ACOUSTICAL METHODS FOR THE MONITORING OF THE MARINE ENVIRONMENT

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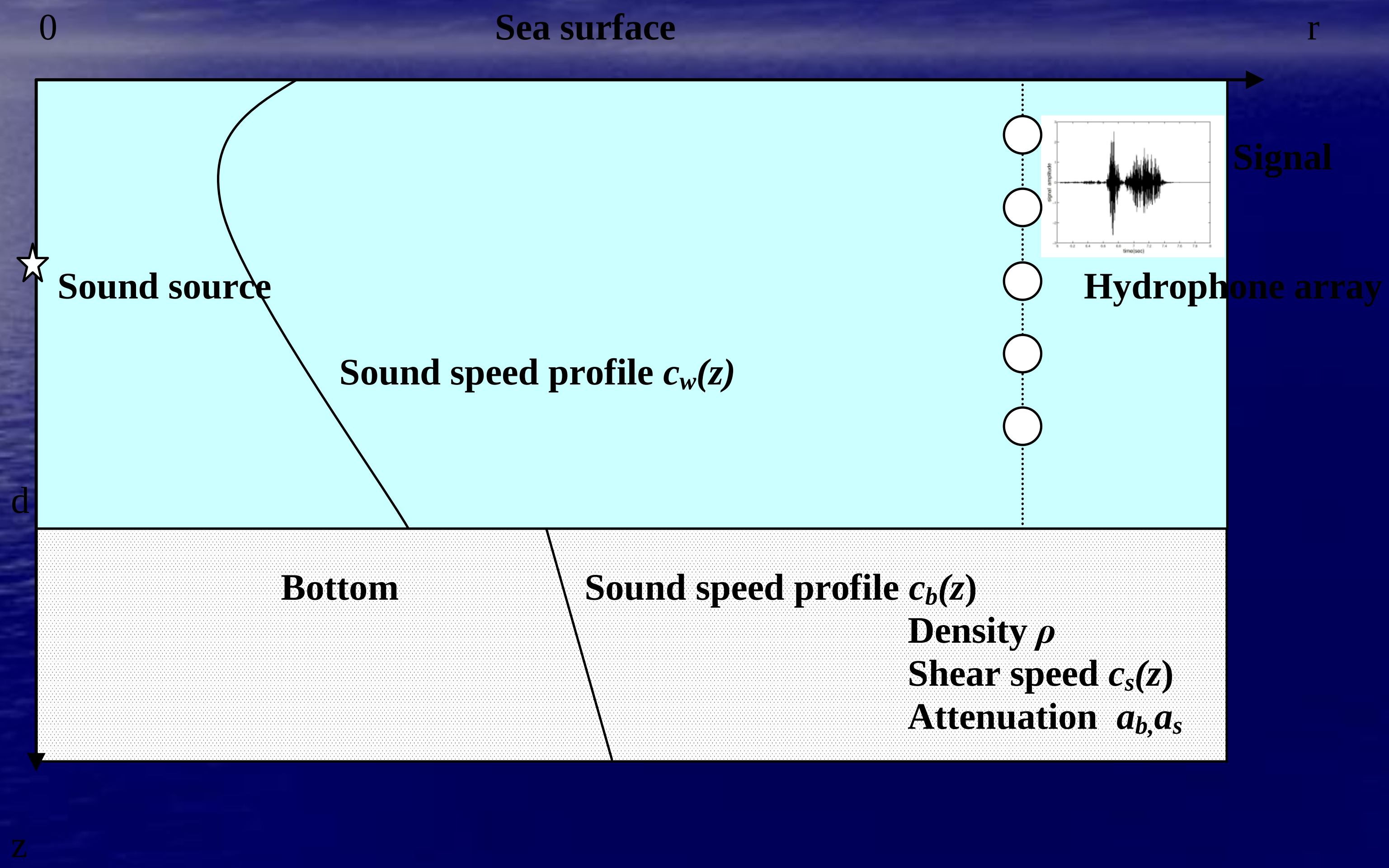
Part II

The geometry at a vertical slice



By representation theorem the acoustic pressure can be expanded in terms of the eigenfunctions of the Depth Problem

$$p(r,z;\omega)=\sum_{n=1}^{N}A_n(r;\omega)u_n(z;\omega)$$

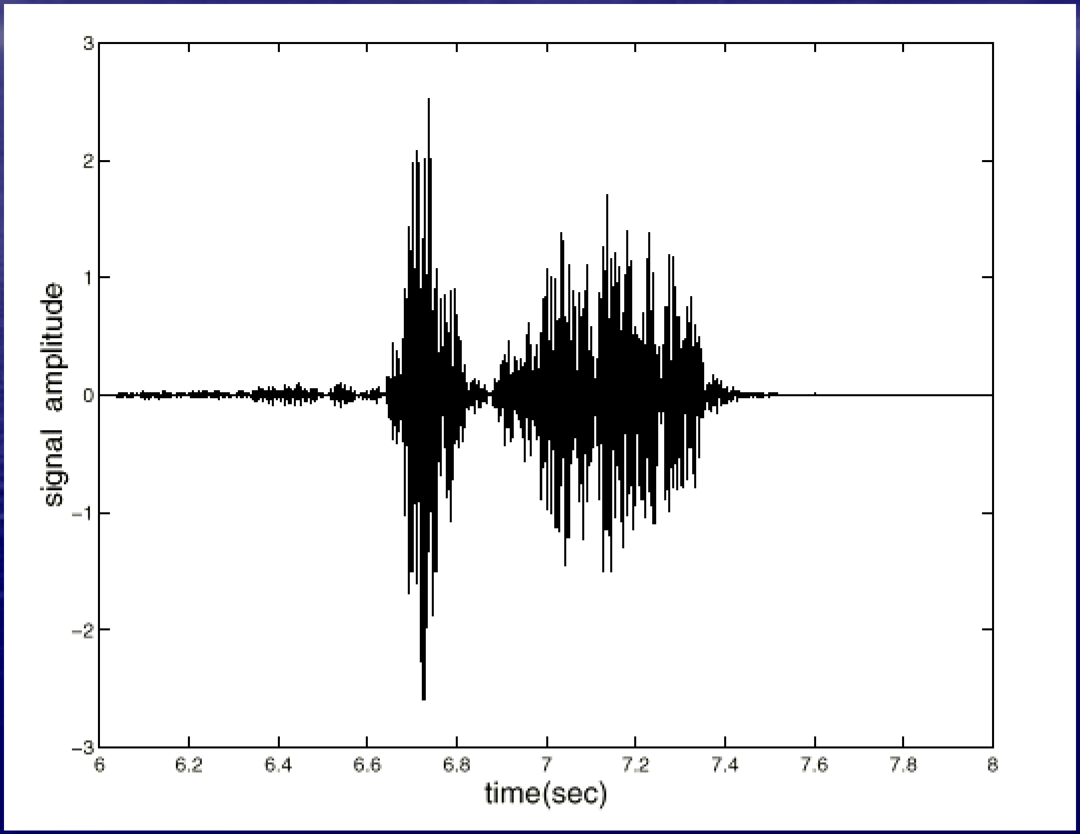
$$p(r,z;\omega)=\frac{i}{4\rho_1}\sum_{n=1}^{N}H_0^{(1)}(\kappa_n r)u_n(z_0;\omega)u_n(z;\omega)$$

