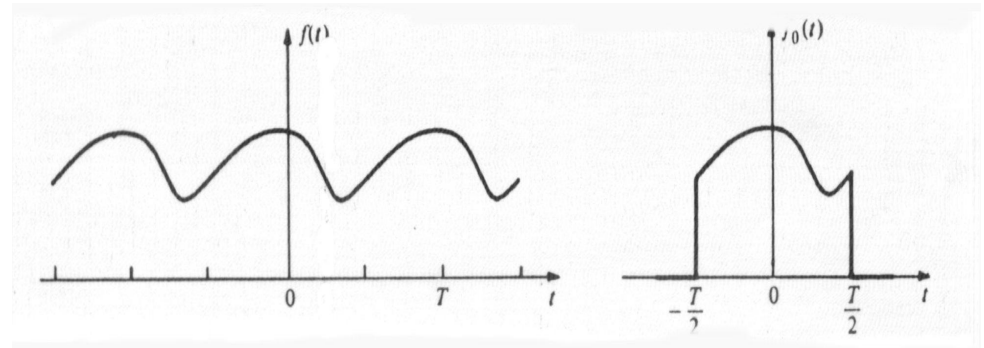
$$= \omega_0 \sum_{n=-\infty}^{\infty} F_0(n\omega_0) \frac{1}{2\pi} e^{in\omega_0 t}$$

$$f(t) = \mathfrak{F}^{-1}(F(\omega)) = \mathfrak{F}^{-1}\left[\omega_0 \sum_{n=-\infty}^{\infty} F_0(n\omega_0) \delta(\omega - n\omega_0)\right] =$$

$$= \omega_0 \sum_{n=-\infty}^{\infty} F_0(n\omega_0) \frac{1}{2\pi} e^{in\omega_0 t}$$

$$a_n = \frac{1}{T} F_0(n\omega_0) = \frac{1}{T} \int_{-T/2}^{T/2} f(t) e^{-in\omega_0 t} dt$$

$$f(t) = \sum_{n=-\infty}^{\infty} a_n e^{in\omega_0 t}$$



$$f(t) = \sum_{n=-\infty}^{\infty} a_n e^{in\omega_0 t}$$

Θεώρημα Συνέλιξης

$$f_1(t) = \sum_{n=-\infty}^{\infty} a_n e^{in\omega_0 t} \quad f_2(t) = \sum_{n=-\infty}^{\infty} b_n e^{in\omega_0 t}$$

$$\frac{1}{T} \int_{-T/2}^{T/2} f_1(t-\tau) f_2(\tau) d\tau = \sum_{n=-\infty}^{\infty} a_n b_n e^{in\omega_0 t}$$

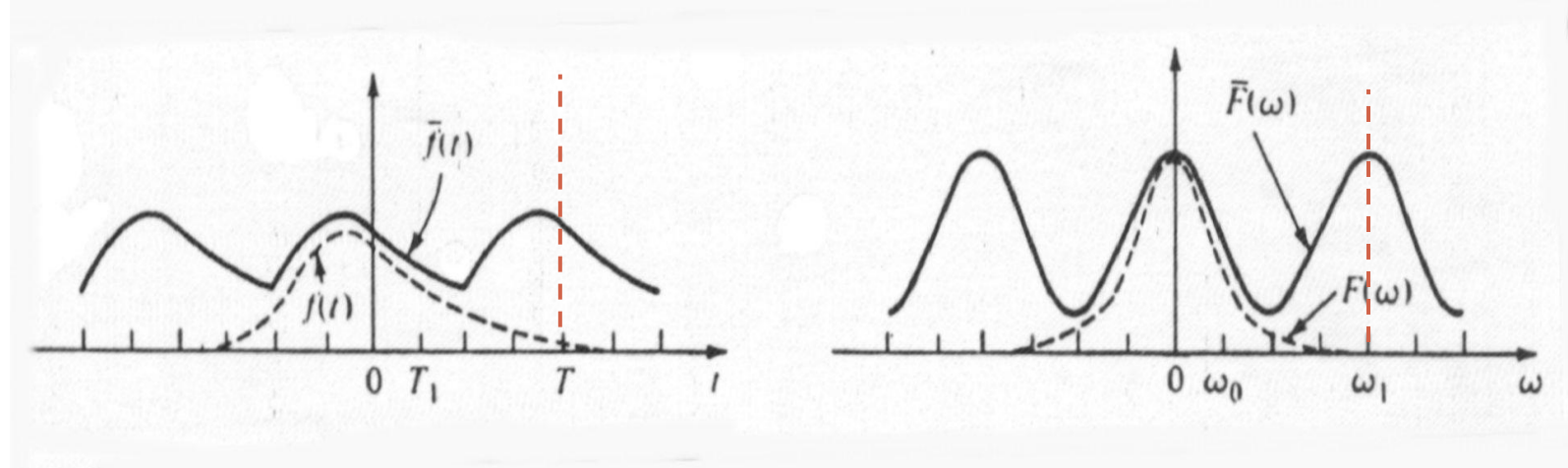
$$\frac{1}{T} \int_{-T/2}^{T/2} f_1(t) f_2(t) e^{-in\omega_0 t} dt = \sum_{m=-\infty}^{\infty} a_m b_{n-m}$$

