ACOUSTICAL METHODS FOR THE MONITORING OF THE MARINE ENVIRONMENT

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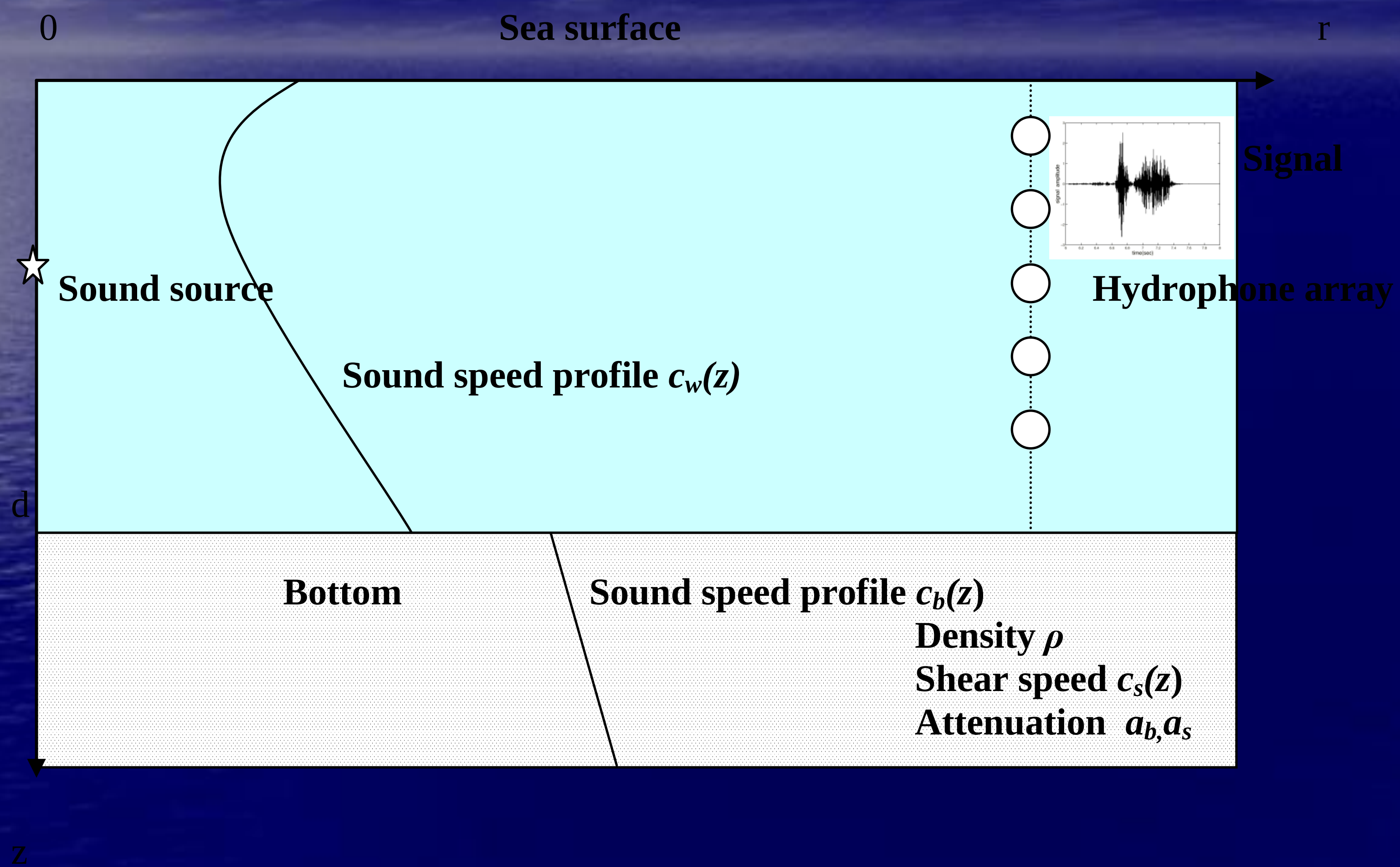
Part III

The geometry at a vertical slice



By representation theorem the acoustic pressure can be expanded in terms of the eigenfunctions of the Depth Problem

$$p(r,z;\omega)=\sum_{n=1}^{N}A_n(r;\omega)u_n(z;\omega)$$

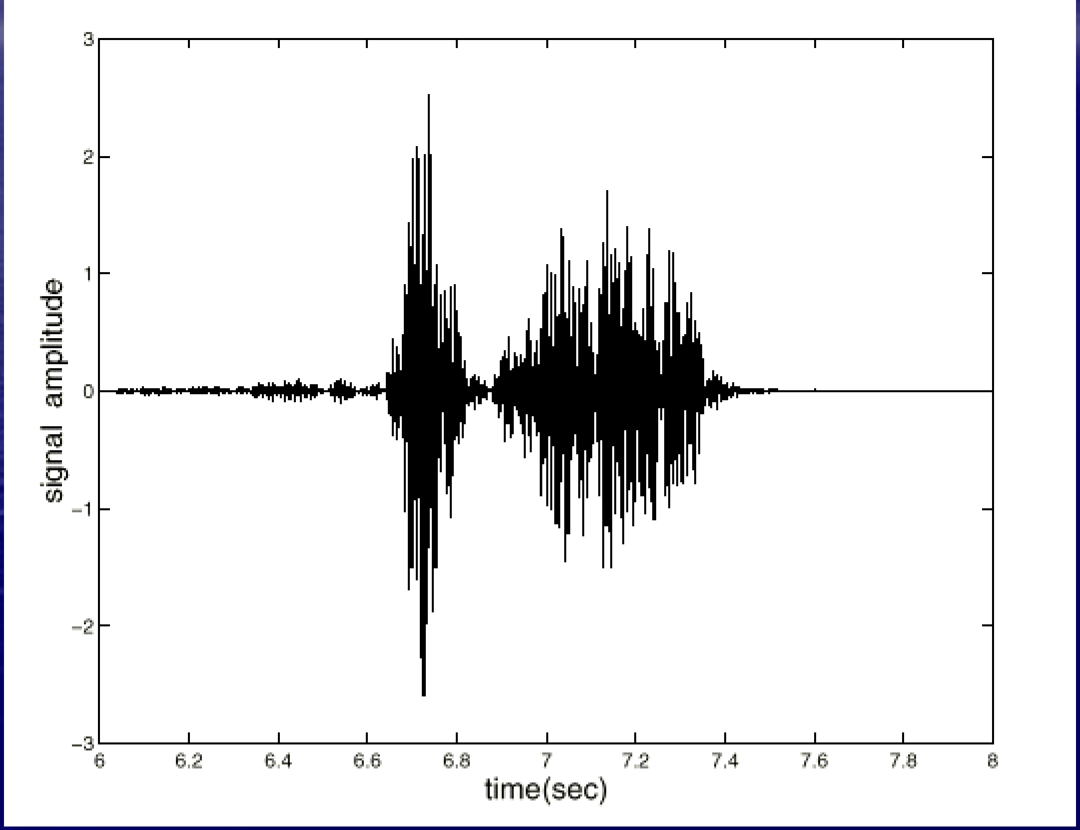
$$p(r,z;\omega)=\frac{i}{4\rho_1}\sum_{n=1}^{N}H_0^{(1)}(\kappa_n r)u_n(z_0;\omega)u_n(z;\omega)$$