Broad-band propagation

$$p'(r,z,\omega) = p(r,z;\omega)S(\omega)$$

The pressure in the time domain

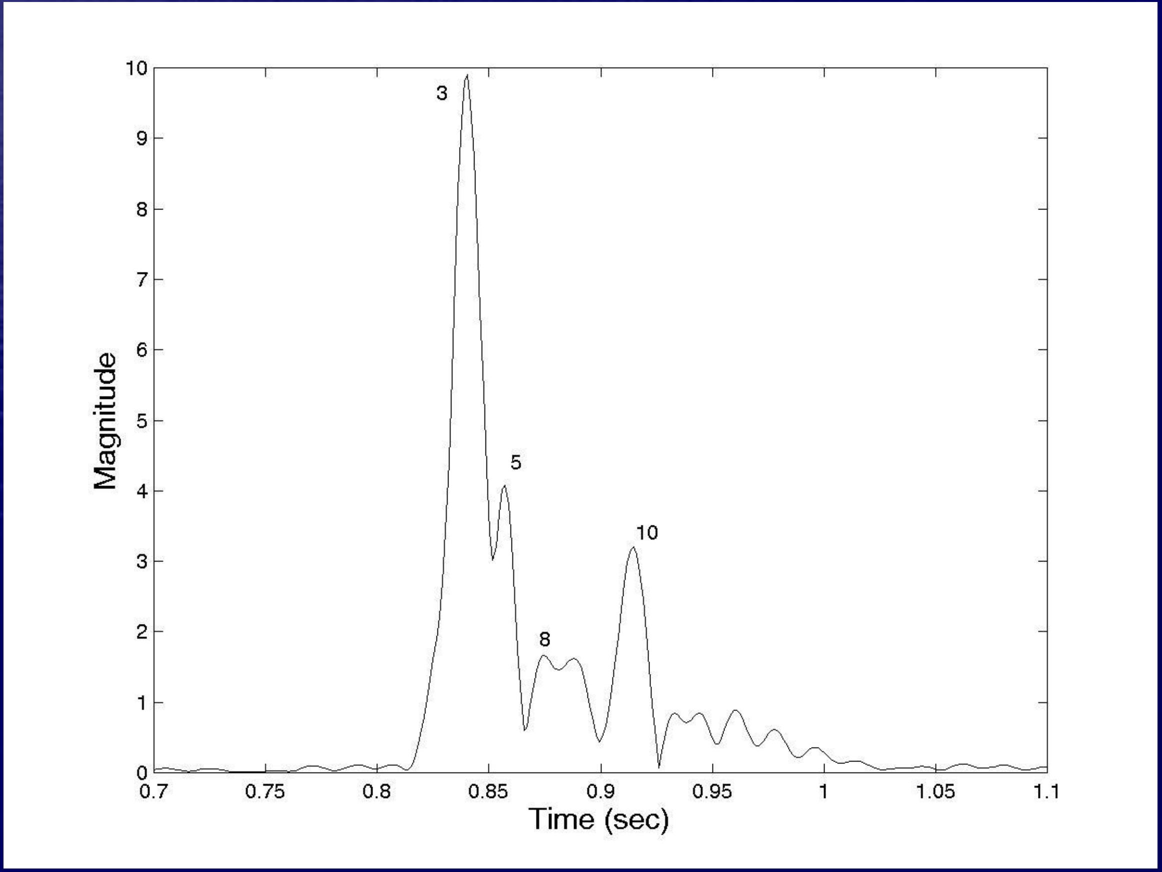
$$p(r,z;t) = \Im^{-1}\left[p'(r,z,\omega);\omega \to t\right]$$