$$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{i2\pi n f_0 t} \quad X_n = \frac{1}{T} \int_{T_0}^{T_0+T} x(t) e^{-i2\pi n f_0 t} dt$$

$$f_0 = \frac{1}{T}$$

# Διακριτός Μετασχηματισμός Fourier

$$\bar{f}(t) = \sum_{n=-\infty}^{\infty} f(t + nT) \quad \bar{F}(\omega) = \sum_{n=-\infty}^{\infty} F(\omega + n\omega_1)$$



$$\bar{f}(t) = \sum_{n=-\infty}^{\infty} f(t + nT) \quad \bar{F}(\omega) = \sum_{n=-\infty}^{\infty} F(\omega + n\omega_1)$$

Poisson Sum Formula

$$\bar{f}(t) = \frac{1}{T} \sum_{n=-\infty}^{\infty} F(n\omega_0) e^{in\omega_0 t} \quad \omega_0 = \frac{2\pi}{T}$$

$$F(n\omega_0) = \int_{-T/2}^{T/2} \bar{f}(t) e^{-in\omega_0 t} dt \quad \text{Συντελεστές Fourier της } T\bar{f}(t)$$

$$f(t) = \sum_{n=-\infty}^{\infty} a_n e^{in\omega_0 t}, \quad a_n = \frac{1}{T} \int_{-T/2}^{T/2} f(t) e^{-in\omega_0 t} dt$$

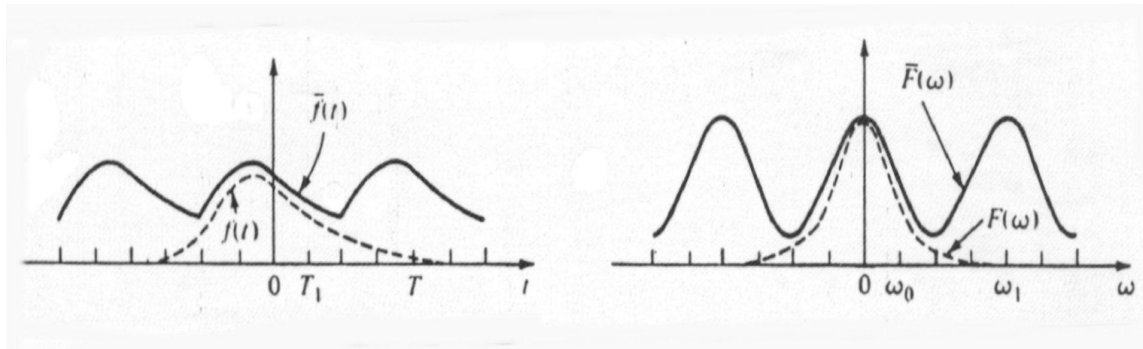
$$\bar{F}(\omega) = \frac{2\pi}{\omega_1} \sum_{n=-\infty}^{\infty} f(nT_1) e^{-inT_1\omega} \quad T_1 = \frac{2\pi}{\omega_1}$$

$$\bar{f}(t) = \sum_{n=-\infty}^{\infty} f(t + nT) \quad \bar{F}(\omega) = \sum_{n=-\infty}^{\infty} F(\omega + n\omega_1)$$