Broad-band propagation

$$p'(r,z,\omega) = p(r,z;\omega)S(\omega)$$

The pressure in the time domain

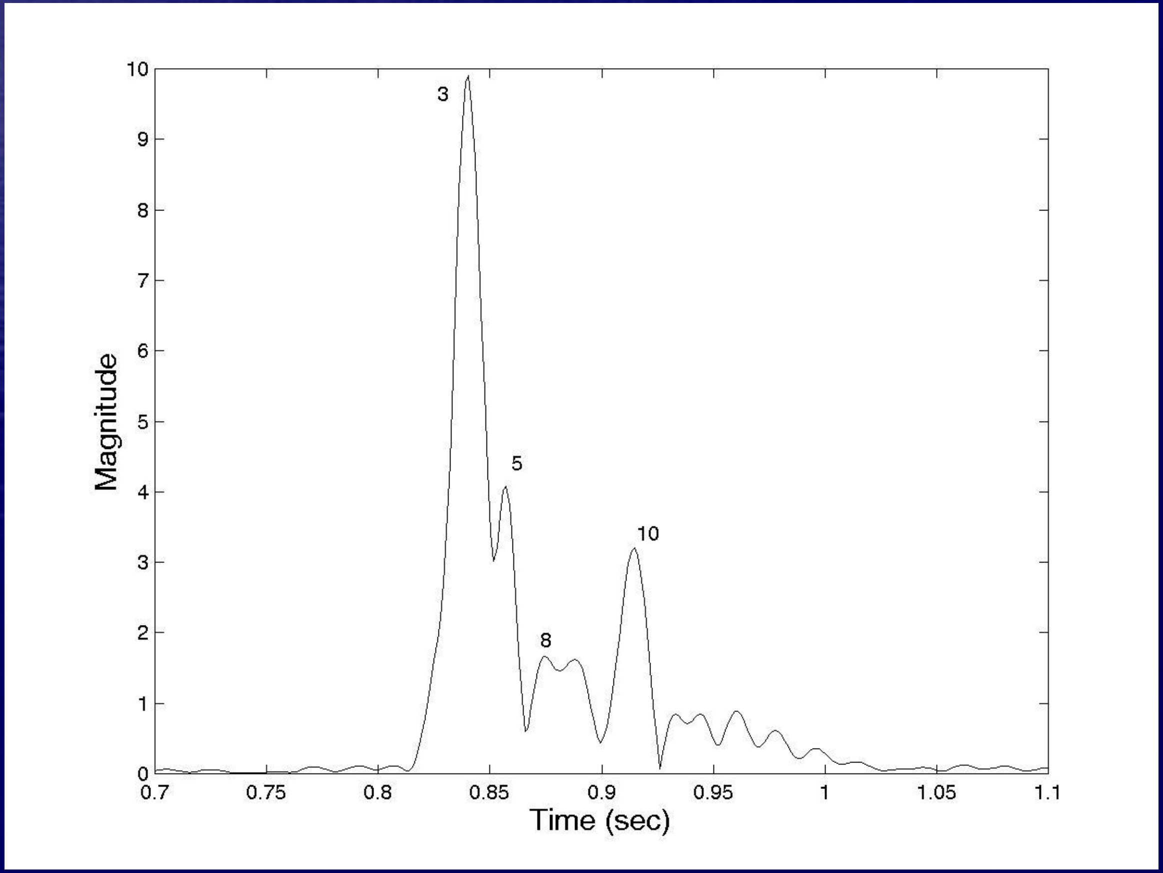
$$p(r,z;t) = \Im^{-1}\left[p'(r,z,\omega);\omega \to t\right]$$



signal amplitude
time(sec)
3
2
1
0
−1
−2
−3
6
6.2
6.4
6.6
6.8
7
7.2
7.4
7.6
7.8
8

Each mode propagates at a different group velocity

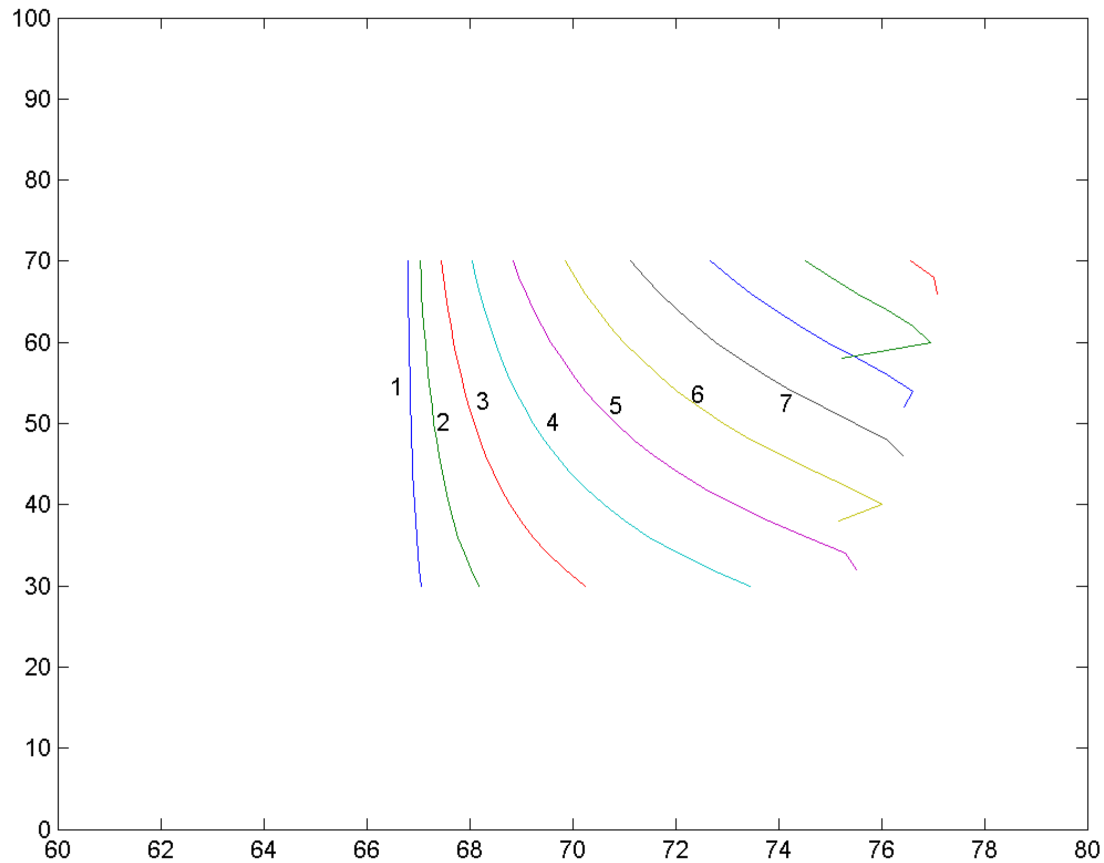
$$v_{gn} = \left.\frac{\partial \omega}{\partial \kappa_n}\right|_{\omega_0}$$



