WORKSHOP
"SDES/SPDES: THEORY, NUMERICS AND THEIR INTERPLAY WITH DATA SCIENCE"
JUNE 26-30, 2019
HERAKLION, CRETE

ABSTRACTS

Wednesday, June 26, 2019

On Nonlinear Filtering and Stochastic PDEs

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Abstract. An introduction to the theory of nonlinear filtering and the related stochastic PDEs will be presented. The filtering equations for partially observed diffusion processes will be obtained, and their solvability in Sobolev spaces will be investigated. The robustness of the nonlinear filters and numerical methods for their calculations will be discussed.

Thursday, June 27, 2019

Recursive procedures with dependent data

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Abstract. The convergence analysis of many well-known algorithms have been done only in the case where the data sequence is i.i.d. We present results about stochastic gradient-type algorithms with a stationary data sequence. We also point out how the theory of Markov chains in random environments can be useful in such a setting.

On the Innovation Conjecture of Nonlinear Filtering

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Abstract. The innovation process plays an important role in the theory of signal processing. A classical conjecture states that the information carried by the innovation process is the same as the information carried by the observation process. It is known that this is not true in general, but there are conditions ensuring that it holds. In the talk we will discuss the innovation problem for nonlinear filtering and using the filtering equations we show the the innovation conjecture holds under natural conditions on the coefficients of the SDEs describing the signal and observation model.

The talk is based on a joint work with Nicolai Krylov.

Friday, June 28, 2019

TBA

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Abstract. –

Using sequential quasi-Monte Carlo for inference in partly observed diffusion processes

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Abstract. SMC (Sequential Monte Carlo) is a class of Monte Carlo algorithms for filtering and related sequential problems. Gerber and Chopin (2015) introduced SQMC (Sequential quasi-Monte Carlo), a QMC version of SMC. In this talk, I will explain how to extend SQMC to the filtering of continuous-time state-space models, where the latent process is a diffusion. Thus, the latent process is infinite-dimensional, but I will explain how to implement SQMC in such a way that it provides good performance despite this. (Joint work with Mathieu Gerber.)

Variance reduction for MCMC methods via martingale representations

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Abstract. In this talk we discuss an efficient variance reduction approach for MCMC algorithms relying on a novel discrete time martingale representation for Markov chains. The proposed approach is fully non-asymptotic and does not require any type of ergodicity or special product structure of the underlying density. By rigorously analyzing the convergence of the proposed algorithm, we show that it's complexity is indeed significantly smaller than one of the original MCMC algorithm. The numerical performance of the new method is illustrated in the case of Gaussian mixtures and logistic regression models.

Variance reduction for dependent sequences via empirical variance minimization with applications

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Abstract. We propose a novel and practical variance reduction approach for additive functionals of Markov chains and more general dependent sequences. Our approach combines the use of control functions with the minimization of an empirical asymptotic variance estimate. We present empirical results carried out on a number of real world benchmarks showing that our variance reduction method achieves signicant improvement as compared to state-of-the-art methods at the expense of a moderate increase of computational overhead. This is joint work with Denis Belomestny, Leonid Iosipoi, Eric Moulines and Sergey Samsonov. The talk will be complemented by the poster of Sergey Samsonov.

Higher Order Langevin Monte Carlo Algorithm

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Abstract. We consider a problem of sampling from a high-dimensional target probability distribution π, under the assumption that its density on $\mathbb{R}^d$ is known up to a normalizing constant. Crucially, the Langevin SDE associated with the target distribution π is assumed to have a locally Lipschitz drift coefficient, such that its second derivative is locally Hölder continuous with exponent $\beta \in (0,1]$. Motivated by the results in [Brosse et al. (2017), arXiv: 1710.05559 [Stat.ME]] and [Sabanis and Zhang (2017), arXiv:1707.05086[math.PR]], a new (unadjusted) Langevin Monte Carlo (LMC) algorithm with improved rates in total variation and Wasserstein distance is proposed. It is shown that the new sampling method converges to the stationary measure with rate $1+\beta/2$ in Wasserstein distance, while the rate is 1 in total variation. In this talk, I will first explain the new algorithm and then discuss some key techniques used to obtain an improved rate of convergence.

