

Χρησιμοποιώντας τη μέθοδο Fourier βρείτε τη λύση των ακόλουθων προβλημάτων:

Άσκηση 11.1.

$$\begin{aligned}u_{xx}(x, y) + u_{yy}(x, y) &= 0 \quad \forall (x, y) \in (0, 1) \times (0, 1), \\u(0, y) &= \sin(\pi y), \quad u(1, y) = 0 \quad \forall y \in [0, 1], \\u(x, 0) &= 0, \quad u(x, 1) = 0 \quad \forall x \in [0, 1].\end{aligned}$$

Άσκηση 11.2.

$$\begin{aligned}u_{xx}(x, y) + u_{yy}(x, y) &= 0 \quad \forall (x, y) \in D, \\u(2 \cos(\theta), 2 \sin(\theta)) &= \cos(\theta) \quad \forall \theta \in [0, 2\pi], \\u(\cos(\theta), \sin(\theta)) &= 1 \quad \forall \theta \in [0, 2\pi],\end{aligned}$$

όπου $D := \{(x, y) \in \mathbb{R}^2 : 1 < \sqrt{x^2 + y^2} < 2\}$.

Άσκηση 11.3.

$$\begin{aligned}u_t &= u_{xx} \quad \forall t > 0, x \in (0, \pi), \\u(t, 0) &= u_x(t, \pi) = 0 \quad \forall t > 0, \\u(0, x) &= (x - \pi)^2 - \pi^2 \quad \forall x \in [0, \pi].\end{aligned}$$

Άσκηση 11.4.

$$\begin{aligned}u_{tt} &= u_{xx} + \cos^2(x) \quad \forall t > 0, x \in (0, \pi), \\u_x(t, 0) &= 0, \quad u_x(t, \pi) = 0 \quad \forall t > 0, \\u(0, x) &= \frac{1}{2} \cos(2x), \quad u_t(0, x) = 0 \quad \forall x \in [0, \pi].\end{aligned}$$

Γ. Ζουράρης