Using Dispersion Curves

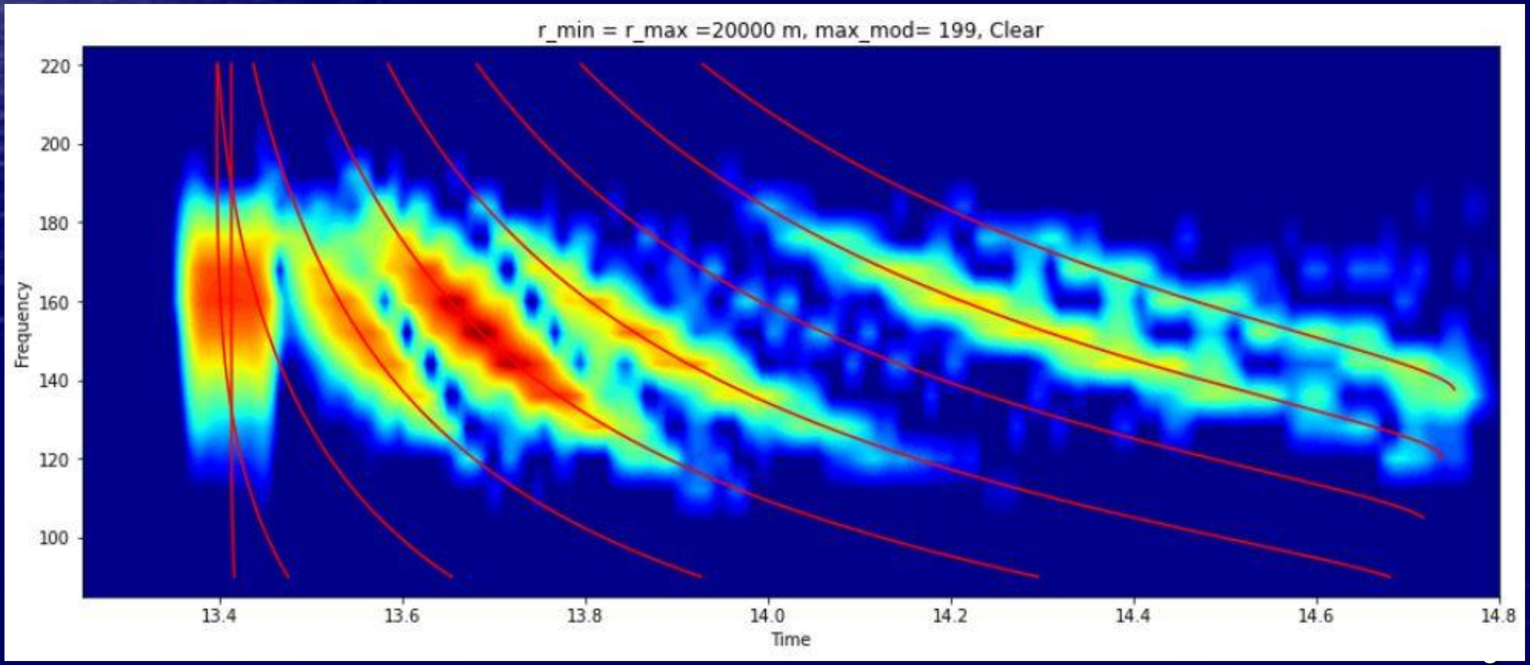
When the identification of modes at the central frequency is difficult, due to the peculiarity of the dispersion characteristics of the medium, the full dispersion curves can be used to resolve the possible ambiguity or to be used as alternative observables for inversion.

Dispersion curves show the arrival time as a function of frequency for each propagating mode


100
90
80
70
60
50
40
30
20
10
0
60
62
64
66
68
70
72
74
76
78
80
1
2
3
4
5
6
7

By 2-D Fourier transform or Wavelet transform in the measured signal, we can obtain the scalogram (spectrogram) of the received signal which indicates the propagation of the modal packets in the environment.