Multilevel ensemble Kalman filtering algorithms

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Abstract. The ensemble Kalman filter (EnKF) is a sequential filtering method based on an ensemble of particle paths and sample moment (Monte Carlo) approximations of true moments required in the filter update step. EnKF is often both robust and efficient, but its performance may suffer in settings where the computational cost of accurate simulations of particles is high. The multilevel Monte Carlo method (MLMC) is an extension of the classical Monte Carlo method which by sampling stochastic realizations on a hierarchy of resolutions may reduce the computational cost of moment approximations by orders of magnitude. In this talk I will present recent results [1, 2] on the combining of MLMC and EnKF to construct the multilevel ensemble Kalman filter (MLEnKF) applicable to the following filtering settings:

(I) finite dimensional state space and discrete time observations,
(II) infinite dimensional state space and discrete time, finite dimensional observations.

Theoretical results and numerical evidence of the performance gain of MLEnKF over EnKF will be presented.

The presentation is based on joint works with Alexey Chernov (Uni Oldenburg) , Kody J. H. Law (Uni Manchester), Fabio Nobile (EPFL), Gaukhar Shaimerdenova (KAUST), and Raul Tempone (RWTH Aachen and KAUST).

References

[1] H. Hoel, K. Law, and R. Tempone. *Multilevel ensemble Kalman filtering.* SIAM J. Numer. Anal. 54(3), 18131839, 2016.
[2] A. Chernov, H. Hoel, K. Law, F. Nobile, and R. Tempone. *Multilevel ensemble Kalman filtering for spatio-temporal processes.* arXiv:1710.07282, 2017.

Couplings and convergence to equilibrium for Langevin dynamics and Hamiltonian Monte Carlo methods

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Abstract. Coupling methods provide a powerful approach to quantify convergence to equilibrium of Markov processes in appropriately chosen Wasserstein distances. This talk will give an overview on two closely relately, recently developed coupling approaches for kinetic Langevin equations and for Hamiltonian Monte Carlo methods (HMC). The approaches do not require convexity, and they yield bounds of kinetic order if the damping coefficient in the Langevin equation or the integration length in HMC are adjusted appropriately. The talk is based on joint work with Arnaud Guillin, Raphael Zimmer and Nawaf Bou-Rabee.

Saturday, June 29, 2019

On the convergence of the Crank Nicolson/FEM for a linear stochastic heat equation

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Abstract. We formulate an initial- and Dirichlet boundary- value problem for a linear stochastic heat equation, in one space dimension, forced by an additive space-time white noise. First, we approximate the mild solution to the problem by the solution of the regularized second-order linear stochastic parabolic problem with random forcing proposed by Allen, Novosel and Zhang (Stochastics Stochastics Rep., 64, 1998). Then, we construct numerical approximations of the solution to the regularized problem by combining the Crank-Nicolson method in time with a standard Galerkin finite element method in space. We derive strong a priori estimates of the modeling error made in approximating the mild solution to the problem by the solution to the regularized problem, and of the numerical approximation error of the Crank-Nicolson finite element method.

On the convergence of the Hamiltonian Monte Carlo algorithm and other irreversible MCMC methods

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Abstract. Hamiltonian Monte Carlo is a very popular MCMC method amongst Bayesian statisticians to get samples from a posterior distribution. This algorithm relies on the discretization of Hamiltonian dynamics which leave the target density invariant combined with a Metropolis step. In this talk, we will discuss convergence properties of this method to sample from a positive target density p on R^d with either a fixed or a random numbers of integration steps. More precisely, we will present some mild conditions on p to ensure φ-irreducibility and ergodicity of the associated chain. We will also present verifiable conditions

which imply geometric convergence. We will conclude with the introduction of new exact continuous time MCMC methods, and in particular the Bouncy Particle Sampler for which new theoretical results will be given.

A strong order $3/4$ method for SDEs with discontinuous drift coefficient

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Abstract. We study strong approximation of scalar SDEs at the final time in the case when the drift coefficient may have discontinuities in space. We present a method based on finitely many evaluations of the driving Brownian motion that achieves an L_p-error rate of at least 3/4 for all $p \in [1, +\infty)$ under piecewise smoothness assumptions on the coefficients. To obtain this result we show in particular that a quasi-Milstein scheme achieves an L_p-error rate of at least 3/4 in the case of coefficients that are both Lipschitz continuous and piecewise differentiable with Lipschitz continuous derivatives, which is of interest in itself. The latter error rates are obtained via a detailed analysis of the average size of increments of the time-continuous quasi-Milstein scheme over time-intervals in which the scheme crosses a point of non-differentiability of the coefficients.