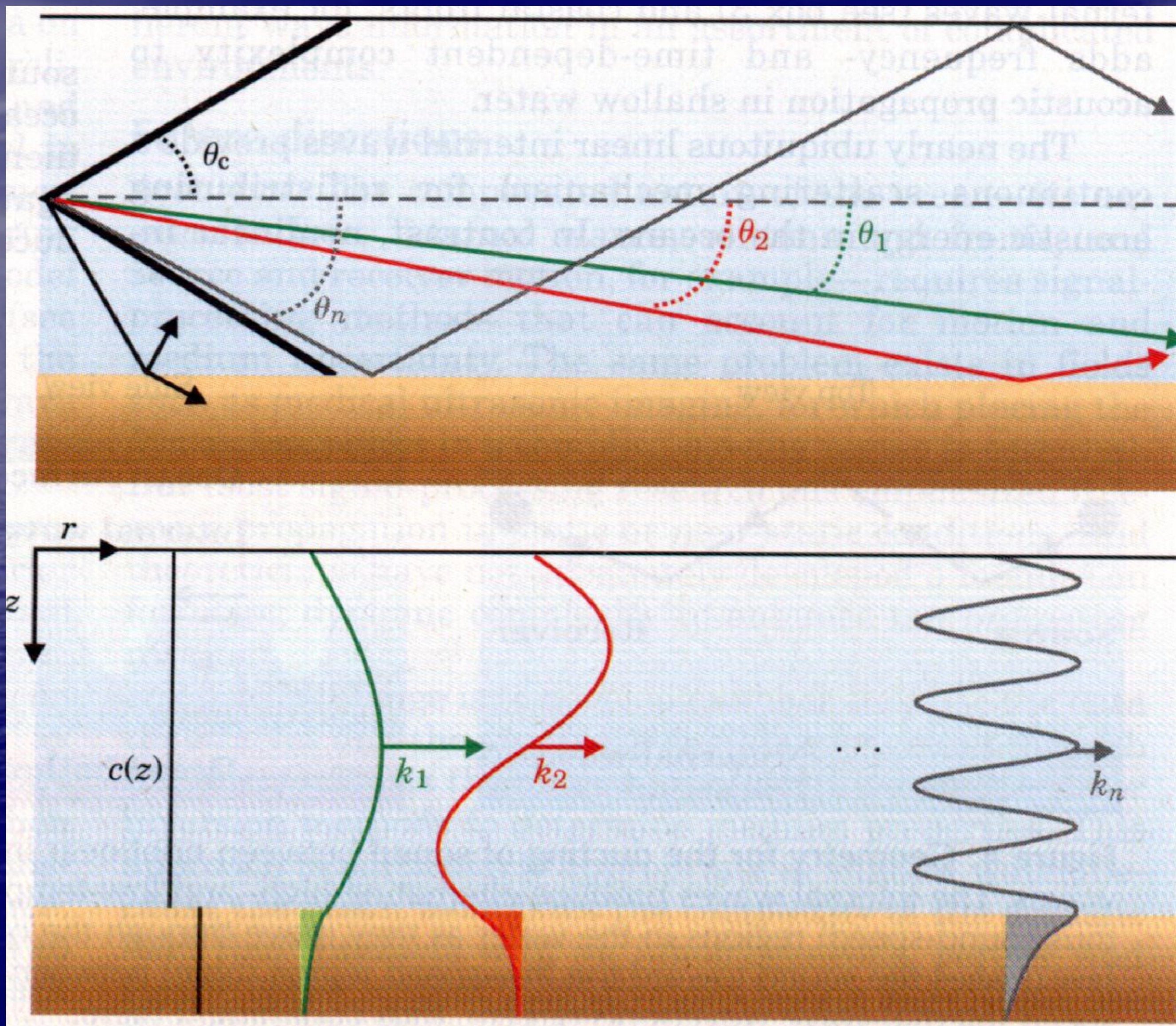


signal amplitude
3
2
1
0
−1
−2
−3
6
6.2
6.4
6.6
6.8
7
7.2
7.4
7.6
7.8
8
time(sec)

Each mode propagates at a different group velocity

$$v_{gn} = \left.\frac{\partial \omega}{\partial \kappa_n}\right|_{\omega_0}$$





$$v_{gn} = \left.\frac{\partial \omega}{\partial \kappa_n}\right|_{\omega_0}$$

$$\tau_n = \left.\frac{\partial \bar{\kappa}_n}{\partial \omega}\right|_{\omega_0} r$$

Methods of Ocean Acoustic Tomography

Linear methods

Ray Inversions

Linearizing with respect to a reference environment

$$c(\vec{x}) = c_0(\vec{x}) + \delta c(\vec{x})$$

$$\delta\tau_n = \int_{\Gamma_n} \frac{\delta c(\vec{x})}{c_0^{\,2}(\vec{x})} ds$$

$$d_n = \int_{\Gamma_n} \frac{v(\vec{x})}{c_0^{\,2}(\vec{x})} ds$$

Ray Inversions

$$c(\vec{x}) = c_0(\vec{x}) + \delta c(\vec{x})$$

$$\delta\tau_n = \int_{\Gamma_n} \frac{\delta c(\vec{x})}{c_0^{\,2}(\vec{x})} ds$$

Model ?

Determination of the eigen-ray path Γ_n

Parameters ?

$\delta c(\vec{x})$

Measurements ?

$\delta\tau_n \longrightarrow d_i$

Definition of an inverse problem of the form:

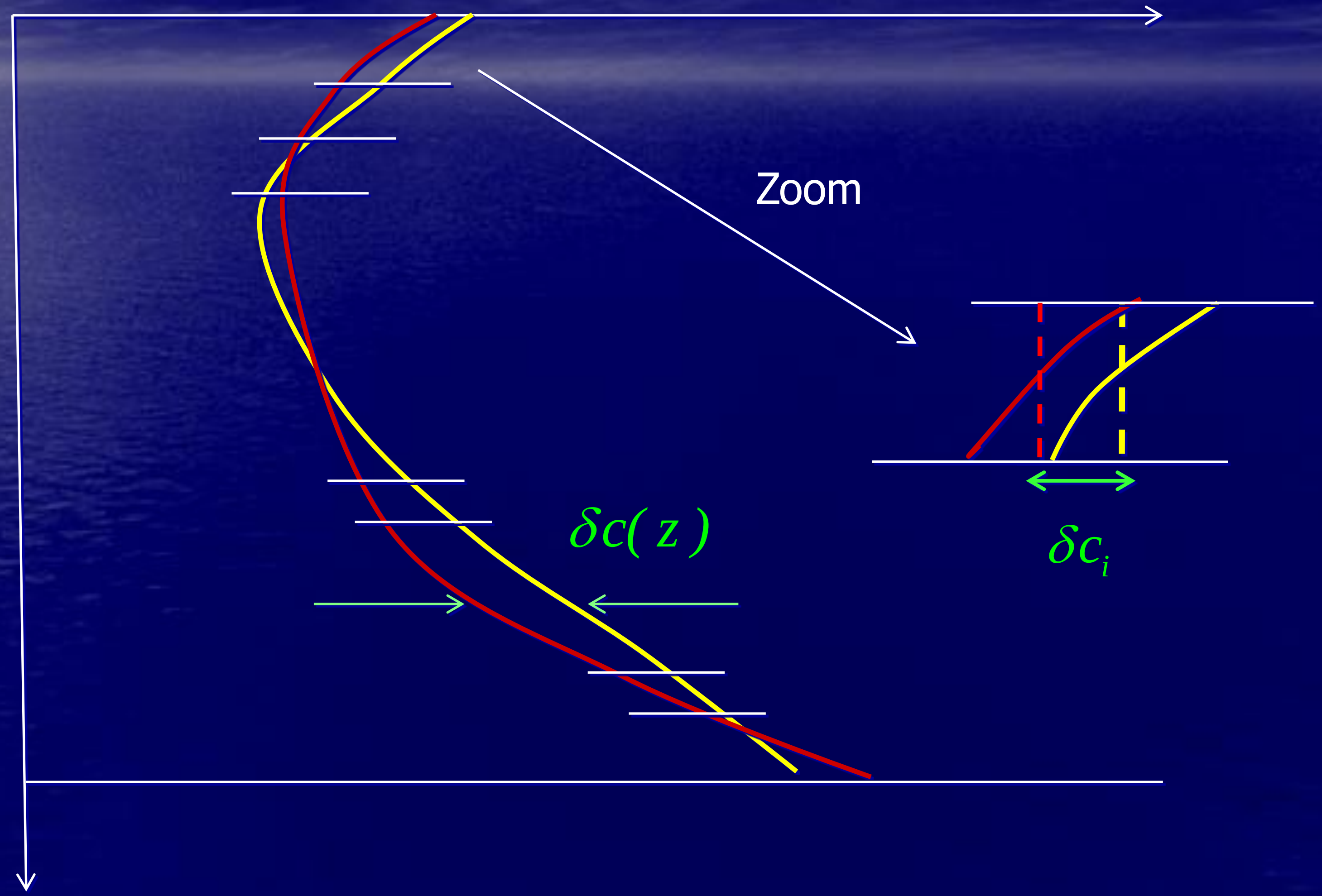
$$d_i = \int G_i(\vec{x}) m(\vec{x}) d\vec{x}$$