Ideally, the dispersion curves should be at the peaks of the propagating modesl

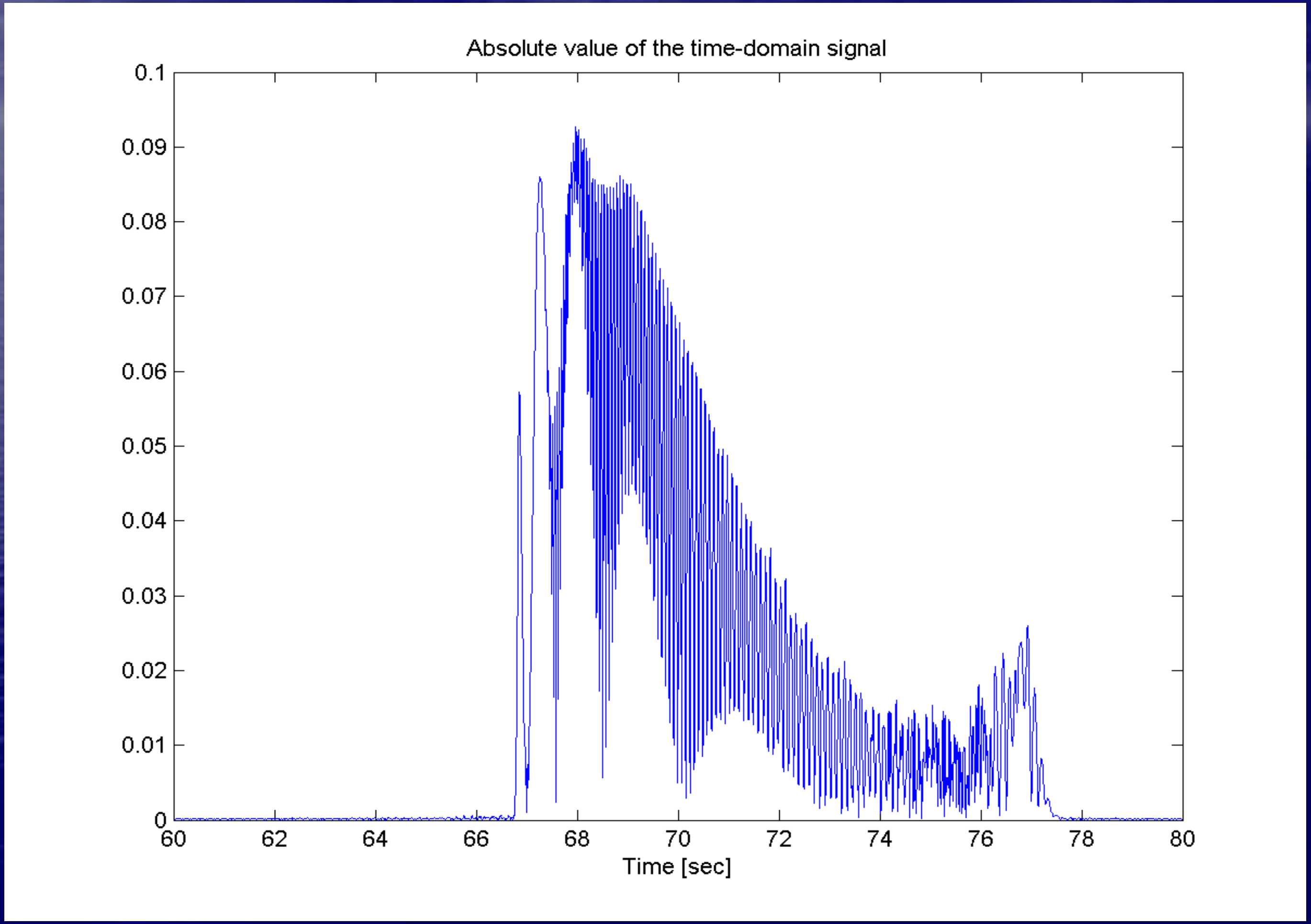


The inversion scheme

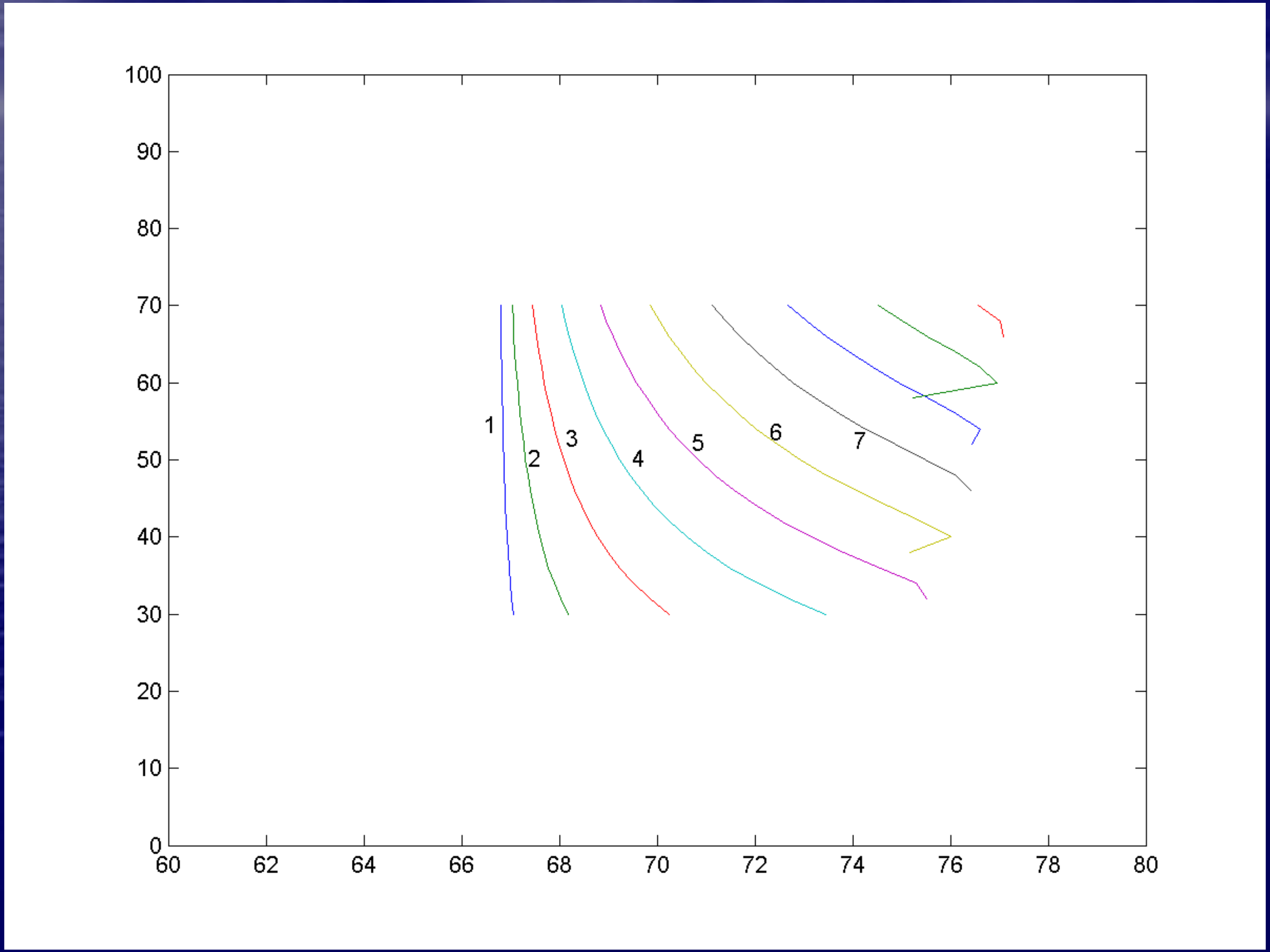
- Calculate the dispersion curves of the measured signal using an appropriate 2-D transform (measured dispersion curves)
- Calculate the dispersion curves for each one of elements of a search space (estimated dispersion curves).
- Define an appropriate cost function to measure the difference between measured and estimated dispersion curves.
- Define the solution of the inverse problem as the environment yielding the minimum distance between estimated and measured dispersion curves.
- All methods applied to standard optimization schemes (Simulated Annealing, Genetic Algorithms, Neural Networks..) can in principle be used in this context as well to optimize the search procedures)

(Potty and Miller J.C.A. 2000, J.A.S.A. 2000, Bonell et al. JASA 2012)