$$\bar{f}(t) = \frac{1}{T} \sum_{n=-\infty}^{\infty} F(n\omega_0) e^{in\omega_0 t} \quad \omega_0 = \frac{2\pi}{T}$$

$$F(n\omega_0) = \int_{-T/2}^{T/2} \bar{f}(t) e^{-in\omega_0 t} dt$$

Διακριτές τιμές της συνάρτησης  $F(\omega)$



Χρειαζόμαστε κάτι ανάλογο για την συνάρτηση  $f(t)$

$$y(t) = \sum_{k=-\infty}^{\infty} c_k e^{ik\omega_0 t}$$

$$T_1 = T / N$$

$$y(mT_1) = \sum_{k=-\infty}^{\infty} c_k e^{ik\omega_0 mT_1} = \sum_{k=-\infty}^{\infty} c_k w_N^{km} \quad w_N = e^{i2\pi / N}$$

$$\omega_0 T_1 = \omega_0 T / N = 2\pi / N$$



$$k = n + rN, n = 0, 1, \dots, N-1 \quad r = \dots -1, 0, 1, \dots$$

$$y(mT_1) = \sum_{n=0}^{N-1} \sum_{r=-\infty}^{\infty} c_{n+rN} w_N^{(n+rN)m} = \sum_{n=0}^{N-1} w_N^{mn} \sum_{r=-\infty}^{\infty} c_{n+rN}$$

$$\bar{c}_N \equiv \sum_{r=-\infty}^{\infty} c_{n+rN}$$

$$y(mT_1) = \sum_{n=0}^{N-1} \bar{c}_n w_N^{mn}, \quad m = 0, \dots, N-1$$

Οι τιμές δειγματοληψίας της  $y(mT_1)$  και οι συντελεστές σχετίζονται από το ως άνω σύστημα  $N$  εξισώσεων

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-i\omega t} dt$$

$$T_1 = T / N, \quad \omega_0 = \frac{2\pi}{T}, \quad \omega_1 = \frac{2\pi}{T_1} = N\omega_0$$

$$\bar{f}(mT_1) = \frac{1}{T} \sum_{n=0}^{N-1} \bar{F}(n\omega_0) w_N^{mn}, \quad w_N = e^{i2\pi/N}$$

$$m = 0, \dots, N-1$$

Σύστημα  $N$  εξισώσεων που μας δίδουν τις τιμές δειγματοληψίας

$$\bar{F}(n\omega_0)$$

Εάν  $F(\omega) = 0$  για  $|\omega| > \sigma$  και  $\omega_1 > 2\sigma$

$$F(\omega) = \bar{F}(\omega) \quad \text{για} \quad |\omega| < \sigma$$

$$A_m = \sum_{n=0}^{N-1} a_n w_N^{mn} \quad m = 0, \dots, N-1, \quad w_N = e^{i2\pi/N}$$