We need the theory to predict $G_i(\vec{x})$

After suitable discretization

$$d_i = \sum_{j=1}^{N} G_{ij} m_j$$

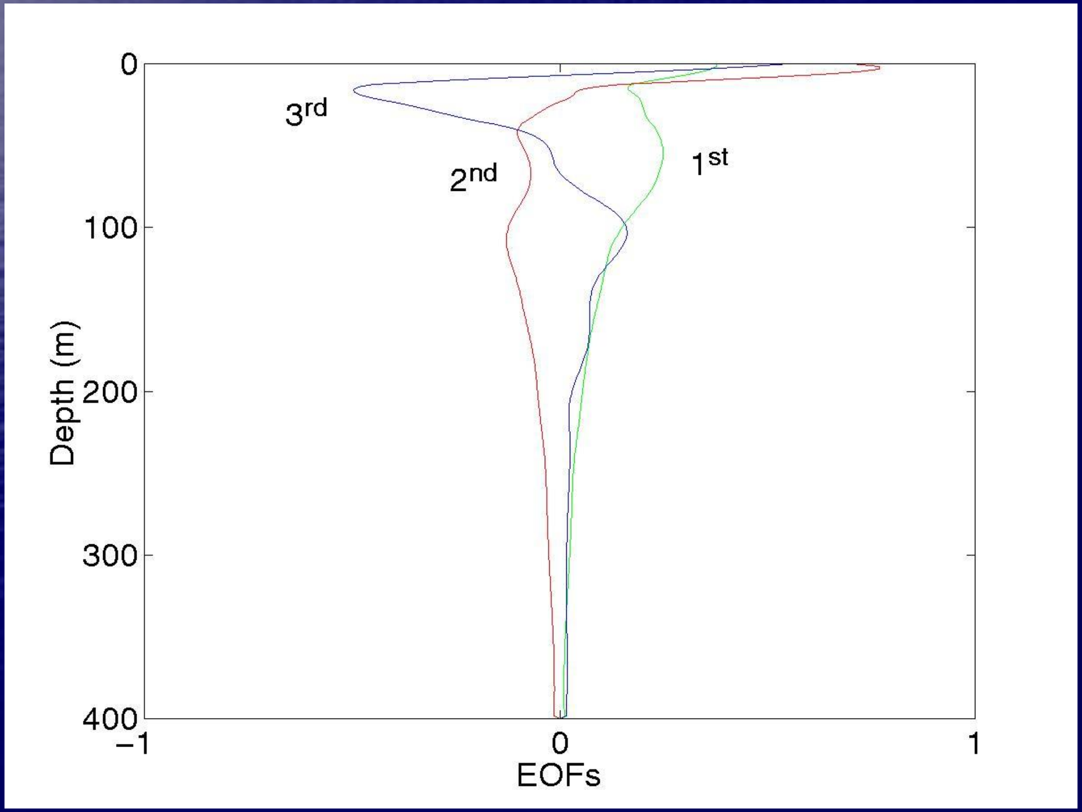
Discrete Linear Inverse Problem


Zoom
$\delta c(z)$
δc_i

Using Empirical Orthogonal Functions

$$\delta c(z) = \sum_{\ell} \theta_{\ell} f_{\ell}(z)$$

$$\delta c(r,z) = \sum_{\ell} \theta_{\ell}(r) f_{\ell}(z)$$



Additional a-priori information !

Necessary Assumption :

Ray arrivals can be resolved and identified !

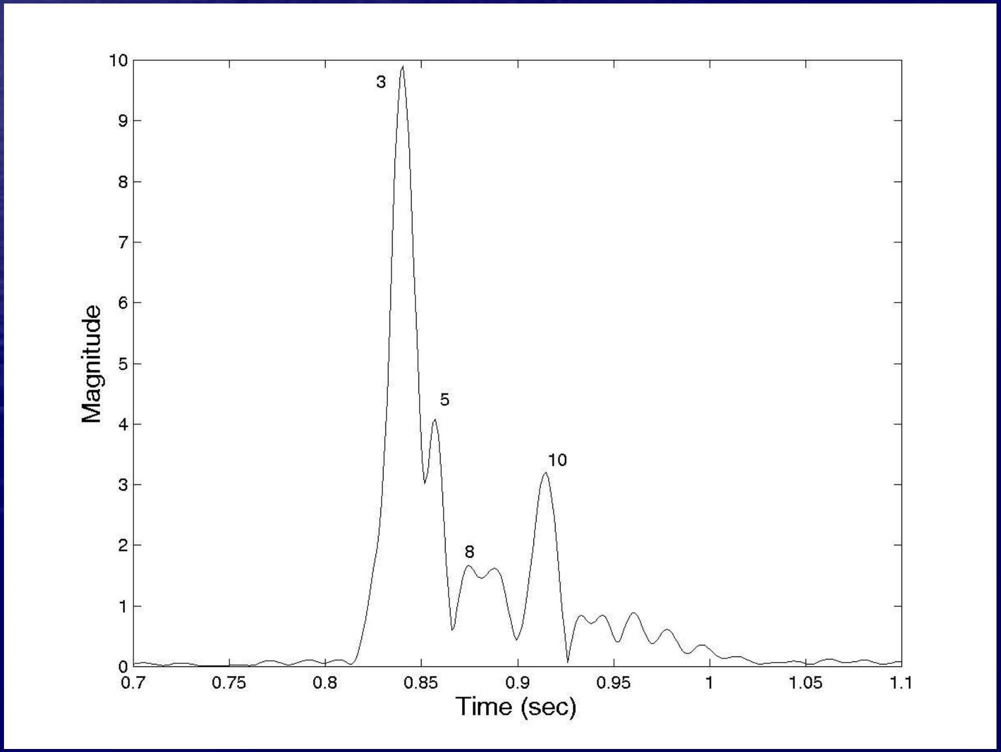
If not

Check for modal arrivals !!

Modal Travel-Time Inversions

If modal arrivals can be resolved and identified !!