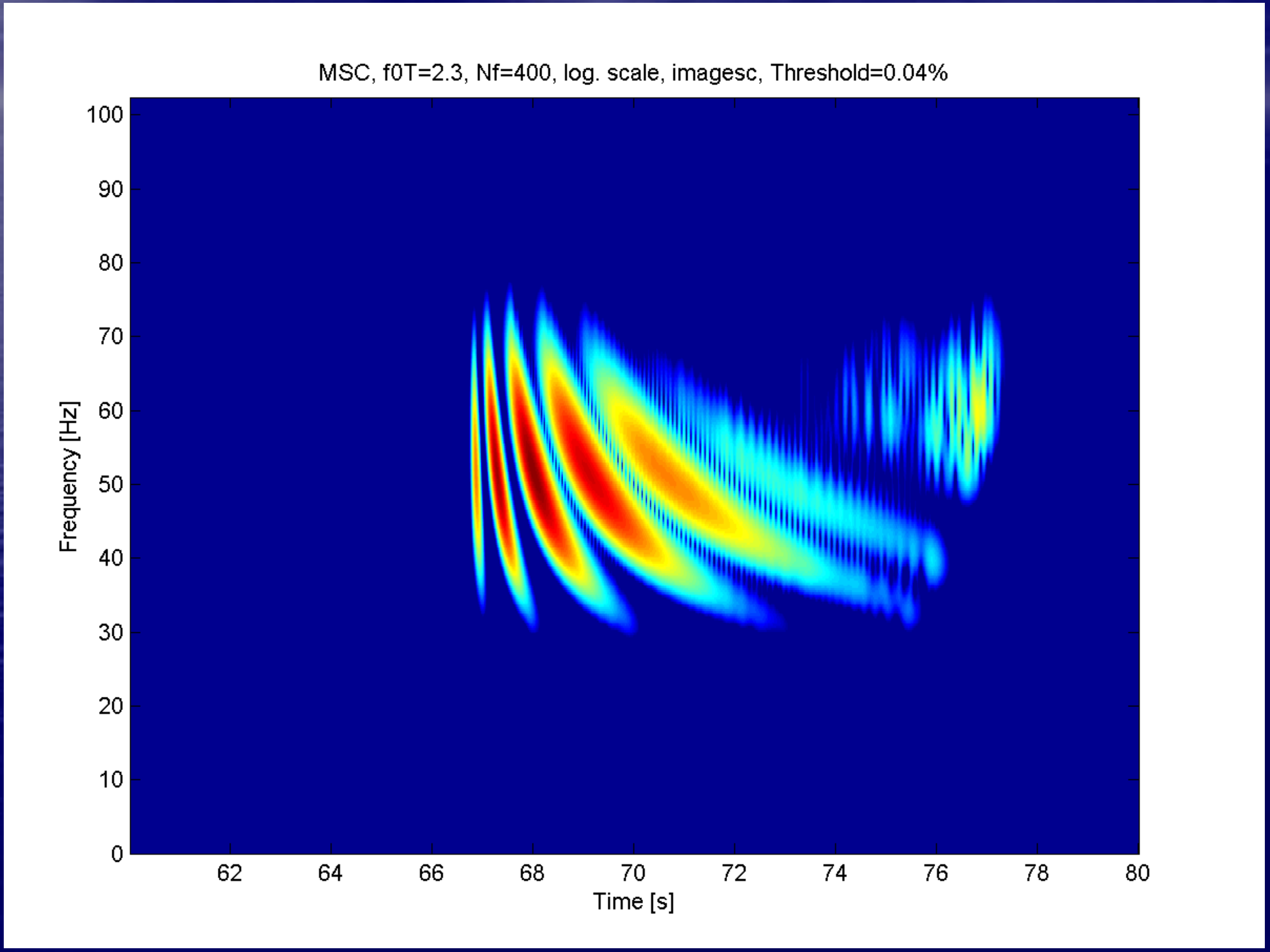
The arrival pattern



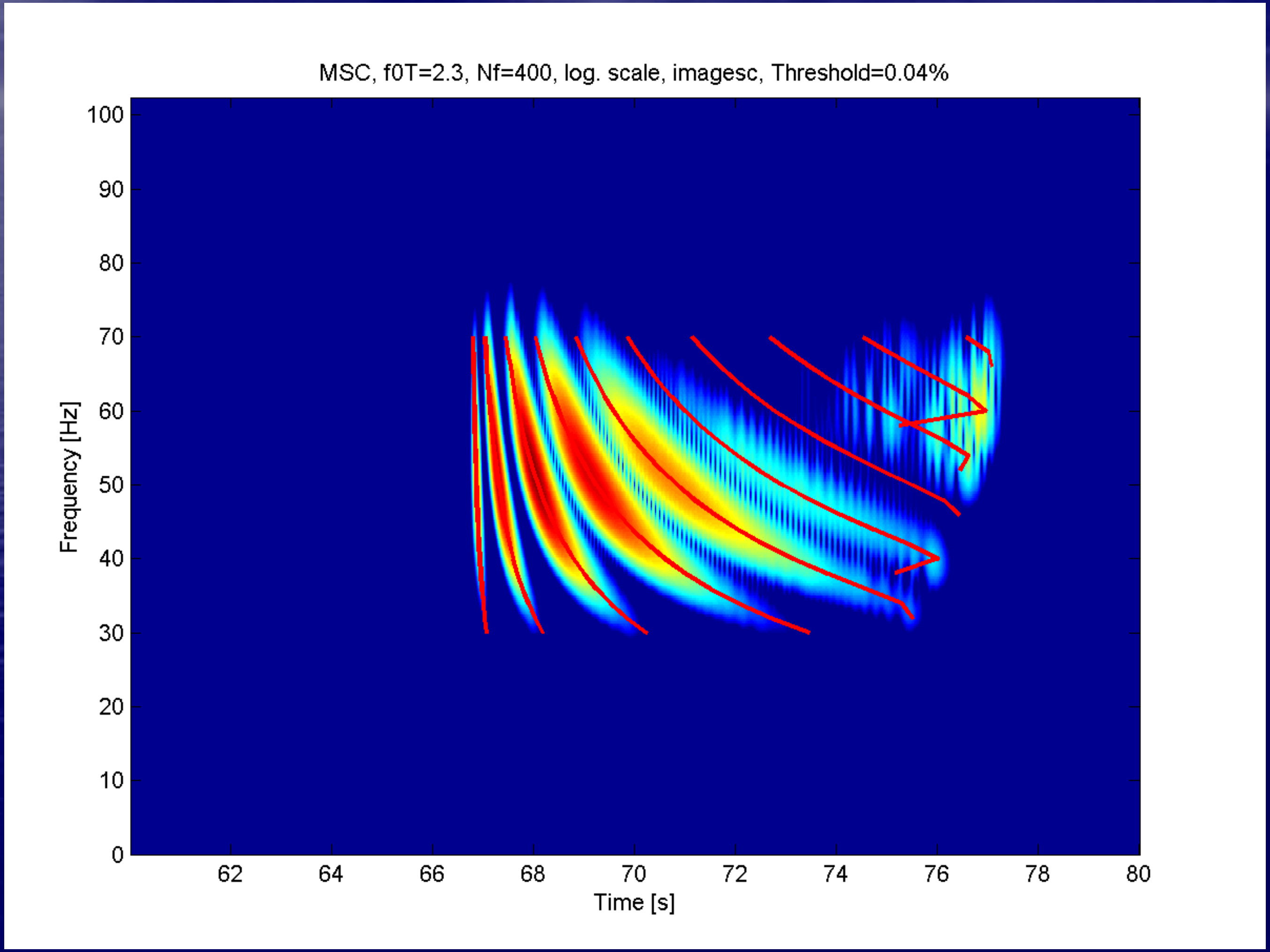
The dispersion curves



The Scalogram using Morlet's wavelet



The Scalogram using Morlet's wavelet



Disadvantages of traditional observables

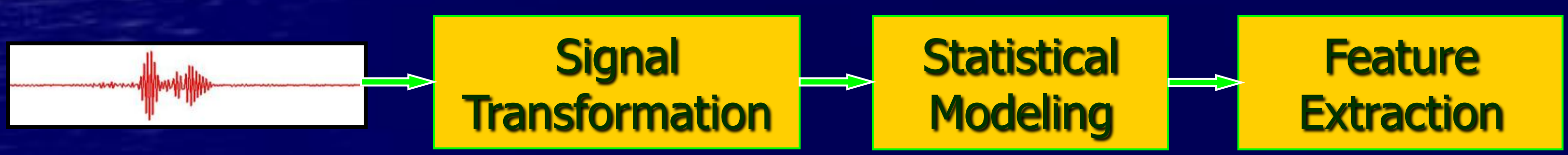
- They are not always identified (e.g. ray arrivals, modal arrivals, modal phase)
- The functional relationship between the traditional observables and the recoverable parameters sometimes is not sensitive enough to the change of observables
- The type of observables is highly related to the geometry of the environment.

A new Approach

- Use characteristics of a transformed version of the acoustic signal.
- Select properties of the transformed signal which uniquely characterize the acoustic signal

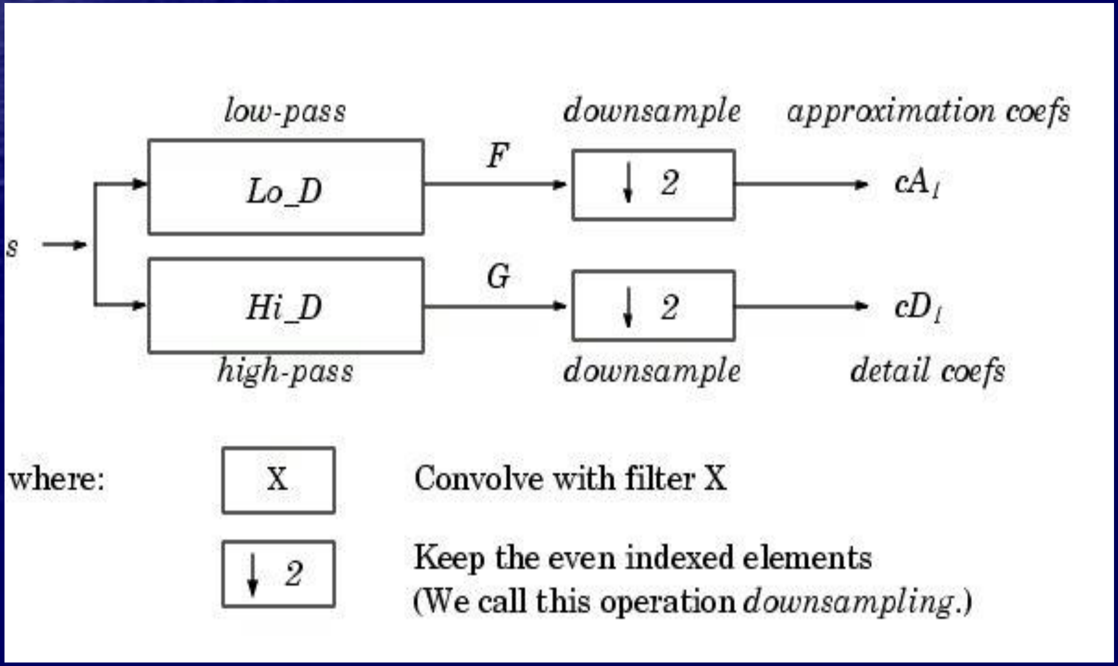
Statistical Signal Characterization

- The acoustic signal is "characterized" by a set of coefficients which describe statistical distributions of specific signal parameters