$$a_n = \frac{1}{N} \sum_{m=0}^{N-1} A_m w_N^{-mn} \quad n = 0, \dots, N-1$$

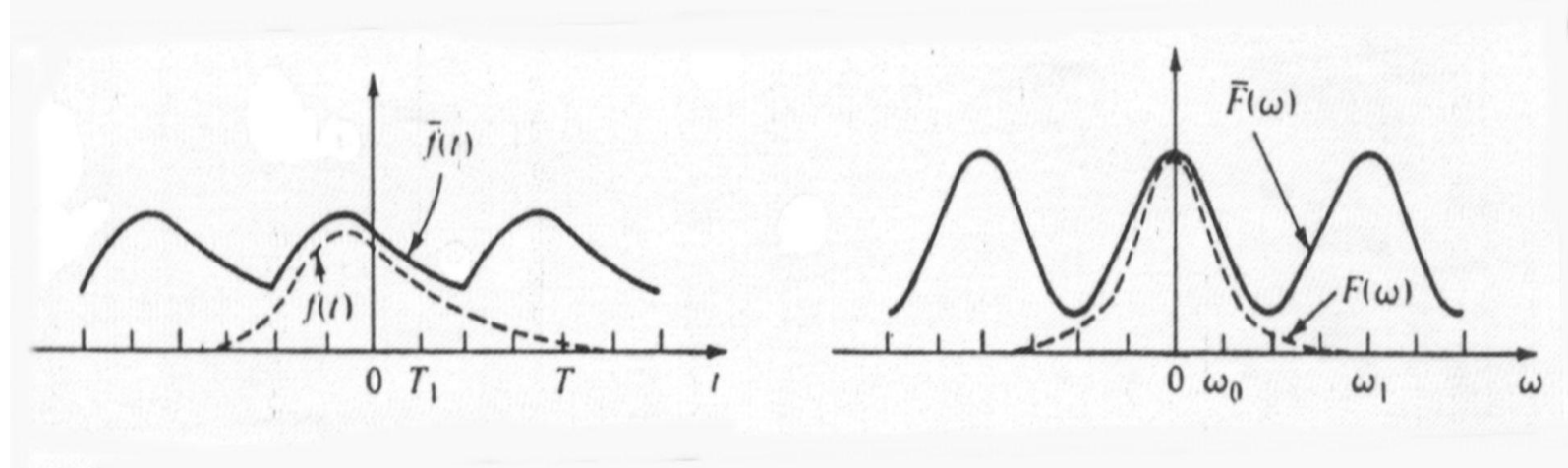
$$a_N \leftrightarrow A_m$$

Διακριτή σειρά Fourier

Περιοδική  $A_{m+N} = A_m, \quad a_{n+N} = a_n$

# Διακριτός Μετασχηματισμός Fourier

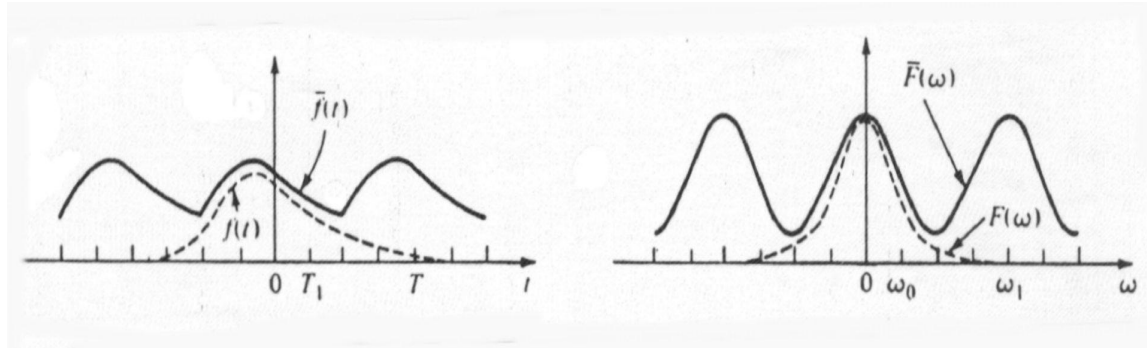
$$\bar{f}(t) = \sum_{n=-\infty}^{\infty} f(t + nT) \quad \bar{F}(\omega) = \sum_{n=-\infty}^{\infty} F(\omega + n\omega_1)$$