Modal packets travel with their respective group velocity $v_{gn} = \left.\frac{\partial \omega}{\partial \kappa_n}\right|_{\omega_0}$



Linearizing with respect to a reference environment we get an expression of the travel time differences in terms of sound speed differences

$$\tau_n = \left.\frac{\partial \overline{\kappa}_n}{\partial \omega}\right|_{\omega_0} r \qquad \delta\tau_n(r) = \left.\frac{\partial \delta\overline{\kappa}_n r}{\partial \omega}\right|_{\omega_0}$$

From theory and assuming low order perturbations

$$\delta\overline{\kappa}_n = \frac{1}{\kappa_n^0}\int_D \frac{1}{\rho(\vec{x})}\left|u_n^0(\vec{x})\right|^2 \frac{k^{02}\delta c(\vec{x})}{c_0(\vec{x})} d\vec{x}$$

By suitable discretization or expression of the sound speed difference in terms of EOFs we define a linear system of the form

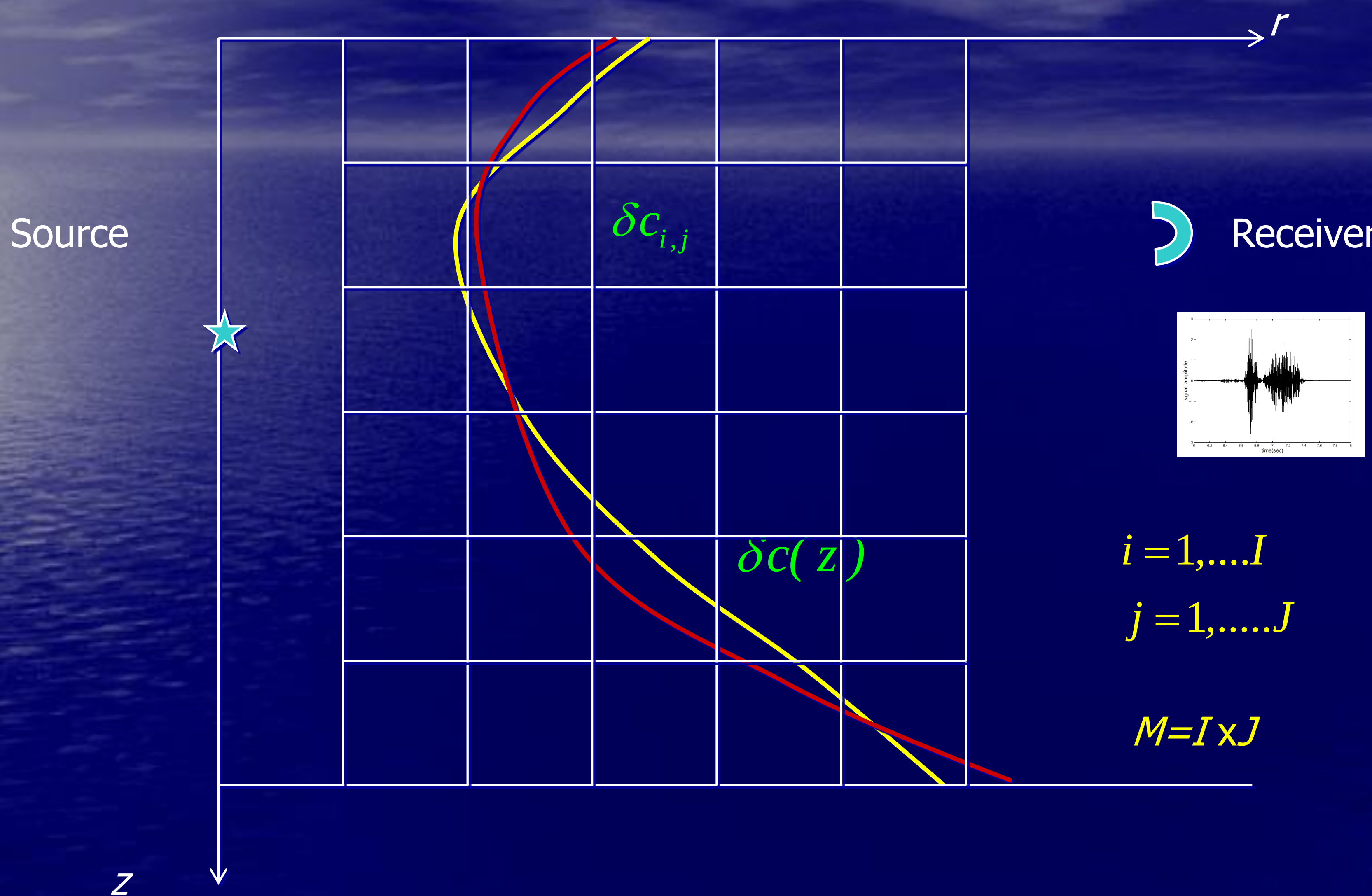
$$\delta\tau_n(r) = \frac{\partial \delta\bar{\kappa}_n r}{\partial\omega}\bigg|_{\omega_0} = \sum_{i=1}^{I}\sum_{j=1}^{J}\left(\frac{\partial Q^0_{n,i,j}}{\partial\omega}\bigg|_{\omega_0}\right)\delta c_{i,j}\Delta z\Delta r$$