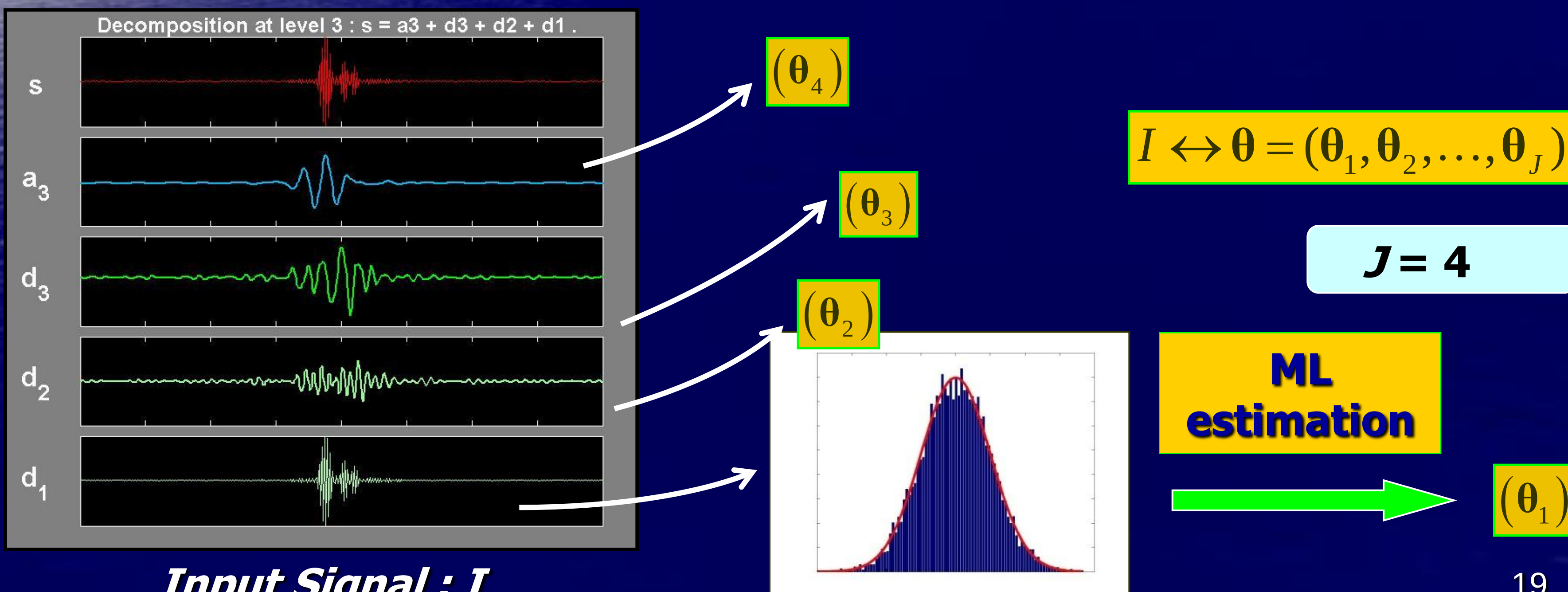
Statistical Signal Characterization

- Apply wavelet transform at various levels
- Extract the wavelet sub-band coefficients



Statistical Signal Characterization

- Fit the marginal distribution of wavelet sub-band coefficients with a member of a family of probability density functions (PDFs) $p(x;\, \boldsymbol{\theta}_i)$
- Feature Extraction := Estimate the model parameters $\boldsymbol{\theta}_i$



Input Signal : I

Statistical Signal Characterization

- The marginal distributions are modeled using *Symmetric Alpha-Stable* (SαS) distributions, which are best described by the Characteristic Function:

$$\phi(\omega) = e^{j\delta\omega - \gamma|\omega|^{\alpha}}$$

Here, $\delta=0$

Statistical Signal Characterization

Each signal is characterized by parameters *a* and *γ* of the detailed coefficients in each one of the *I* bands of decomposition plus the approximate coefficients of the I^{th} band.

Thus, the signal is characterized by *2x(I+1)* observables *:*

$$\left(a_1 , \gamma_1 , a_2 , \gamma_2 , \ldots . a_{\mathrm{I}} , \gamma_{\mathrm{I},} a_{\mathrm{I}+1} , \gamma_{\mathrm{I}+1,} \right)$$

Statistical Signal Characterization

- Similarity measurement : Comparison between two signals using statistical features.
- Use the Kullback-Leibler Divergence (KLD or Relative Entropy) between two PDFs

$$d_i = D(p(X;\boldsymbol{\theta}_\mathbf{q}) \,||\, p(X;\boldsymbol{\theta}_\mathbf{i})) = \int p(x;\boldsymbol{\theta}_\mathbf{q}) \log \frac{p(x;\boldsymbol{\theta}_\mathbf{q})}{p(x;\boldsymbol{\theta}_\mathbf{i})} dx$$

Statistical Signal Characterization

- Kullback-Leibler Distance between normalized Characteristic Functions

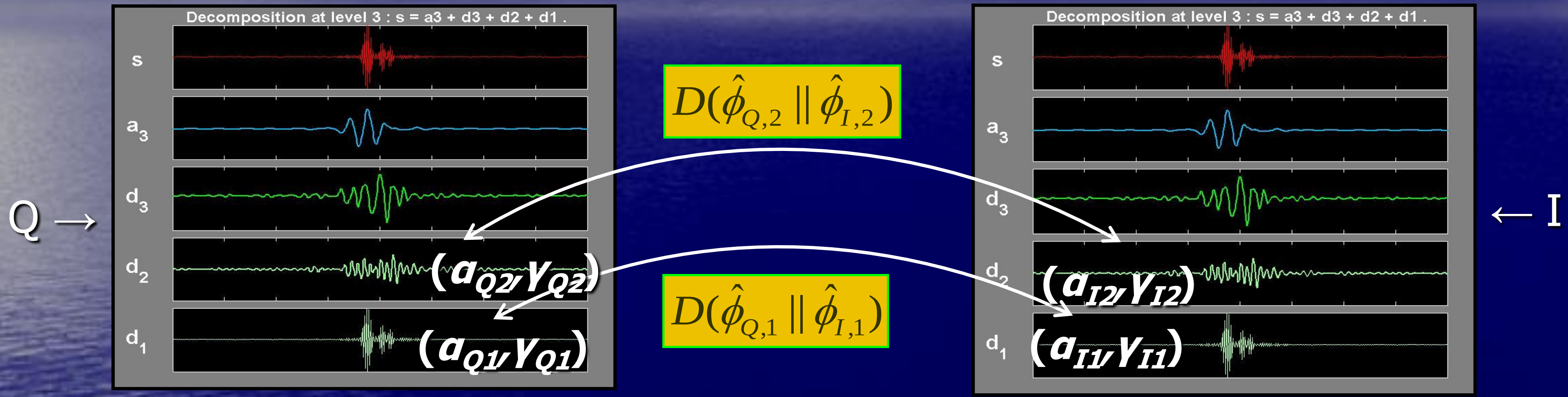
$$D(\hat{\phi}_q \,\|\, \hat{\phi}_i) = \ln\left(\frac{c_{\hat{\phi}_i}}{c_{\hat{\phi}_q}}\right) - \frac{1}{\alpha_{\hat{\phi}_q}} + \left(\frac{\gamma_{\hat{\phi}_i}}{\gamma_{\hat{\phi}_q}}\right)^{\alpha_{\hat{\phi}_i}} \cdot \frac{\Gamma\left((\alpha_{\hat{\phi}_i}+1)/\alpha_{\hat{\phi}_q}\right)}{\Gamma\left(1/\alpha_{\hat{\phi}_q}\right)}$$