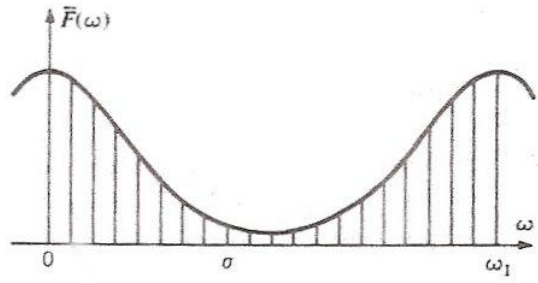
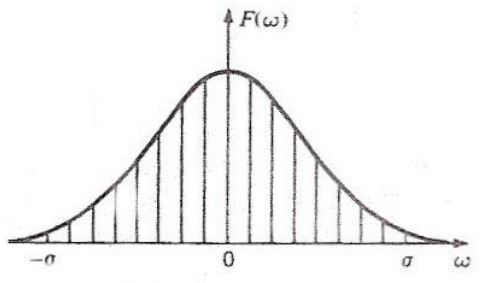
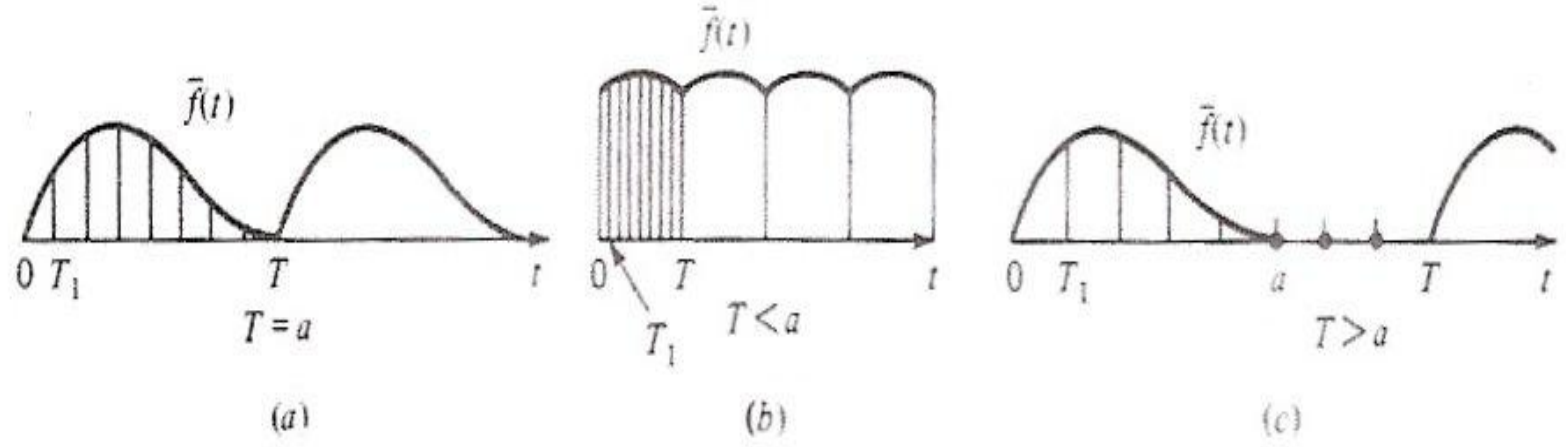
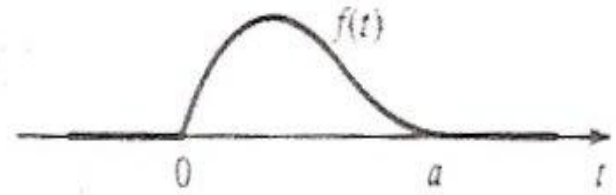
$$\bar{f}(mT_1) = \sum_{k=-\infty}^{\infty} f(mT_1 + kT), \quad \bar{F}(n\omega_0) = \sum_{r=-\infty}^{\infty} F(n\omega_0 + r\omega_1)$$

$$\frac{1}{T} \bar{F}(n\omega_0) \leftrightarrow \bar{f}(mT_1)$$

$$\bar{F}(n\omega_0) = T_1 \sum_{m=0}^{N-1} \bar{f}(mT_1) w_N^{-mn} \quad w_N = e^{i2\pi/N}$$