$$d_n \rightarrow \delta\tau_n$$

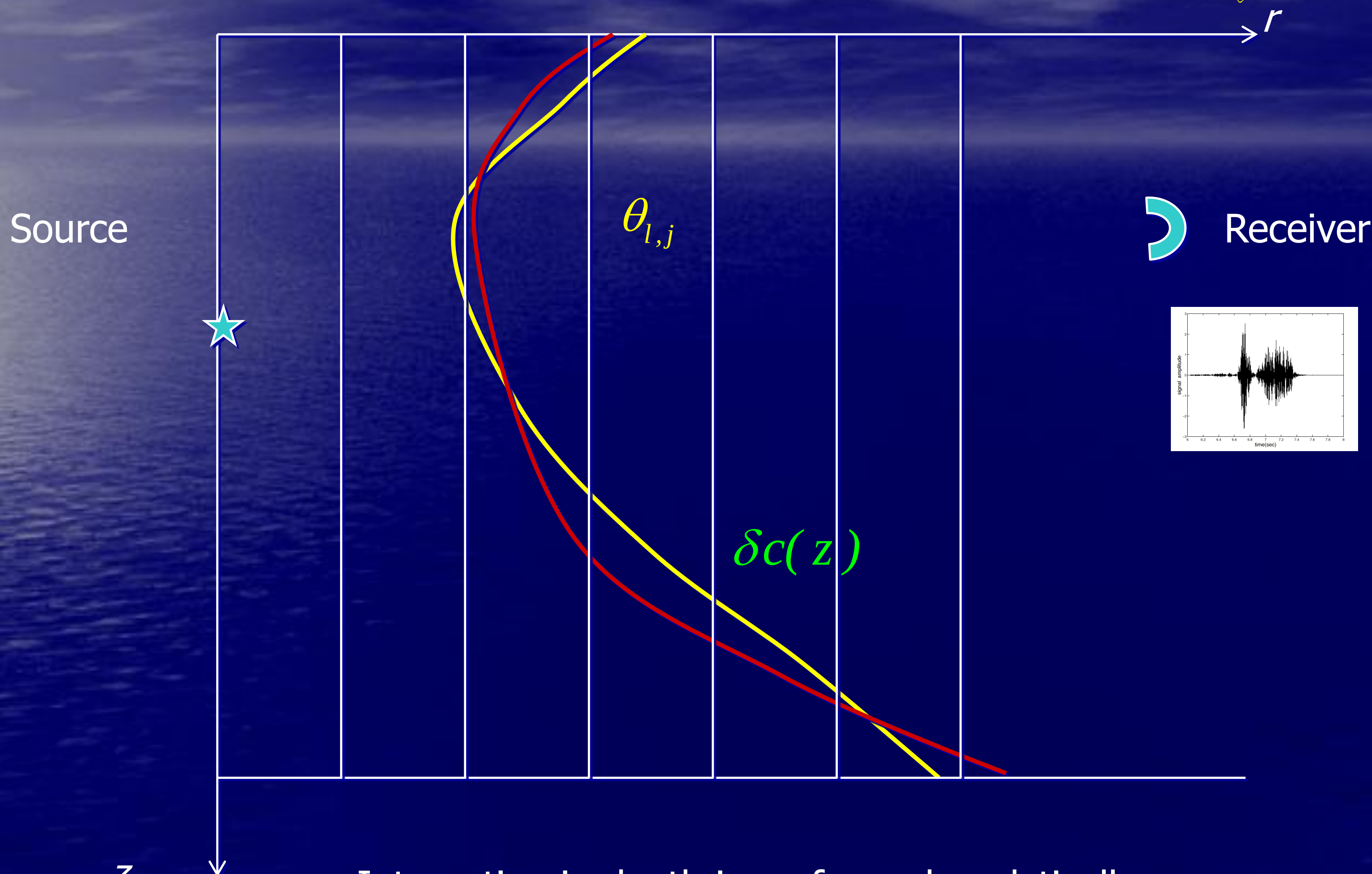
$n=1,\ldots\ldots N$

$$d_n = \sum_{k=1}^{M} G_{nk} m_k$$

N : number of identifiable modal arrivals


r
Source
Receiver
$\delta c_{i,j}$
$\delta c(z)$
$i = 1,....I$
$j = 1,.....J$
$M=I \times J$
z

Using EOFs $\delta c(r,z)=\sum_{\ell}\theta_{\ell}(r)f_{\ell}(z) \Rightarrow \delta c(r_j,z)=\sum_{\ell}\theta_{\ell,j}f_{\ell}(z)$



Integration in depth is performed analytically

Advantages of linear methods :

- Easy to implement
- Computationally efficient and fast

Disadvantages of linear methods :

- Require good a-priori knowledge of the environment
- The kernel matrix sometimes is characterized by bad condition

Non linear methods

Measurements (Data)

- We need to define the "observables"
- They have to be easily measured or inferred after some post-processing of the measured data.
- They should be sensible to changes of the environmental parameters.

Measurements (Data)

- Measurements are always the acoustic field either at a single hydrophone or on an array of hydrophones.
- The data to be used in the inverse problem can be

Measurements (Data)

– The field at a specific frequency or at a frequency band (acoustic pressure)
– The ray or modal arrivals (time of arrival)
– The dispersion curves (characteristic of modal propagation)
– The modal “phase” for each one of the propagating modes
– Statistical or probabilistic features of the signal

Matched Field Inversions

If a vertical array of hydrophones is available, the full-field can be used for inversions

Matching at a single frequency :

Perform Discrete Fourier Transform to obtain the field in the frequency domain at each one of the *N* hydrophones

$$F_n(\vec{x};\omega) = \sum_{k=0}^{K} p_n(\vec{x};t_k)w(t_k)e^{-i\omega t_k}$$

- Collect measurements of the acoustic field in $N \geq 1$ hydrophones
- Select a propagation model and use it to compute the acoustic field for candidate model parameters
- Use a suitable processor to cross-correlate the measured with modelled/predicted replica fields and search for the *highest* correlation

Optimization Process !!