The talk is based on joint work with Larisa Yaroslavtseva (University of Ulm).

Approximation of stochastic equations with irregular drift

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Abstract. Recently, the numerical approximation of stochastic equations with irregular drift has attracted the attention of many researchers. In a series of articles it was shown that if the drift f is α-Hölder then the strong rate of convergence of the Euler-Maruyama is $\frac{\alpha}{2}$. In this talk we will show that if one exploits the noise in an efficient way it can be shown that the rate of convergence is in fact much better than $\frac{\alpha}{2}$. Namely, we will see that the rate is $\frac{1}{2}$ provided that f is only Dini continuous. In dimension $d = 1$, the same conclusion holds for f merely measurable and bounded. Moreover, we will see that if f is α-Hölder continuous, then the rate of convergence is in fact $\frac{1+\alpha}{2}$. Our results also extend to equations driven by fractional noise. This talk is based on joint works with Máté Gerencsér and Oleg Butkovsky.

Boundary regularity of stochastic PDEs

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Abstract. A surprising observation by Krylov is that Dirichlet problems for SPDEs can admit very poor boundary behavior: for any $\alpha > 0$ there is a simple linear equation whose solution is not α-Hölder continuous. We discuss how to obtain a positive counterpart of this: i.e. we show that for any given equation one can find an $\alpha > 0$ such that the solution is α-Hölder continuous.

Pricing American options by exercise rate optimization and Markovian projections

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Abstract. This talk addresses the problem of pricing American basket options in a multivariate setting, which includes among others, the Bachelier and the Black-Scholes models.

In high dimensions, nonlinear partial differential equation methods for solving the problem become prohibitively costly due to the curse of dimensionality.

In the first part, we present a novel method [1] for the numerical pricing of American options based on Monte Carlo simulation and the optimization of exercise strategies. Previous solutions to this problem either explicitly or implicitly determine so-called optimal *exercise regions*, which consist of points in time and space at which a given option is exercised. In contrast, our method determines the *exercise rates* of randomized exercise strategies. We show that the supremum of the corresponding stochastic optimization problem provides the correct option price. By integrating analytically over the random exercise decision, we obtain an objective function that is differentiable with respect to perturbations of the exercise rate even for finitely many sample paths. The global optimum of this function can be approached gradually when starting from a constant exercise rate. Numerical experiments on vanilla put options in the multivariate Black–Scholes model and a preliminary theoretical analysis underline the efficiency of our method, both with respect to the number of time-discretization steps and the required number of degrees of freedom in the parametrization of the exercise rates. Finally, we demonstrate the flexibility of our method through numerical experiments on max call options in the classical Black–Scholes model, and vanilla put options in both the Heston model and the non-Markovian rough Bergomi model.

In the last part of the presentation, we proposed to use a stopping rule that depends on the dynamics of a low-dimensional Markovian projection of the given basket of assets [2]. It is shown that the ability to approximate the original value function by a lower-dimensional approximation is a feature of the dynamics of the system and is unaffected by the path-dependent nature of the American basket option. Assuming that we know the density of the forward process and using the Laplace approximation, we first efficiently evaluate the diffusion coefficient corresponding to the low-dimensional Markovian projection of the basket. Then, we approximate the optimal early-exercise boundary of the option by solving a Hamilton-Jacobi-Bellman partial differential equation in the projected, low-dimensional space. The resulting near-optimal early-exercise boundary is used to produce an exercise strategy for the high-dimensional option, thereby providing a lower bound for the price of the American basket option. A corresponding upper bound is also provided. These bounds allow assessing the accuracy of the proposed pricing method. Indeed, our approximate early-exercise strategy provides a straightforward lower bound for the American basket option price. Following a duality argument due to Rogers, we derive a corresponding upper bound solving only the low-dimensional optimal control problem. Numerically, we show the feasibility of the method using baskets with dimensions up to fifty. In these examples, the resulting option price relative errors are only of the order of a few percents.

References

[1] C. Bayer, R. Tempone and S. Wolfers, *Pricing American Options by Exercise Rate Optimization*, *arXiv:1809.07300* (2018).

[2] C. Bayer, J. Häppölä and R. Tempone, *Implied Stopping Rules for American Basket Options from Markovian Projection*, Quantitative Finance, **19**(3) (2019), 371–390.