$f(t)$   
 $\bar{f}(t)$



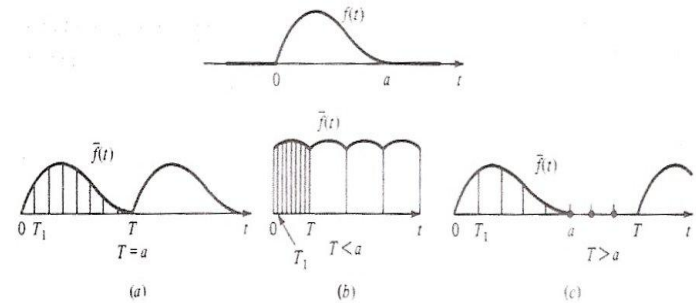
$$T = a, N = N_0$$

$$F(\omega) = 0 \quad |\omega| > \sigma \quad \omega_1 > 2\sigma$$

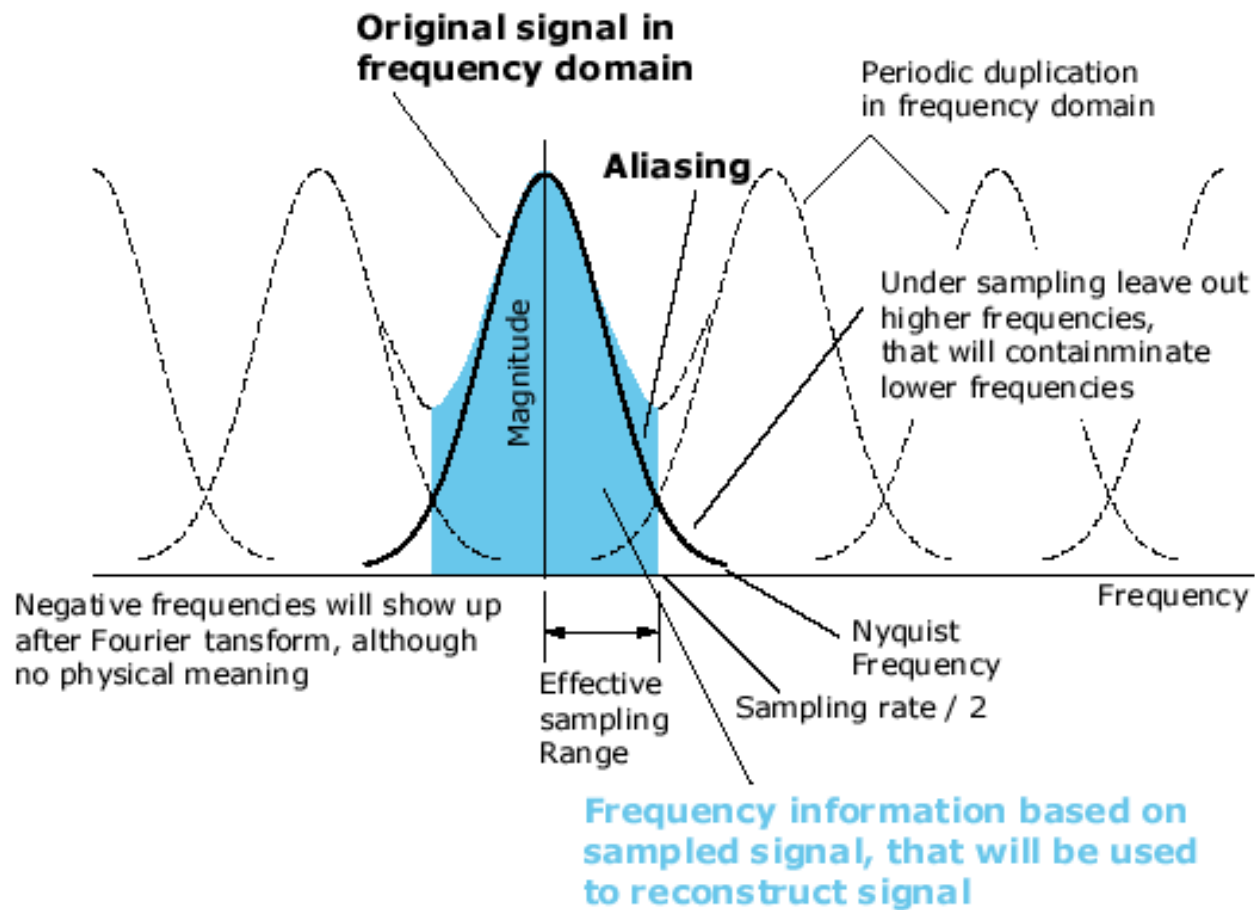
$$F(\omega) = \bar{F}(\omega) \quad |\omega| < \sigma$$

$$F(n\omega_0) \approx \begin{cases} \bar{F}(n\omega_0) & \gamma\alpha |n| \leq \frac{N}{2} \\ 0 & \gamma\alpha |n| < \frac{N}{2} \end{cases}$$

$$F(n\omega_0) \approx \begin{cases} \bar{F}(n\omega_0) & \gamma\alpha \quad 0 \leq n \leq \frac{N}{2} \\ \bar{F}(N\omega_0 + n\omega_0) & \gamma\alpha \quad |n| < \frac{N}{2} \end{cases}$$







### Under Sampling