Measured in *N* hydrophones

$$\mathbf{F} = (F_1, F_2, \ldots F_N)^T$$

Modeled for *N* hydrophones

$$\hat{\mathbf{F}} = (\hat{F}_1, \hat{F}_2, \ldots \hat{F}_N)^T$$

$C = \langle \mathbf{F}\mathbf{F}^{+} \rangle$

Compare

$w = \langle \hat{\mathbf{F}}\hat{\mathbf{F}}^{+} \rangle$

$$L(\mathbf{m}) = \mathbf{w}^{+} C \mathbf{w}$$

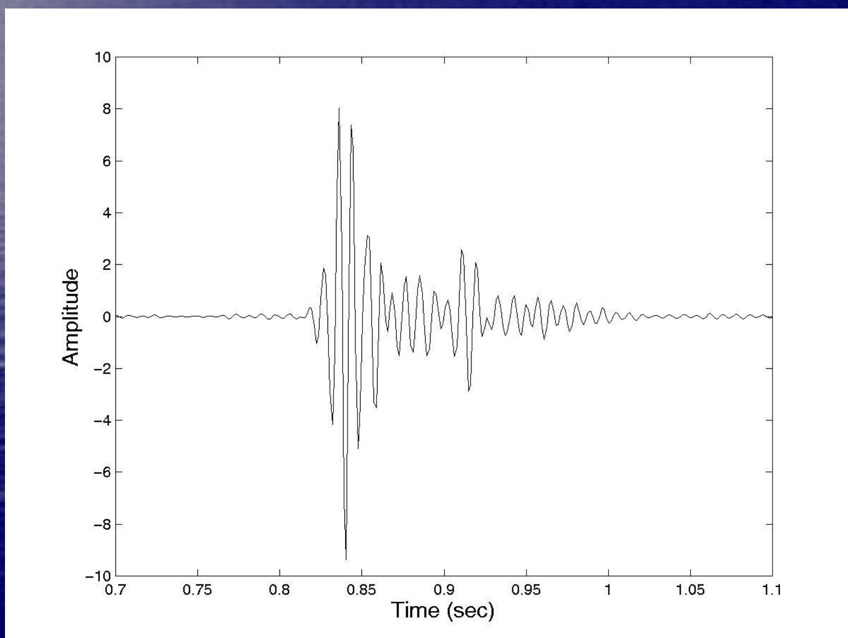
Bartlett processor

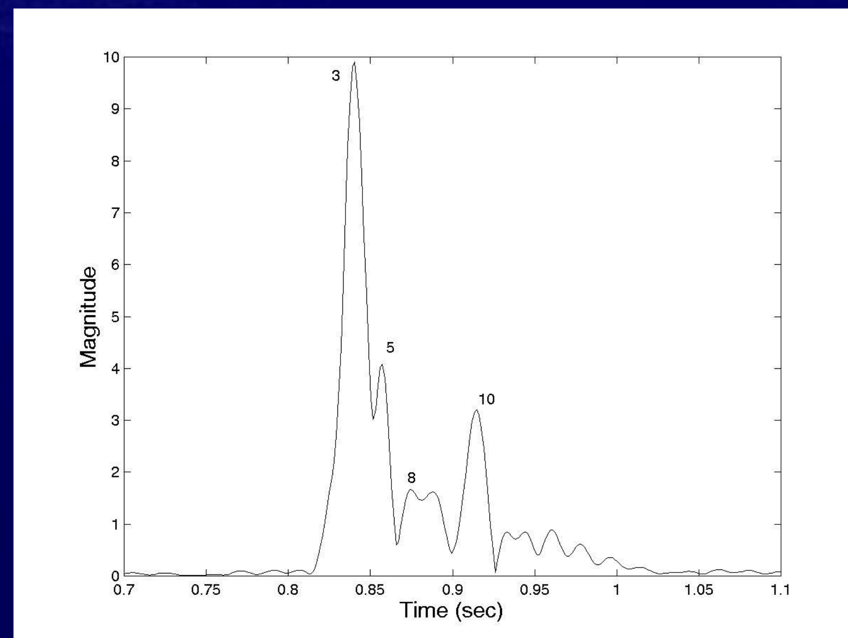
$\mathbf{m}^{est}$ will be the vector maximizing the Processor

For normalized data, maximum of the $L(\mathbf{m})$ is 1

Inversions based on travel time

Necessary Condition : Identification of the type of signal peaks:





Not always easy !

Inversions based on modal travel time

Hypothesis : Identification of N modal arrivals

Replica fields are produced based on the pre-specified search space of the model parameters.

Use of the group velocities calculated for the modes of the replica fields to identify the modeled arrivals of the propagating modes ,

$$\widehat{t}_n, n = 1,\ldots N$$

Define the actual modal arrivals t_n and compare the corresponding times with the arrival times of the replica signals.

$$\delta t_n = t_n - \widehat{t}_n$$

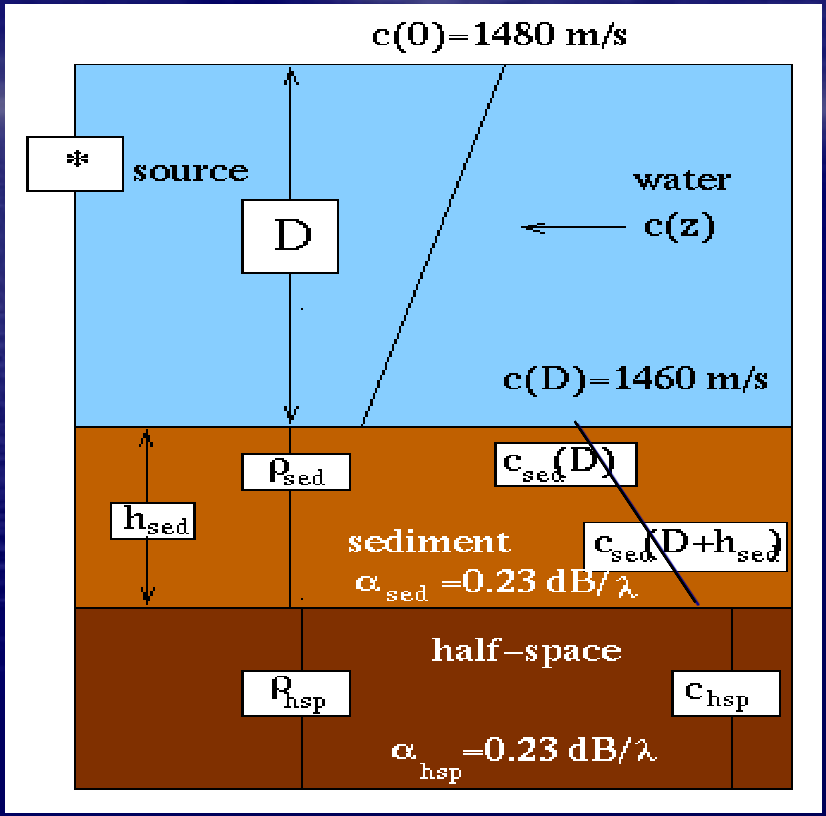
The inverse problem is formulated as an implicit non-linear problem

$$\delta t_n = g(\mathbf{m})$$

Define an appropriate processor *P* to solve the inverse problem as an optimization process