- Each signal is represented by four sub-bands d_i denotes the detailed coefficients and α denote the approximate coefficients

$$(\mathbf{d}_1, \mathbf{d}_2, \mathbf{d}_3, \mathbf{a}_3) \qquad \mathbf{d}_i = (\gamma_i, \alpha_i)$$

- We assume that sub-band coefficients are mutually independent

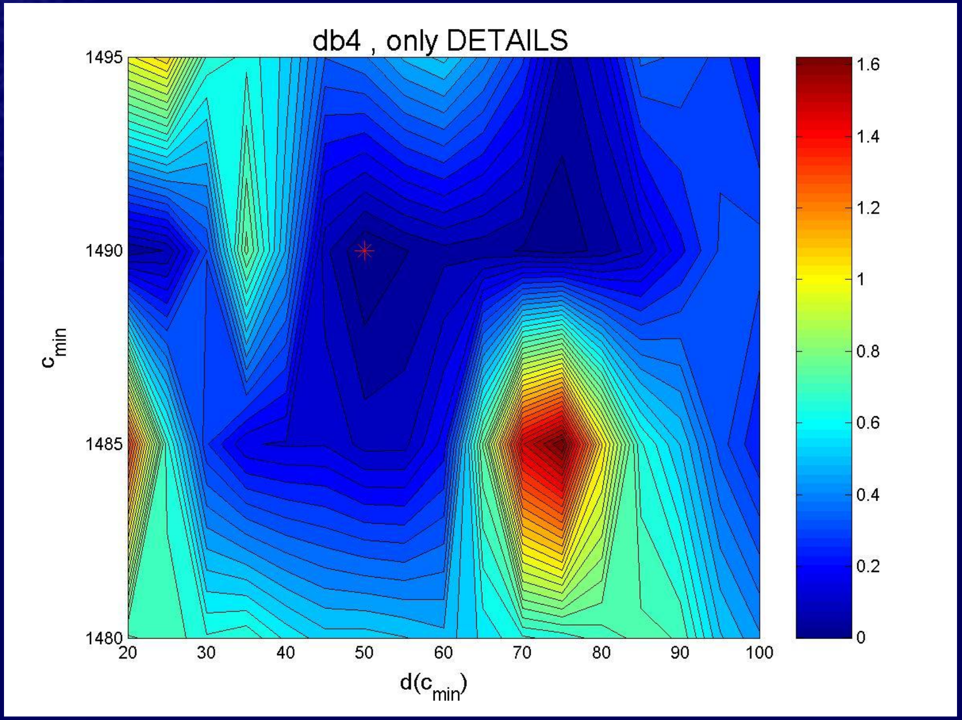
Statistical Signal Characterization



$$D(Q \,||\, I) = \sum_{k=1}^{J} D(\hat{\phi}_{Q,k} \,||\, \hat{\phi}_{I,k})$$

Taroudakis et al.
JASA **119**, 1396-1405 (2006)

It has been shown that the Kullback-Leibler Divergence **(KLD)** in connection with the statistical modeling, has the sensitivity properties which are necessary in order that this function is used for the characterization of an underwater signal.



Taroudakis et al.
ICTCA05 (2005)

Summary of non-linear methods in acoustical oceanography

Recoverable Parameters

$$m = \left(c_i , c_{si} , \rho_i , a_{pi} , a_{spi} d_i , z_0 , z , R \right)$$

Measurements at a single Hydrophone (S) or at an array of Hydrophones (A)

A-priori information : Number of layers, Search space, EOFs

Summary of non-linear methods in acoustical oceanography

Solution methods

	Data	Post processing analysis	Propagation Model	Processor
Matched-Field (A)	The acoustic field at ω	Fourier transform	All	Bartlett or equivalent
Matched-Travel time (S)	Travel time for each mode or ray	Identification of modal or ray arrivals	Normal-Mode Ray	Least square
Dispersion Analysis (S)	Dispersion curves	2-D Fourier transform or wavelet transform	Normal-Mode	Various
Statistical (S)	Statistical parameters of the wavelet transform	Wavelet transform – Multilayer analysis- Statistical Description	All	KLD

How to perform an efficient search (optimization)

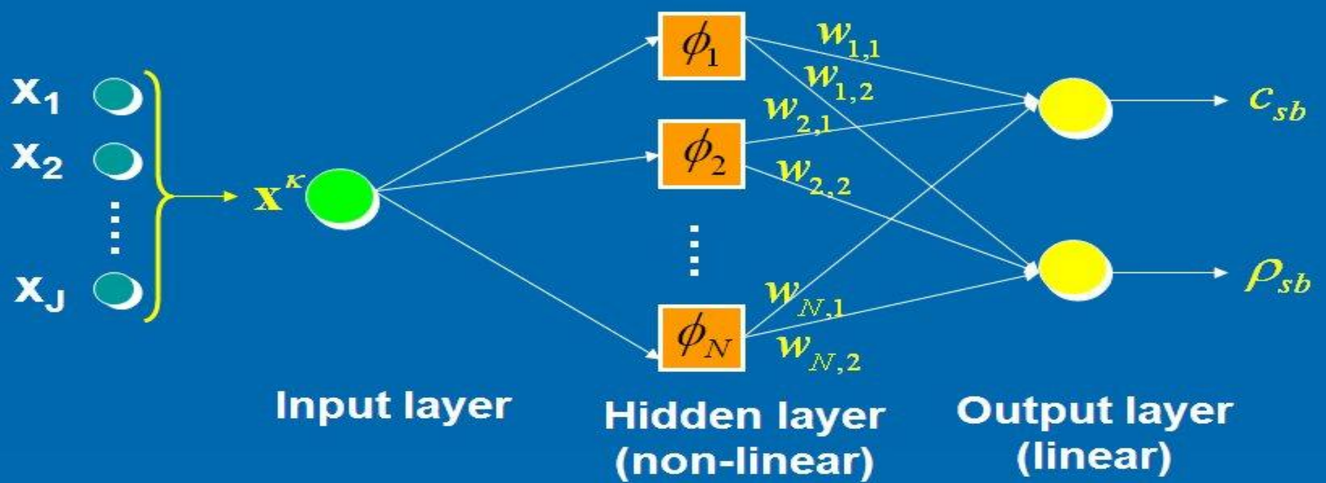
- Simulated Annealing
- Genetic Algorithms
- Methods developed for the individual problems
- Machine learning (Neural Networks)
- Pure statistical-probabilistic processing

Inversion Procedure

- A **R**adial **B**asis **F**unctions **N**eural **N**etwork (RBF NN)
- A Genetic Algorithm with a-posteriori statistical analysis of the population

Using RBF NN

RBF Architecture



Input layer: $\mathbf{x}^k = \{\mathbf{x}_i = (\alpha_i, \gamma_i)\}$ $i = 1:J$ = number of decomposition levels

Output layer: $c_{sb} = \sum_{j=1}^{N} w_{j,1}\phi_j(\mathbf{x})$ $\quad \rho_{sb} = \sum_{j=1}^{N} w_{j,2}\phi_j(\mathbf{x})$

(N = number of training samples)

Using RBF NN

- Hidden units: use Radial Basis functions

$$\phi_j\left(\left\|\mathbf{x}-\boldsymbol{\mu}^j\right\|\right),\ j=1,\ldots N$$

- Linear activation function:

$$\phi_j(\mathbf{x})=\exp\left(-\frac{\left\|\mathbf{x}-\boldsymbol{\mu}^j\right\|^2}{2\sigma_j^2}\right)$$