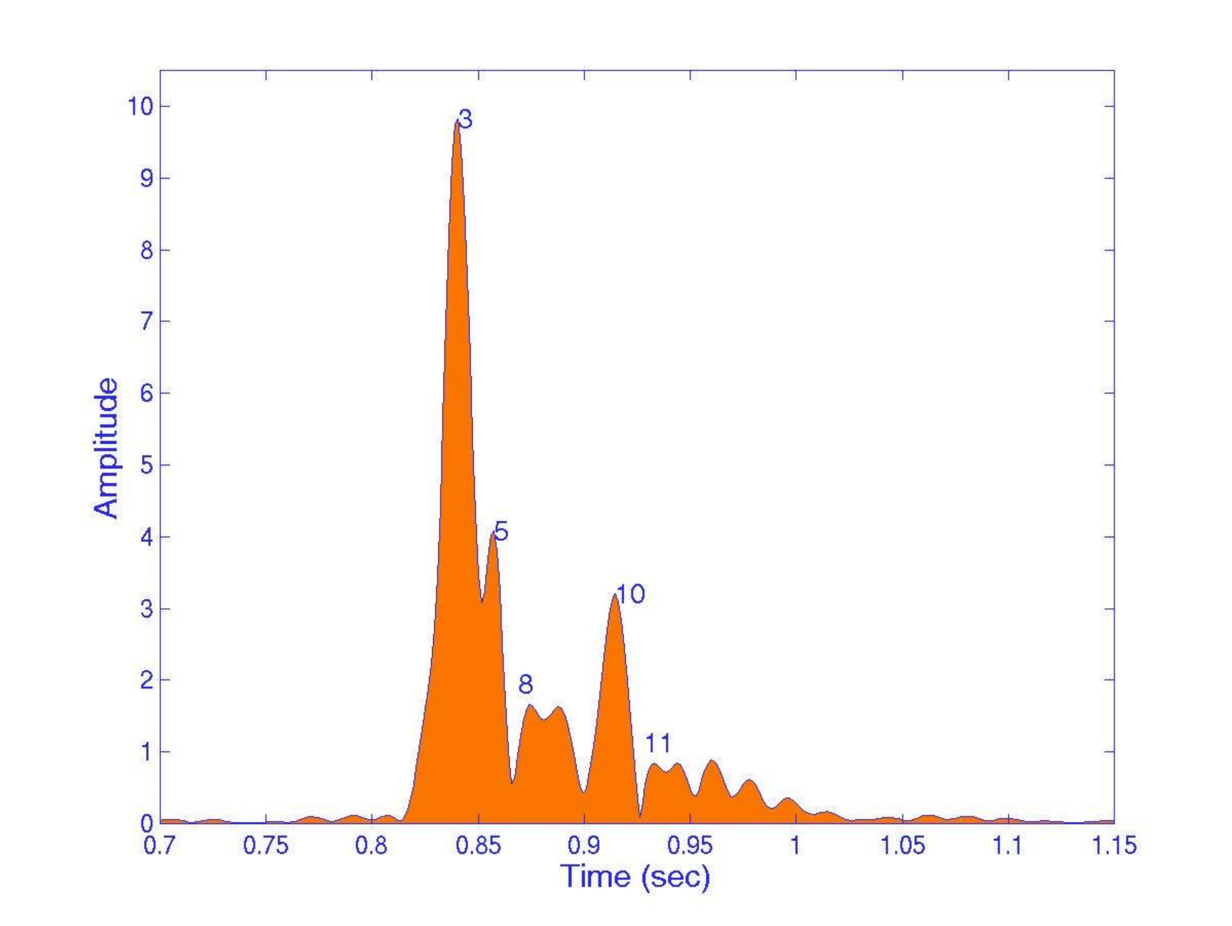
$$P(\delta t_n) = \sqrt{\frac{1}{N}\sum_{n=1}^{N}\delta t_n{}^2}$$

Test case

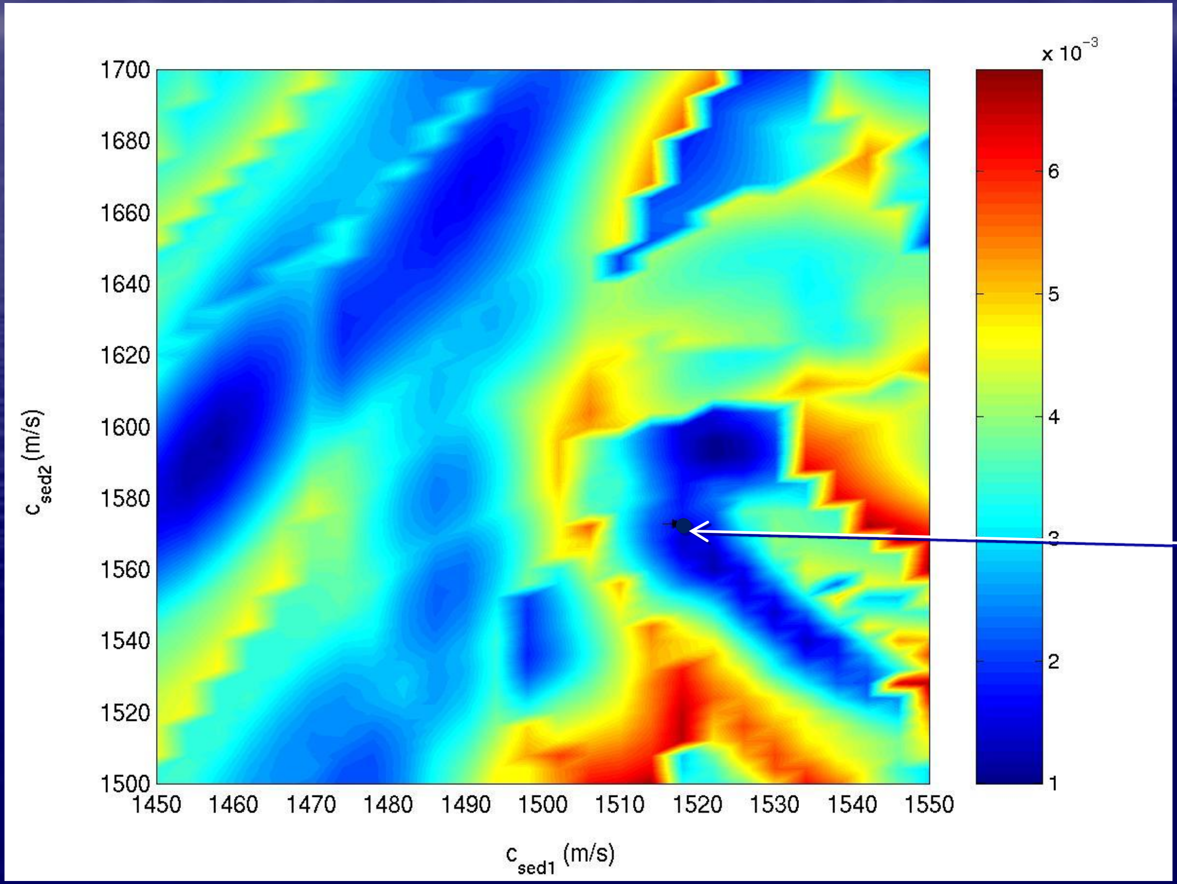


A description of the test environment based on the benchmark case WAa of the Vancouver 97 Workshop

Arrival pattern (f_0 =112 Hz)



The cost function over the whole search space for the sediment properties



Minimum of the cost function

Inversion results

Parameter	Actual	Reference	Recovered
$C_{sed}(D)$	1516,2	1506,1	1517,44
$C_{sed}(D+h)$	1573,2	1539,5	1569,84

A systematic search over the search space has been performed

No specific optimization algorithms have been applied