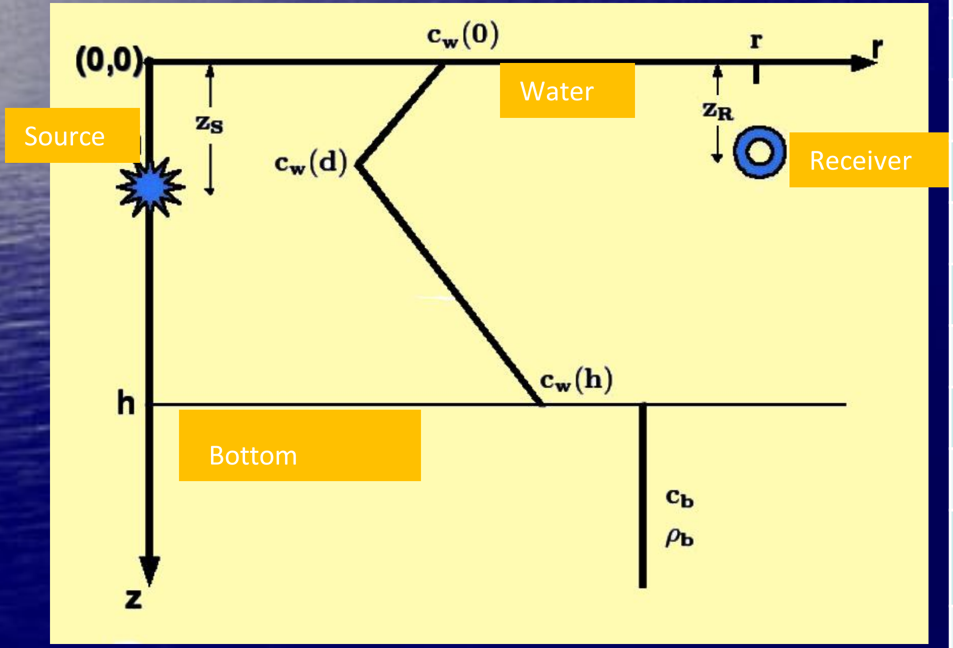
  - σ_j is a spread parameter
  - the output depends on the distance of the input $\mathbf{x}$ from the center $\boldsymbol{\mu}^j$

Using RBF NN

- Parameters that have to be learned for an RBF NN with the given architecture:
  - The centers μ^j of the RBF activation functions
  - The spreads σ_j (we use activation functions with the same spread parameter)
  - The weights $w_{i,j}$ from the hidden to the output layer
- Apply a learning algorithm for determining the above RBF network parameters.

Experimental Results using RBF NN

- The shallow-water environment



Description	Symbol	True Value	Search Space
Water Depth	h	200 m	
Source Depth	z_0	100 m	
Receiver Depth	z_R	100 m	
Range	R	5000 m	
Central Frequency	f_0	100 Hz	
Bandwidth	Δf	40 Hz	
Sound speed	$c_w(0)$	1500 m/sec	
	$c_w(d)$	1490 m/sec	
	$c_w(h)$	1515 m/sec	
Depth of the c_{min}	d	50 m	
Sound speed in bottom	c_b	1600 m/sec	[1550,1650]
Bottom density	ρ_b	1200 kg/m3	[1170,1240]

Experimental Results using RBF NN

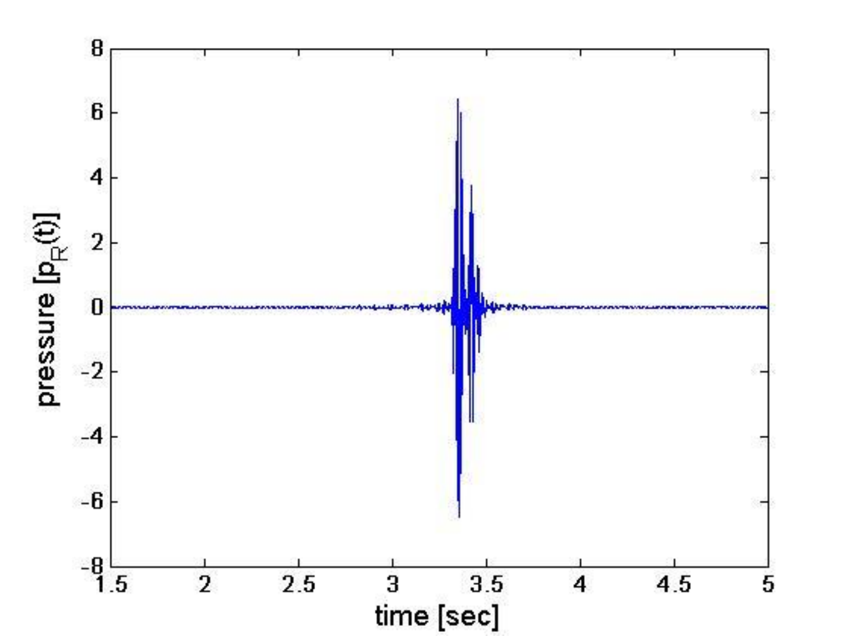
- Synthetic signals' database

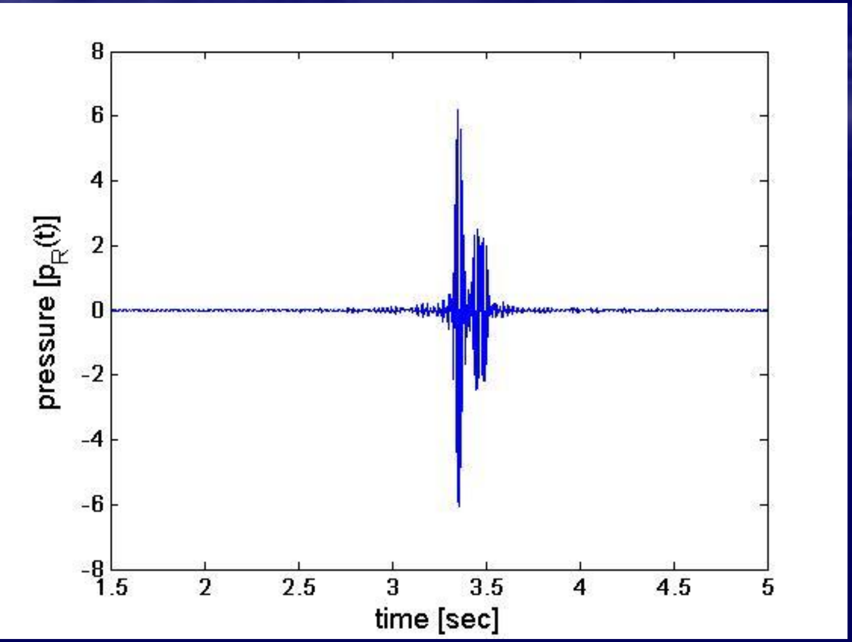
$$\{c_{sb} \in [1550:5:1650]\ \ m/\sec,\ \rho_{sb} \in [1170:1:1240]\ \ kg/m^3\}$$

$$\Rightarrow 1491\, signals\ \Rightarrow input\ \mathbf{x} = (\alpha_1, \gamma_1, \ldots, \alpha_4, \gamma_4)$$

- Decompose each signal with a 3-level DWT (with db4 wavelet)
- RBF NN construction using the estimated *SaS* parameters of a subset of M=200,300 training signals obtained from distinct environments

Experimental Results using RBF NN





True :

$(c_{sb}, \rho_{sb}) = (1570, 1185)$

Estimated :

$(\hat{c}_{sb}, \hat{\rho}_{sb}) = (1569.4, 1181.8)$

True :

$(c_{sb}, \rho_{sb}) = (1600, 1200)$

Estimated :

$(\hat{c}_{sb}, \hat{\rho}_{sb}) = (1600.7, 1201.8)$